Rock and Soil Damage-fracture Space Mechanics: Physical State Indexes

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Abstract

Each of the current analytical methods used in rock and soil CT mechanics is "observation", wasting the precious CT quantitative information. Utilizing the set theory and the measure theory, the concepts of the λ level damage-fracture ratio and the λ level damage-fracture rate, the λ1-λ2 level intercepted section ratio and rate, the λ level damage-fracture content are defined based on the CT number, simultaneously furthering to research the relation among the λ level damage-fracture rate, the CT number and the density, the relation among the bulk strain, the density damaged increment and the CT number, the relation among stress, strain and the density damaged increment.

Keywords: The λ level damage-fracture ratio, the λ level damage-fracture rate, The λ1-λ2 level intercepted section ratio, The λ1-λ2 level intercepted section rate, The measure, The perfect space, the damage-fracture space, The rock and soil damage-fracture space mechanics

1. Introduction

X-radial computerized topography analysis is called CT for short. Computerized tomography can inspect the inner change of the material and the structure without damage, simultaneously having the higher resolving power. The characteristics of utilizing the CT number to research the mechanics of the rock and soil material are that the experimental expenditure is large and the quantitative information on each elements can be presented. Obviously, the resource of the CT datum should be utilized very cherishingly because of its rarity and the abundant information contained. However, the current research on the evolution process of the damage and meso-crack is observed and analyzed only from the view of the size, mean and variance of the CT number, the gray variety of the CT image[1]-[4], the statistic frequency of the CT number[5], the damage variable[5], the damage evolution equation[6], the damage evolution rate[7], the damage contour line[8], the threshold value of the CT number[9], the structural tensor of the soil mass[9] and the density damaged increment[10] etc., a systematic and characteristic researching method has not formed so far. The characteristics of these methods are that the quantitative information of CT resolving power elements averaged the mean information, applying the rare quantitative information to the qualitative description, all these result in wasting the resources of the CT image. On the other hand, the direct observation of the CT image generates the error of the visual angle. The same kind of the research begins early [11]-[12], but the research methods are the same to the above, without big breakthrough [13]-[14]. So the CT research of the current rock and soil mechanics actually locates at "the observation stage of the CT image".

Paper [15] defined the concepts of the perfect degree and the damage-fracture degree of a certain point in the rock and soil sample, the λ level perfect field and the damage-fracture field, the λ_1-λ_2 level intercepted sector etc., simultaneously defined the measures of the λ level perfect field and the damage-fracture field, the λ_1-λ_2 level intercepted sector, also established the concepts of perfect space and damage-fracture space of the rock and soil sample, and studied some of their basic qualities.

This article which utilizes the knowledge of the set theory and the measure theory defines the conceptions such as the λ level damage-fracture ratio and the damage-fracture rate, the λ_1-λ_2 level intercepted sector ratio and rate, the λ level damage content about rock based on the CT number, furthers to research the relation among the λ level damage-fracture rate, the CT number and the density, the relation among the bulk strain, the density damaged increment and the CT number, the relation among the strain, stress and the density damaged increment.

Through utilizing modern mathematics quantitatively analyze the property of the rock and soil CT mechanics, these two papers synchronously establish a stable foundation for the creation of "the rock and soil damage-fracture space mechanics". Therefore, these series of the articles ideally realize the gradual transition and the uniform description of the meso-mechanics and the macro-mechanics, achieve the uniform description of the continuum mechanics and porous media or fissured media mechanics.

The article suites not only for the rock mass material but also for the soil mass material, but the following examples regard rock material as the object described, simultaneously confines the following discussion to the damage-fracture space, the concepts are not illustrated can refer to the referenced article [15].

The new conceptions not explained the derivation in this paper all are independently originated by the author, all belong to the original job.

2. The measurements of the damage-fracture field and the intercepted sector

The size of a set can be estimate by measure, the problems about what is the measure and what is the measurable space can be consulted through the mathematic measure theory[16] and paper[15].

The definition of λ level damage-fracture field is

$$d_\lambda = \{(x,y,z) | \ (x,y,z) \in \Omega \ and \ d(x,y,x) \geq \lambda\}$$

Its measure is used to estimate the size usually, namely

m （d_λ）

The measure of the damage-fracture field enhances with the increasing of the stress.

