Relative Entropy Sorting Method Based on the Preference Information of Alternative

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Abstract—Multiple attribute decision making (MADM) problems are to find a desirable solution from a finite number of feasible alternatives assessed on multiple attributes, both quantitative and qualitative. In the recent years, MADM has received a great deal of attention from researchers. This paper studies the multi-attribute decision-making problem of power communication resources investment, where both the attributes values of the alternatives and the subjective preference information of the alternatives are interval values. We provide a decision-making approach based on the distance of relative entropy. In addition, for the attributes weights are completely unknown or partially known, we calculate them by minimizing deviation of subjective and objective information. The deviation between subjective assessment and objective information is determined based on the order of the solutions is determined by the relative entropy distances from objective and subjective preferences of each solution. Finally, an example is illustrated to examine the effective of our method.

Keywords—multiple attributes decision-making; interval value; relative entropy; attributes weights

I. INTRODUCTION

Currently, the multi-attribute decision making theory and methods based on the explicit program evaluation have been more perfect [1,3]. However, due to the complexity and the decision-makers limited cognition of objective things, the decisions made by the objective information alone will lead it not accurate. So multiple attribute decision making problem with subjective evaluation of program has attracted people's attention gradually [4,7]. Reference[4] is studied on multi-attribute decision making problems which attribute weights are completely unknown and preference information is provided in the form of interval number ,and it proposes a decision-making method based on the objective programming model. Reference[6] is studied on multi-attribute decision making problems which attribute weights are known partially and preference information is provided in the form of complementary judgment matrix and reciprocal judgment matrix. Reference[7] is studied on multi-attribute decision making problems which attribute weights can not fully be ascertained, program property and preference information are provided in the form of triangular fuzzy number. Reference[8] is studied on multi-attribute decision making problems which program property and preference information is provided in the form of intuitionistic fuzzy numbers. Reference[9] is studied on multi-attribute decision making problems which program property and preference information of the program is provided, and it proposes a decision-making method based on the grey relational analysis;

Entropy as the best measure of uncertainty is widely used in various disciplines. In recent years, the principle of entropy optimization is applied successfully in the multi-attribute decision making problems [1], and it has achieved certain results. According to the relative entropy theory, reference[10] proposes a method to determine the weights of experts. According to the relative entropy theory, reference[11] proposes a combination weighting approach based on the multi-attribute decision making. According to the relative entropy which comes from the comparison between the evaluation solution, the ideal solution and negative ideal, reference[12] proposes a sort method of multi-attribute decision making. For multi-granularity uncertain linguistic and multi-attribute group decision making problems with incomplete policymakers attribute weight information, reference[13] proposes a possible-degree sort method based on the relative entropy. For the consistency problem of complementary judgment matrix, reference[14] proposes a relative entropy sorting method.
based multiplicative and additive. For uncertainty multi-
attribute decision making problems that decision matrix
elements is the fuzzy values of three parameters interval
values, reference[15] proposes a relative closeness sort
method based on the relative entropy. On the basis of grey
system, reference[16] proposes a relative entropy gathering
model based on the gray relational analysis. For uncertain
multiple attribute decision making problems that decision
matrix element is interval number, reference[17] proposes
a closeness sorting method based on relative entropy.

However, reference[17] does not consider the interval
multi-attribute decision making problems with uncertain
and subjective evaluation information about the program.
For this problem, this paper presents an interval multiple
attribute decision analysis method based on relative
entropy. And on the cases of that the weight information
is unknown completely or is known partially, the optimal
model is build to solved the problems respectively. The
method of this paper is without the comparison of interval
numbers and the calculations are more simple. Finally,
through the study of the case, the effectiveness of the
proposed method is illustrated.

II. PRELIMINARIES

Definition 1: Set \( R \) to real number, where
\( a = [a^L, a^U] \), \( a^L \leq a^U \) is known as the closed
interval. In particular, when \( a^L = a^U \), \( a \) is degraded to
the certain number.

Definition 2: According to the information theory ,
there are two systems A and B and the extent of the
difference between their state \( A_i \) and \( B_j \) (i=1, 2, ..., n)
is the available to measure by the Kullback-Leibler
distance [18].

\[
C_i = A_i \log \frac{A_i}{B_i} + (1 - A_i) \log \frac{1 - A_i}{1 - B_i}
\]

The degree of difference between the two systems A
and B is

\[
C = \sum_{i=1}^{n} \{A_i \log \frac{A_i}{B_i} + (1 - A_i) \log \frac{1 - A_i}{1 - B_i}\}
\]

The objective preference value about attribute \( G_i \) of
program makers is \( \alpha_j \) and the subjective preference value
is \( \alpha_i \). This paper definite the relative entropy distance
of \( \alpha_j \) and \( \alpha_i \) as follows:

\[
d(\alpha_j, \alpha_i) = \sum_{j=L,R} [a_{ij} \log \frac{a_{ij}}{\alpha_j} + (1 - a_{ij}) \log \frac{1 - a_{ij}}{1 - \alpha_j}]
\]

In the actual decision-making process, in order to make
the decision more accurate and reasonable, it should make
the differences between the subjective preference of the
program and objective preference for the various attributes
the smallest.

