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Homogeneous Systolic Pyramid Automata with *n*-Dimensional Layers

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Abstract

Cellular automata were investigated not only in the viewpoint of formal language theory, but also in the viewpoint of pattern recognition. Cellular automata can be classified into some types. A systolic pyramid automata is also one parallel model of various cellular automata. A homogeneous systolic pyramid automaton with *n*-dimensional layers (*n-HSPA*) is a pyramid stack of *n*-dimensional arrays of cells in which the bottom *n*-dimensional layer (level 0) has size an (a≥1), the next lowest (a-1)n, and so forth, the (a-1)st *n*-dimensional layer (level (a-1)) consisting of a single cell, called the root. Each cell means an identical finite-state machine. The input is accepted if and only if the root cell ever enters an accepting state. An *n-HSPA* is said to be a real-time *n-HSPA* if for every *n*-dimensional tape of size a^n ($a \geq 1$) it accepts the *n*-dimensional tape in time a-1. Moreover, a 1- way *n*-dimensional cellular automaton (1-*nCA*) can be considered as a natural extension of the 1-way two- dimensional cellular automaton to *n*-dimension. The initial configuration is accepted if the last special cell reaches a final state. A 1-*nCA* is said to be a real- time 1-*nCA* if when started with *n*-dimensional array of cells in nonquiescent state, the special cell reaches a final state. In this paper, we propose a homogeneous systolic automaton with *n*-dimensional layers (*n-HSPA*), and investigate some properties of real-time *n-HSPA*. Specifically, we first investigate a relationship between the accepting powers of real-time *n-HSPA*'s and real-time 1-*nCA*'s. We next show the recognizability of *n*-dimensional connected tapes by real-time *n-HSPA*'s.

Keywords: cellular automaton, diameter, finite automaton, *n*-dimension, parallelism, pattern recognition, real time.

1. Introduction and Preliminaries

The question of whether processing *n*-dimensional digital patterns is much more difficult than (*n*-1) dimensional ones is of great interest from the theoretical and practical standpoints. Thus, the study of n-dimensional automata as a computational model of n-dimensional pattern processing has been meaningful[4-23]. Cellular automata were investigated not only in the viewpoint of formal language theory, but also in the viewpoint of pattern recognition. Cellular automata can be classified into some types [2]. A systolic pyramid automaton is also one parallel model of various cellular automata. In this paper, we propose a homogeneous systolic automaton with *n*-dimensional layers (*n-HSPA*), and investigate some properties of real-time *n-HSPA*.

Let Σ be a finite set of symbols. An *n-dimensional* tape over Σ is an $(n-1)$-dimensional array of elements of Σ. The set of all n-dimensional tapes over Σ is denoted by $\Sigma^{(n)}$. Given a tape $x \in \Sigma^{(n)}$, for each $j(1 \leq j \leq n)$, we let $l_j(x)$ be the length of x along the jth axis. When $1 \leq i_j \leq l_j(x)$ for each $j(1 \leq j \leq n)$, let $x(i_1, i_2, \ldots, i_n)$ denote the symbol in x with coordinates $(i_1, i_2, \ldots, i_n)$. We concentrate on the input tape x with $l_1(x) = l_2(x) = l_3(x) = \cdots = l_n(x)$. A homogeneous systolic pyramid automaton with n-dimensional layers (*n-HSPA*) is a pyramidal stack of n-dimensional arrays of cells in which the bottom n-dimensional layer (level 0) has size a^n ($a \geq 1$), the next lowest $(a-1)^n$, and so forth, the $(a-1)$st n-dimensional layer (level $(a-1)$) consisting of a single cell, called the root . Each cell means an identical finite-state machine, $M = (Q, \Sigma, \delta, \#, F)$, where Q is a finite set of states, $\Sigma \subseteq Q$ is a finite set of input states, $\# \in Q - \Sigma$ is the *quiescent state*, $F \subseteq Q$ is the set of *accepting states*, and δ: $Q^{2^n+1} \rightarrow Q$ is the *state transition function*, mapping the current states of M and its 2^n son cells in a $2 \times 2 \times \cdots \times 2$ block on the n-dimensional layer below into M's next state. The input is accepted if and only if the root cell ever enters an accepting state. An *n-HSPA* is said to be a real-time *n-HSPA* if for every n-dimensional tape of size a^n ($a \geq 1$) it accepts the n-dimensional tape in time $a-1$. By $£^R$[*n-HSPA*] we denote the class of the sets of all the n-dimensional tapes accepted by a real-time *n-HSPA*[1]. A 1-*way n-dimensional cellular automaton* (1-*nCA*) can be considered as a natural extension of the 1-way two-dimensional cellular automaton to n dimensions [3]. The initial configuration of the cellular automaton is taken to be an $l_1(x) \times l_2(x) \times \cdots \times l_n(x)$ array of cells in the nonquiescent state. The initial configuration is accepted if the last special cell reaches a final state. A 1-*nCA* is said to be a real-time 1-*nCA* if when started with an $l_1(x) \times l_2(x) \times \cdots \times l_n(x)$ array of cells in the nonquiescent state, the special cell reaches a final state in time $l_1(x) + l_2(x) + \cdots + l_n(x) - 1$. By $£^R$[1-*nCA*] we denote the class of the sets of all the n-dimensional tapes accepted by a real-time 1-*nCA* [3].