Ditto, the size of the intercepted sector

$$d_{\lambda_1-\lambda_2} = \{(x,y,z) | \lambda_1 \leq d(x,y,z) \leq \lambda_2, 0 \leq \lambda_1 \leq \lambda_2 \leq 1\}$$

can also be estimated by the measure, namely

m （$d_{\lambda_1-\lambda_2}$）

3. The λ level damage-fracture ratio and rate of the rock

The concept of the λ level damage-fracture ratio of the rock is defined by utilizing the mathematic measure theory.

On the damage-fracture space $(\Omega, d_{\lambda_1-\lambda_2})$ of the rock, the λ level measure damage-fracture ratio is defined as the following,

$$D_e^\lambda = \frac{\mathrm{m}(d_\lambda)}{\mathrm{m}(\overline{d}_\lambda)}$$

according to the λ level damage-fracture field. Its shortened form is called the λ level damage-fracture ratio. Where, $m(*)$ represents the measure of the set $(*)$, $\mathrm{m}(d_\lambda)$ can be simply understood to be the product of the number and the length, area or volume of the CT resolving power elements of the rock sample's λ level damage-fracture field. Then the rock sample's λ level damage-fracture ratio can be simply understood to be that the area or volume of the damage-fracture field divides by the non-damage-fracture field's area or volume. The another formula of the λ level damage-fracture ratio can also be concisely presented as following,

$$D_e^\lambda = \frac{\mathrm{m}(d_\lambda)}{\mathrm{m}(\text{strong } p_{1-\lambda})}.$$

Where, $\mathrm{m}(\text{strong } p_{1-\lambda})$ equates the strong $(1-\lambda)$ level perfect field in the perfect space $(\Omega, p_{\lambda_1-\lambda_2})$.

Simultaneously, on the damage-fracture space $(\Omega, d_{\lambda_1-\lambda_2})$ of the rock, the λ level measure damage-fracture rate of the rock is defined as the following,

$$D_n^\lambda = \frac{\mathrm{m}(d_\lambda)}{\mathrm{m}(\Omega)}$$

according to the λ level damage-fracture field. Its shortened form is called the λ level damage-fracture rate. Then the rock sample's λ level damage-fracture rate can be simply understood to be that the area or volume of the damage-fracture field divides by the complete region's area or volume.

For example, as to the profile of the rock sample showed by figure 1 in the referenced paper[15], the damage-fracture ratios of its 0.5 level damage-fracture field and 0.7 level damage-fracture field (containing air) can be respectively presented as following,

$$D_e^{0.5} = \frac{\mathrm{m}(d_{0.5})}{\mathrm{m}(\overline{d}_{0.5})} = \frac{20659 \times s}{52661 \times s} = 0.392$$

$$D_e^{0.7} = \frac{\mathrm{m}(d_{0.7})}{\mathrm{m}(\overline{d}_{0.7})} = \frac{16949 \times s}{56371 \times s} = 0.301$$

Where, s is the area of the CT resolving power elements. Here is the same to the referenced paper[15], the sampling is continuously done from row 105 to row 480, sampling at every column intervals points from column 42 to column 430, the same to the following context. The damage-fracture rate of its 0.5 level damage-fracture field and 0.7 level damage-fracture field (including air) can be respectively presented as following,

$$D_n^{0.5}=\frac{\mathrm{m}(d_{0.5})}{\mathrm{m}(\Omega)}=\frac{20659\times s}{73320\times s}=0.282$$

$$D_n^{0.7}=\frac{\mathrm{m}(d_{0.7})}{\mathrm{m}(\Omega)}=\frac{16949\times s}{73320\times s}=0.232$$

The λ level damage-fracture ratio and rate bear certain resemblances to the concepts of the porosity ratio and porosity in the soil mechanics, possessing very clear physical meaning. The relation between the λ level damage-fracture ratio and the λ level damage-fracture rate equates the relation between the porosity ratio and porosity, as follows.

$$D_n^{\lambda}=\frac{D_e^{\lambda}}{1+D_e^{\lambda}}$$

$$D_e^{\lambda}=\frac{D_n^{\lambda}}{1-D_n^{\lambda}}$$

With the outside loads increased, the damage-fracture field expands ceaselessly, therefore D_e^{λ} and D_n^{λ} are enhanced with the increasing of the outside loads.