III. POWER COMMUNICATION RESOURCE INVESTMENT
DECISION PROBLEM CONSIDER THE SUBJECTIVE
INTERVAL EVALUATION

Power communication resource investment scale is big
and involved with complicated related factors, so it is
difficult to accurately estimate the cost to the investment
schemes and possible benefits. In reality, it generally
adopts the method of combining the expert group
evaluation and the financial accounting, so the scheme
evaluation value is usually a interval data. Here are norms
expressing such problems.

Considering m feasible investment program \( A_1 \),
\( A_2 \), ..., \( A_m \), valuation attributes \( G_1 \), \( G_2 \), ..., \( G_n \) . The
subjective evaluation information of experts to plan \( A_i \)
given in the form of interval numbers, as
\( \alpha_i = [\alpha_i^L, \alpha_i^U] \), i=1, 2, ..., m. The attribute value of plan
\( A_i \) under the valuation attribute \( G_j \) is interval
number \( [x_{ij}^L, x_{ij}^R] \) and the decision matrix is \( A \). Try to
synthesize the evaluations of subjective and objective to
determined the optimal investment program.

\[
A = \begin{pmatrix}
x_{11}^L & x_{11}^R & K & x_{m1}^L & x_{m1}^R \\
x_{12}^L & x_{12}^R & M & x_{m2}^L & x_{m2}^R \\
& & & & \\
& & & & \\
& & & & 
\end{pmatrix}
\]

IV. POWER COMMUNICATION RESOURCE INVESTMENT
DECISION METHOD BASED ON RELATIVE ENTROPY SORTING

For those power communication resource investment
problems considering subjective evaluation, this paper uses
the difference between the subjective information and
objective evaluation information of investment program,
which is based on a model constructed by relative entropy
distance, and presents constrained nonlinear programming
model, of which attribute weights are determined, goal is
minimizing the difference between subjective and objective
and the known part weight information is constraint. The decision-making process steps are as
follows:

Step1. Standardization process of interval decision
matrix should be conducted. Then the normalization
matrix is denoted \( Y = ([y_{ij}^L, y_{ij}^R])_{m \times n} \).

Step2. Build a linear programming model to solve the
optimal attribute weights. In the actual decision-making
process, due to the complexity of objectives and the
limitations of policy makers, clear attribute weight
information is difficult to be determined, there will be a
situation of attribute weight information is incomplete,
even completely unknown. For this type of decision-
making problems, it is necessary to give a reasonable
method to determine the weight. To this end, this paper
provides an evaluation difference minimization model based on relative entropy.

A. If the attribute weights partially known, there are six cases:\[19,20]:

\[
\begin{align*}
 w_i' &\geq w_j' ; \quad \alpha w_i - w_j' \geq \alpha' \wedge w_i' \geq \beta w_j' ; \\
\chi w_i' - w_j &\leq w_j' + \varepsilon ; \\
\delta w_i' - w_j &\leq (\theta _j + \epsilon _j)w_j or \theta _j \leq \frac{w_i}{w_j} ; w_j' \neq 0 ; \\
\epsilon w_i' - w_j &\geq w_k - w_j' ; \quad j \neq k \neq l .
\end{align*}
\]

Here, \( \alpha , \beta , \chi , \theta , \epsilon \) are non-negative constants. Thus it can establish the following single objective optimization model:

\[
\begin{align*}
\min D(w) &= \sum_{i=1}^{m} \sum_{j=1}^{n} d(y_{ij}, \alpha _i)w_j \\
\text{s.t. } w &\in W \sum_{j=1}^{n} w_j = 1 , w_j \geq 0 , j = 1, \ldots , n
\end{align*}
\]

(1)

\[
d(y_{ij}, \alpha _i) = \sum_{j=1}^{n} [y_{ij} \log \frac{y_{ij}}{\alpha _i} + (1-y_{ij}) \log \frac{1-y_{ij}}{1-\alpha _i}]
\]

presents the relative entropy distance of \( a_{ij} \) and \( \alpha _i \) as follows, where \( a_{ij} \) is the objective preference value about attribute \( G_j \) of program makers and \( \alpha _i \) is the subjective preference value. \( W \) represents some of the attributes of known weight information.