2. Main Results

We mainly investigate a relationship between the accepting powers of real-time *n-HSPA*'s and real-time 1-*nCA*'s. The following theorem implies that real-time *n-HSPA*'s are less powerful than real-time 1-*nCA*'s.

Theorem 2.1. $£^R$[*n-HSPA*] $\subsetneq$ $£^R$[1-*nCA*].

Proof : Let $V = \{x\ x \in \{0,1\}^{(n)} \mid l_1(x) = l_2(x) = \cdots = l_n(x) \& [\forall i_1, \forall i_2, \ldots, \forall i_{n-1}\ (1 \leq i_1 \leq l_1(x), 1 \leq i_2 \leq l_2(x), \ldots, 1 \leq i_{n-1} \leq l_{n-1}(x))[x(i_1, i_2, \ldots, i_{n-1}, 1) = x(i_1, i_2, \ldots, i_{n-1}, l_n(x))]]\}$.

It is easily shown that $V_1 \in £^R$[1-*nCA*]. Below, we show that $V \notin £^R$[*n-HSPA*]. Suppose that there exists a real-time *n-HSPA*($n = 3$) accepting V . For each $t \geq 4$, let

$W(n) = \{ x \in \{0,1\}^{(3)} | l_1(x) = l_2(x) = \cdots = l_n(x) \& [x(1, 2, 1), (t, t-1, t)] \in \{0\}^{(3)}\}$.

Eight sons of the root cell $A_{(t-1,1,1,1)}$ of M $A_{(t-2,1,1,2)}$, $A_{(t-2,1,2,2)}$, $A_{(t-2,2,1,2)}$, $A_{(t-2,2,2,2)}$, $A_{(t-2,1,1,3)}$, $A_{(t-2,1,2,3)}$, $A_{(t-2,2,1,3)}$, $A_{(t-2,2,2,3)}$ are denoted by C_{UNW}, C_{USW}, C_{USE},C_{UNE}, C_{DNW}, C_{DSW}, C_{DSE}, C_{DNE}, respectively. For each x in $W(n)$, $x(UNW)$, $x(USW)$, $x(USE)$, $x(UNE)$, $x(DNW)$, x(USW), $x(USE)$, $x(UNE)$ are the states of *CUNW, CUSW, CUSE, CUNE, CDNW, CDSW, CDSE, CDNE*, at time t-2, respectively. Let $\sigma(x)$ = ($x(UNW)$, $x(USW)$, $x(DNW)$, $x(DSW)$), $\gamma(x)$ = ($x(USE)$, $x(UNE)$, $x(DSE)$, $x(DNE)$). and $\rho(x)$ = ($x(UNW)$, $x(USW)$, $x(DNW)$, $x(DSW)$, $x(USE)$, $x(UNE)$, $x(DSE)$, $x(DNE)$). Then, the following two propositions must hold:

Proposition 2.1. (i) *For any two tapes x, $y \in W(n)$ whose* 1*st*(1−3) *planes are same*, $\sigma(x) = \sigma(y)$. (ii) *For any two tapes x, $y \in W(n)$ whose n-th*(1−3) *planes are same*, $\gamma(x) = \gamma(y)$

[**Proof :** From the mechanism of each cell, it is easily seen that the states of C_{UNW}, C_{USW}, C_{DNW}, C_{DSW} are not influenced by the information of $x(1-3)_t$'s. From this fact, we have (i). The proof of (ii) is the same as that of (i). ☐]

Propositon 2.2. *For any two tapes x, $y \in W(t)$ whose* 1*st* (1-3) *planes are different*, $\sigma(x) \neq \sigma(y)$.