Simultaneously, the crack scale falling into the scope reviewed decreases gradually with the decreasing of the λ value, the zone merged into the damage-fracture field is also increasing gradually, D_e^{λ} and D_n^{λ} increases with the λ value reduced. Namely, $D_e^{\lambda_2}\geq D_e^{\lambda_1}$ and $D_n^{\lambda_2}\geq D_n^{\lambda_1}$ come into existence when $\lambda_2<\lambda_1$.

If the volume of the λ level damage-fracture field and the λ level non-damage-fracture field can be worked out, presented by $\mathrm{V}(d_{\lambda})$ and $\mathrm{V}(\overline{d_{\lambda}})$ respectively (notes, the volume of d_{λ} and $\overline{d_{\lambda}}$ sometimes don't exist). Then the concepts of the λ level damage-fracture ratio and the λ level damage-fracture rate can be defined by the volume ratio and rate as the following,

$$D_e^{\lambda}=\frac{\mathrm{V}(d_{\lambda})}{\mathrm{V}(\overline{d_{\lambda}})}=\frac{\mathrm{V}(d_{\lambda})}{\mathrm{V}(\text{strong } p_{1\text{-}\lambda})}$$

$$D_n^{\lambda}=\frac{\mathrm{V}(d_{\lambda})}{\mathrm{V}(\Omega)}$$

They are the volume damage-fracture ratio and rate in the rock and soil mechanics respectively.

Obviously, the concepts of the volume damage-fracture ratio and rate in the classical rock and soil mechanics are special examples of the concepts of the measure damage-fracture ratio and rate defined in this article.

4. The λ1-λ2 intercepted section ratio and rate of the rock

Ditto, the λ_1-λ_2 intercepted section ratio and rate can be defined.

On the damage-fracture space $(\Omega, d_{\lambda_1-\lambda_2})$ of the rock sample, the λ_1-λ_2 intercepted section ratio is defined as the following according to the λ_1-λ_2 intercepted section $d_{\lambda_1-\lambda_2}$,

$$D_e^{\lambda_1-\lambda_2}=\frac{\mathrm{m}(d_{\lambda_1-\lambda_2})}{\mathrm{m}(\overline{d_{\lambda_1-\lambda_2}})}$$

The λ_1-λ_2 intercepted section rate of the rock sample is defined as following,

$$D_n^{\lambda_1-\lambda_2}=\frac{\mathrm{m}(d_{\lambda_1-\lambda_2})}{\mathrm{m}(\Omega)}$$

For example, as to the profile of the rock sample showed by figure 1 in the paper[15], the intercepted section ratios of the 0.5-0.9 level and 0.9-1.0 level intercepted sections can be respectively presented as following,

$$D_e^{0.5-0.9}=\frac{\mathrm{m}(d_{0.5-0.9})}{\mathrm{m}(\overline{d_{0.5-0.9}})}=\frac{6214\times s}{67106\times s}=0.093$$

$$D_e^{0.9-1.0}=\frac{\mathrm{m}(d_{0.9-1.0})}{\mathrm{m}(\overline{d_{0.9-1.0}})}=\frac{14445\times s}{58875\times s}=0.245$$

The intercepted section rates of the intercepted section of its 0.5-0.9 level and 0.9-1.0 level can be respectively presented as following,

$$D_n^{0.5-0.9}=\frac{\mathrm{m}(d_{0.5-0.9})}{\mathrm{m}(\Omega)}=\frac{6214\times s}{73320\times s}=0.085$$

$$D_n^{0.9-1.0}=\frac{\mathrm{m}(d_{0.9-1.0})}{\mathrm{m}(\Omega)}=\frac{14445\times s}{73320\times s}=0.197$$