B. If the attribute weights is completely unknown, we can establish the following single objective optimization model:

\[
\begin{align*}
\min D(w) &= \sum_{i=1}^{m} \sum_{j=1}^{n} d(y_{ij}, \alpha _i)w_j \\
\text{s.t. } w &\in W \sum_{j=1}^{n} w_j = 1 , w_j \geq 0 , j = 1, \ldots , n
\end{align*}
\]

(2)

Solve the single objective optimization model, make it normalized to obtain optimal attribute weight vector \( w_j \):

\[
w_j = \frac{\sum_{i=1}^{n} d(y_{ij}, \alpha _i)}{\sum_{j=1}^{n} \sum_{i=1}^{n} d(y_{ij}, \alpha _i)} , \quad j = 1, 2, \ldots , n
\]

(3)

Step3. The relative entropy distances from objective and subjective preferences of each solution is calculated as follows:

\[
d_i = \sum_{j=1}^{n} d(y_{ij}, \alpha _i)w_j , i = 1, \ldots , m
\]

(4)

Where \( d_i \) sums up the overall relative entropy distances between objective and subjective preferences given by the decision makers of all characteristic from solution \( A_i \).

Solution \( A_i \) is ordered by the size of \( d_i \). If \( d_i \) is much bigger, it means that the objective preference given by decision makers is much closer to the subjective preference. Accordingly, the corresponding solution is much superior.

V. CASE STUDY

As for communication resource government, Electric Power Company of Henan needs to consider huge data and a variety of resources such as transmission network, switched network, service network and access network and so on. In order to advance the date processing and the efficiency, the company should contrast the five alternative power communication investment resource \( A_i (i=1, \ldots , 5) \) and pick out the best one. What we will integrate the varieties of resources into are safety efficacy \( G_1 \), economic efficacy \( G_2 \) and manage efficacy \( G_3 \), according to which we evaluate investment cases. For each scheme, investor’s subjective preference value is \( \alpha _1 = [0.30, 0.70] \), \( \alpha _2 = [0.20, 0.90] \), \( \alpha _3 = [0.10, 0.45] \), \( \alpha _4 = [0.25, 0.55] \), \( \alpha _5 = [0.20, 0.80] \). \( Y \) for the standardization of decision matrix, specific conditions as follows:

\[
Y = \begin{bmatrix}
0.214, 0.220 & 0.166, 0.178 & 0.184, 0.190 \\
0.206, 0.225 & 0.220, 0.229 & 0.182, 0.191 \\
0.195, 0.204 & 0.192, 0.198 & 0.220, 0.231 \\
0.181, 0.190 & 0.195, 0.205 & 0.185, 0.195 \\
0.175, 0.184 & 0.193, 0.201 & 0.201, 0.211
\end{bmatrix}
\]

Based on the method to determine the optimal scheme of Calculating steps are as follows:

A. If the attribute weight is partially known, assume that the attribute weight information is interval number, then \( W \) is:

\[
w_1 = [0.3350, 0.3755] \quad w_2 = [0.3009, 0.3138] \quad w_3 = [0.3194, 0.3363]
\]

By formula (1) to establish the following linear programming problem:

\[
\begin{align*}
\min D(w) &= 1.3570w_1 + 1.3728w_2 + 1.4043w_3 \\
\text{s.t. } w &\in W \sum_{j=1}^{n} w_j = 1 , w_j \geq 0 , j = 1, 2, 3
\end{align*}
\]

get the optimal weight vector:
With the formula (4) calculate the relative entropy distance both objective and subjective preferences.

\[ d_i = 0.2490, \quad d_2 = 0.5689, \quad d_3 = 0.0748, \quad d_4 = 0.1198, \quad d_5 = 0.3644. \]

According to the value of solution to rank, \( d_3 < d_4 < d_i < d_5 < d_2 \), among this five investment programmers, \( A_3 > A_4 > A_1 > A_5 > A_2 \), we can know the result \( A_3 \) is optimal.

B. If the attribute weight is completely unknown, you can use Formula (3) to gain the most optimal weight vector \( w = (0.3282, 0.3321, 0.3397) \), then use formula (4) to calculate the relative entropy distance between objective and subjective preferences.

\[ d_1 = 0.2512, \quad d_2 = 0.5698, \quad d_3 = 0.0748, \quad d_4 = 0.1193, \quad d_5 = 0.3631. \]

According to \( d_i \) to rank, \( d_3 < d_4 < d_1 < d_5 < d_2 \), therefore, among this five investment programmers, the result is \( A_3 > A_4 > A_1 > A_5 > A_2 \). So, scheme \( A_3 \) is optimal.

VI. CONCLUSIONS

This paper studies the multi-criteria decision-making problem of power communication resources investment, where both the attributes values of the alternatives and the subjective preference information of the alternatives are interval values. We provide a decision-making approach based on the distance of relative entropy and use single objective optimization model to determine the attribute weights, without comparing the size of the interval number. In this way, the calculation is simpler. In addition, with due consideration of objective information and policy makers subjective intention, we can make the decision-making process more reasonable.

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