[**Proof :** Suppose to the contrary that $\sigma(x) = \sigma(y)$. We consider two tapes x', $y' \in W(t)$ satisfying the following :
(i) $x(1-3)_1$ and $x(1-3)_t$, are equal to $x(1-3)_1$ of x, respectively
(ii) $y'(1-3)_1$ is equal to $y(1-3)_1$, and $y'(1-3)_t$ is equal to $x(1-3)_1$.

As is easily seen, $x' \in V$ and so x' is accepted by M. On the other hand, from Proposition 2.1(ii), $\gamma(x') = \gamma(y')$. From Proposition 2.1(i), $\sigma(x) = \sigma(x')$, $\sigma(y) = \sigma(y')$. It follows that y' must be also accepted by M. This contradicts the fact that y' is not in V . ☐]

Proof of Theorem 2.1 (*continued*) : Let $p(t)$ be the number of tapes in $W(t)$ whose 1st (1-3) planes are different, and let $Q(t) = \{ \sigma(x) \mid x \in W(t)\}$, where k is the number of states of each cell of M. Then, $p(t) = 2^{t^2}$, and $Q(t) \leq k^4$. It follows that $p(n) > Q(t)$ for large t. Therefore, it follows that for large t, there must be two tapes x,y in $W(t)$ such that their 1st (1-3) planes are different and $\sigma(x) = \sigma(y)$. This contradicts Proposition 2.2, so we can conclude that $V \notin £^R[3\text{-}HSPA]$. In the case of n-dimension, we can show that $V \notin £^R[n\text{-}HSPA]$ by using the same technique. This completes the proof of Theorem 2.1. ☐

We next show the recognizability of n-dimensional connected tapes by real-time *n-HSPA*'s by using the name technique of Ref.[3]. Let x in $\{0,1\}^{(n)}$. A maximal subset P of $\mathbf{N}^n$ satisfying the following conditions is called a 1-*component* of x.

(i)For any $(i_1,i_2, \ldots ,i_n \in P$, we have $1 \leq i_1 \leq l_1(x)$, $1 \leq i_2 \leq l_2(x), \ldots, 1 \leq i_n \leq l_n(x)$, and $x(i_1,i_2, \ldots ,i_n) = 1$.

(ii) For any $(i_1,i_2, \ldots ,i_n)$, $(i_1', i_2', \ldots , i_n') \in P$, there exists a sequence $(i_{1,0},i_{2,0}, \ldots ,i_{n,0}),(i_{1,1},i_{2,1}, \ldots ,i_{n,1}), \ldots, (i_{1,n},i_{2,n}, \ldots ,i_{n,n})$ of elements in P such that $(i_{1,0},i_{2,0}, \ldots ,i_{n,0}) = (i_1,i_2, \ldots ,i_n)$, $(i_{1,n},i_2,n, \ldots ,i_{n,n}) = (i'_1,i'_2, \ldots ,i'_n)$, and $|i_{1,j} - i_{1,j-1}| + |i_{2,j} - i_{2,j-1}| + \ldots + |i_{n,j} - i_{n,j-1}| \leq 1 (1 \leq j \leq n)$. A tape $x \in \{0, 1\}^{(n)}$ is called *connected* if there exists exactly one 1-component of x.

Let T_c be the set of all the n-dimensional connected tapes. Then, we have

Theorem 2.2. $T_c \notin £^R[n\text{-}HSPA]$.

3. Conclusion

We investigated a relationship between the accepting powers of homogeneous systolic pyramid automaton with *n*-dimensional layers(*n-HSPA*) and one-way *n*-dimensional cellular automata (1-*nCA*) in real time, and showed that real-time *n-HSPA*'s are less powerful than real time 1-*nCA*'s.

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