5. The specific gravity of the λ level non-damage-fracture field

On the damage-fracture space $(\Omega, d_{\lambda_1-\lambda_2})$ of the rock sample, the specific gravity of the λ level non-damage-fracture field is defined as the following according to the λ level damage-fracture field,

$$G_{\lambda}=\frac{\rho_{\bar{d}}\ \mathrm{m}(\overline{d_{\lambda}})}{\rho_w\ \mathrm{m}(\overline{d_{\lambda}})}=\frac{\rho_{\bar{d}}}{\rho_w} \text{ or } G_{\lambda}=\frac{\gamma_{\bar{d}}\ \mathrm{m}(\overline{d_{\lambda}})}{\gamma_w\ \mathrm{m}(\overline{d_{\lambda}})}=\frac{\gamma_{\bar{d}}}{\gamma_w}$$

The rock's specific gravity of the λ level non-damage-fracture field can be simply understood to be that the mass or weight of the rock's λ level non-damage-fracture field divide by the mass or weight of water with the same volume (the water in the 4 centigrade), namely the density or bulk-weight of the rock's λ level non-damage-fracture field divide by the density or bulk-weight of water with the same volume (the water in the 4 centigrade).

6. The λ level damage-fracture content of the rock

The rock sample is assumed to be saturated, on the damage-fracture space $(\Omega, d_{\lambda1-\lambda2})$ of the rock sample, the λ level damage-fracture content is defined as the following according to the λ level damage-fracture field, which can be represented by w_λ,

$$w_\lambda = \frac{\rho_d \ \mathrm{m}(d_\lambda)}{\rho_{\bar{d}} \ \mathrm{m}(\overline{d_\lambda})} = \frac{\gamma_d \ \mathrm{m}(d_\lambda)}{\gamma_{\bar{d}} \ \mathrm{m}(\overline{d_\lambda})}$$

Where, the natural density and natural bulk-weight of the non-damage-fracture part and the damage-fracture part are presented by $\rho_{\bar{d}}$ and ρ_d, $\gamma_{\bar{d}}$ and γ_d respectively.

The λ level damage-fracture content of the saturated rock sample can be simply understood to be that the mass or weight of the damage-fracture field divides by the mass or weight of the non-damage-fracture field, usually showed by the percent or the decimal fraction.

As to a certain proper level λ_0, the λ_0 level damage-fracture field is probable to appropriately correspond to the size of the crack containing water, if the water content of rock or soil parts without the crack equates zero, namely the crystal water inside the rock or soil is not considered, the λ_0 level damage-fracture content equals to its water content.

The following formulas obviously come into existence,

$$w_\lambda = \frac{\rho_\Omega \ \mathrm{m}(\Omega) - \rho_{\bar{d}} \ \mathrm{m}(\overline{d_\lambda})}{\rho_{\bar{d}} \ \mathrm{m}(\overline{d_\lambda})} = \frac{\rho_\Omega \ \mathrm{m}(\Omega)}{\rho_{\bar{d}} \ \mathrm{m}(\overline{d_\lambda})} - 1 = \frac{\rho_d \ \mathrm{m}(d_\lambda)}{\rho_\Omega \ \mathrm{m}(\Omega) - \rho_d \ \mathrm{m}(d_\lambda)}$$

$$= \frac{\gamma_\Omega \ \mathrm{m}(\Omega) - \gamma_{\bar{d}} \ \mathrm{m}(\overline{d_\lambda})}{\gamma_{\bar{d}} \ \mathrm{m}(\overline{d_\lambda})} = \frac{\gamma_\Omega \ \mathrm{m}(\Omega)}{\gamma_{\bar{d}} \ \mathrm{m}(\overline{d_\lambda})} - 1 = \frac{\gamma_d \ \mathrm{m}(d_\lambda)}{\gamma_\Omega \ \mathrm{m}(\Omega) - \gamma_d \ \mathrm{m}(d_\lambda)}$$

Where, the average density and bulk-weight of the complete field are presented by ρ_Ω and γ_Ω respectively.

Having these concepts, the uniform description of the crannied media mechanics and loose media mechanics, namely the rock mechanics and the soil mechanics are established on the basis of the CT number mechanics.

On the damage-fracture space $(\Omega, d_{\lambda1-\lambda2})$ of the rock sample, because of $D_e^\lambda = \frac{\mathrm{m}(d_\lambda)}{\mathrm{m}(\overline{d_\lambda})}$, $D_e^\lambda \ \mathrm{m}(\overline{d_\lambda}) = \mathrm{m}(d_\lambda)$, simultaneously $w_\lambda = \frac{\rho_d \ \mathrm{m}(d_\lambda)}{\rho_{\bar{d}} \ \mathrm{m}(\overline{d_\lambda})}$, so $\mathrm{m}(d_\lambda) = w_\lambda \frac{\rho_{\bar{d}} \ \mathrm{m}(\overline{d_\lambda})}{\rho_d}$. Therefore, $D_e^\lambda \ \mathrm{m}(\overline{d_\lambda}) = w_\lambda \frac{\rho_{\bar{d}} \ \mathrm{m}(\overline{d_\lambda})}{\rho_d}$ can be concluded, which can be further predigested to obtain the relation between the λ level damage-fracture ratio and the λ level damage-fracture content as following,

$$D_e^\lambda = w_\lambda \frac{\rho_{\bar{d}}}{\rho_d}$$

7. The relation among the λ level damage-fracture rate, the CT number and the density

According to the Takashi's research results in 1983[12], the value of an arbitrary pixel in the CT image can be presented by the CT number H, this CT number derives of the linear combination of the absorption coefficient that the X radial is absorbed by the media, namely,

$$H = a\mu + b$$

In the formula, a and b are constant, μ is the linear absorption coefficient of the X radial.

On the damage-fracture space $(\Omega, d_{\lambda1-\lambda2})$ of the rock sample, it can be seen from the definition of the λ level damage-fracture rate,

$$\mu = (1 - D_n^\lambda)\mu_{\bar{d}} + D_n^\lambda \mu_d \tag{1}$$

In the formula, μ, $\mu_{\bar{d}}$ and μ_d are the X radial linear absorption coefficients of the complete field, the λ level non-damage-fracture field and the λ level damage-fracture field respectively.

The absorption coefficient of the unit mass rock sample assumed as μ^m, generally speaking, μ^m is only related with the wavelength of the incidence X radial. The relation between the X radial linear absorption coefficient of the object and the absorption coefficient of the unit mass is as the following,

$$\mu = \rho\mu^m \tag{2}$$

The μ in the formula (2) is replaced by the one in formula (1) acquires the following one,

$$\rho\,\mu^m = (1 - D_n^\lambda)\,\rho_{\bar{d}}\,\mu_{\bar{d}}^m + D_n^\lambda\,\rho_d\,\mu_d^m$$

In this formula, $\mu_{\bar{d}}^m$ and μ_d^m present the X radial linear absorption coefficients of the unit mass of the λ level non-damage-fracture field and the λ level damage-fracture field respectively. The average density of the λ level non-damage-fracture field and the λ level damage-fracture field can be showed by $\rho_{\bar{d}}$ and ρ_d respectively. It can draw the following conclusion from the last two formulas simplified,

$$\mu = \mu_{\bar{d}} + D_n^{\lambda}(\mu_d - \mu_{\bar{d}})$$

$$\rho\,\mu^m = \rho_{\bar{d}}\,\mu_{\bar{d}}^m + D_n^{\lambda}(\rho_d\,\mu_d^m - \rho_{\bar{d}}\,\mu_{\bar{d}}^m)$$

According to these two formulas, the theory relation between the λ level damage-fracture field and the CT number or density as following,

$$D_n^{\lambda} = \frac{\mu - \mu_{\bar{d}}}{\mu_d - \mu_{\bar{d}}} = \frac{H - H_{\bar{d}}}{H_d - H_{\bar{d}}}$$

$$D_n^{\lambda} = \frac{\rho\mu^m - \rho_{\bar{d}}\,\mu_{\bar{d}}^m}{\rho_d\,\mu_d^m - \rho_{\bar{d}}\,\mu_{\bar{d}}^m}$$

In these formulas, $H_{\bar{d}}$ and H_d present the average CT numbers of the λ level non-damage-fracture field and the λ level damage-fracture field respectively. This formula is the equative relation between the λ level damage-fracture field and the average CT number of each field.

The discussion below this paragraph only exists when the damage-fracture field just equals to the macroscopic crack, moreover the inside of the crack is completely filled with air.

As to a certain level λ_0, the damage-fracture field just equals to the macroscopic crack, moreover the inside of the crack is completely filled with air, the density and the CT number of air can be expressed by ρ_d and H_d respectively, here $\rho_d << \rho_{\bar{d}}$, $H_d = -1000$, so the last two formulas are simplified as the following,

$$D_n^{\lambda} = \frac{H_{\bar{d}} - H}{1000 + H_{\bar{d}}} \qquad D_n^{\lambda} = \frac{\rho_{\bar{d}}\,\mu_{\bar{d}}^m - \rho\mu^m}{\rho_{\bar{d}}\,\mu_{\bar{d}}^m}$$

The total density of the rock sample is as the following,

$$\rho = (1 - D_n^{\lambda})\,\rho_{\bar{d}} + D_n^{\lambda}\,\rho_d$$

Considering $\rho_d << \rho_{\bar{d}}$, so $\rho = (1 - D_n^{\lambda})\,\rho_{\bar{d}}$, if the expression of the λ level damage-fracture rate is introduced into the last formula, it can be obtained as the following,

$$\rho = (1 - \frac{H_{\bar{d}} - H}{1000 + H_{\bar{d}}})\,\rho_{\bar{d}} = \frac{1000 + H}{1000 + H_{\bar{d}}}\,\rho_{\bar{d}}$$

$$\rho = (1 - \frac{\rho_{\bar{d}}\,\mu_{\bar{d}}^m - \rho\mu^m}{\rho_{\bar{d}}\,\mu_{\bar{d}}^m})\,\rho_{\bar{d}} = \rho\,\frac{\mu^m}{\mu_{\bar{d}}^m}$$

So, $\mu^m = \mu_{\bar{d}}^m \qquad D_n^{\lambda} = \dfrac{\rho_{\bar{d}} - \rho}{\rho_{\bar{d}}}$

By the way, if $0 = \lambda_1 \le \lambda_2 \le \cdots \le \lambda_n = 1$, the following two formulas exist,

$$\mu = D_n^{\lambda_1-\lambda_2}\,\mu_{\lambda_1-\lambda_2} + D_n^{\lambda_2-\lambda_3}\,\mu_{\lambda_2-\lambda_3} + \cdots + D_n^{\lambda_{n-1}-\lambda_n}\,\mu_{\lambda_{n-1}-\lambda_n}$$

$$\rho = D_n^{\lambda_1-\lambda_2}\,\rho_{\lambda_1-\lambda_2} + D_n^{\lambda_2-\lambda_3}\,\rho_{\lambda_2-\lambda_3} + \cdots + D_n^{\lambda_{n-1}-\lambda_n}\,\rho_{\lambda_{n-1}-\lambda_n}$$

Where, $D_n^{\lambda_i-\lambda_{i+1}}$, $\mu_{\lambda_i-\lambda_{i+1}}$ and $\rho_{\lambda_i-\lambda_{i+1}}$ present the intercepted section rate, the X radial linear absorption coefficient of the unit mass and density of the $\lambda_i - \lambda_{i+1}$ level intercepted section respectively.

8. The density damaged increment of the meso-scale element

The discussion below this paragraph only exists when the damage-fracture field just equals to the macroscopic crack, moreover the inside of the crack is completely filled with air**.**

Every physical quantities of the above definitions only reflect the properties of the rock sample in a certain state. For investigating the changing procedure that the physical states of a certain meso-scale element alter with the stress, the density damaged increment of the meso-scale element is defined below [10].

For any meso-scale element which is arbitrarily obtained from the rock sample, as to a certain level λ_0, the damage-fracture field just equals to the macroscopic crack, moreover the inside of the crack is completely filled with air, the following is already existed,

$$\rho = \frac{1000 + H}{1000 + H_{\bar{d}}}\,\rho_{\bar{d}}$$

According to these, the density damaged increment of the meso-scale element is defined as the following,

$$\Delta D(x, y, z) = \frac{\rho_i - \rho_0}{\rho_0} = \frac{H_i - H_0}{1000 + H_0}$$

Ditto, the CT number of a certain point (x, y, z) so called here is the CT number of the CT resolving power elements in which the point in this coordinate locates.

9. The relation among the volume strain, the density damaged increment and the CT number [10]

As far as the fissured rock mass is concerned, the essence of the meso-scopic volume deformation of the rock is the change of the rock pore and fissure, because the volume deformation of the mineral is small, it can be ignored. The volume deformation is namely the density being changed from the density angle.

Considering a meso-scale element whose mass is m , whose initial volume is v_0 , the bulk strain of the meso-scale element is presented as the following,

$$\varepsilon_v = \frac{v_i - v_0}{v_0} = \frac{v_i/m - v_0/m}{v_0/m} = \frac{\rho_0}{\rho_i} - 1$$

It is expressed by the density damaged increment, namely.

$$\varepsilon_v = \frac{\Delta D}{1+\Delta D}$$

10. The relation among the stress, strain and the density damaged increment

Supposed the tensor of the effective stress in the rock and soil mass is σ_{ij} , it can be decomposed to be these two parts, the effective spherical tensor of the stress σ_m and the partial tensor of the stress S_{ij} ,

$$\sigma_{ij} = S_{ij} + \delta_{ij}\sigma_m \quad (i, j = x, y, z)$$

Here $\sigma_m = \sigma_{ii}/3$, the tensor of the strain ε_{ij} can also be decomposed to be these two parts, the effective spherical tensor of the bulk strain ε_v and the partial tensor of the strain e_{ij} ,

$$\varepsilon_{ij} = e_{ij} + \delta_{ij}\varepsilon_v/3 \quad (i, j = x, y, z)$$

Here $\varepsilon_v = \varepsilon_{ii}$. The bulk strain ε_v is comprised by the compressed and expansion bulk strain ε_{vc} and the shrink and expansion duo to shearing bulk strain, namely

$$\varepsilon_v = \varepsilon_{vc} + \varepsilon_{vs}$$

Therefore, the constitutive relation of the rock mass expresses as the following,

$$\begin{cases} e_{ij} = \dfrac{S_{ij}}{2G} \\ \varepsilon_{vc} = \dfrac{0.435\ C_c}{(1+e_0)} \ln \sigma_m \\ \varepsilon_{vs} = -(\dfrac{q}{\sigma_m} - \eta)\dfrac{(d e_{ij} d e_{ij})^{\frac{1}{2}}}{\lambda} \end{cases}$$

The relation between the compressed and expansion bulk strain and the density damaged increment is showed as the following,

$$\varepsilon_{vc} = \frac{\Delta D_c}{1+\Delta D_c}$$

Here ΔD_c is the density damaged increment based on the CT number corresponding to the triaxial compression test. While the relation between the shrink and expansion duo to shearing bulk strain and the density damaged increment is showed as the following,

$$\varepsilon_{vs} = \frac{\Delta D_s}{1+\Delta D_s}$$

Where ΔD_s is the density damaged increment based on the CT number corresponding to the pure shear test. So the following exists,

$$\varepsilon_v = \frac{\Delta D_c}{1+\Delta D_c} + \frac{\Delta D_s}{1+\Delta D_s}$$

On the basis of these, the constitutive equation based on the theory of the density damaged increment is expressed as the following,

$$\begin{cases} e_{ij} = \dfrac{S_{ij}}{2G} \\ \dfrac{\Delta D_c}{1+\Delta D_c} = \dfrac{0.435\ C_c}{(1+e_0)} \ln \sigma_m \\ \dfrac{\Delta D_s}{1+\Delta D_s} = -(\dfrac{q}{\sigma_m} - \eta)\dfrac{(d e_{ij} d e_{ij})^{\frac{1}{2}}}{\lambda} \end{cases}$$

11. Conclusions

The paper which utilizes the set theory and the measure theory, defines the concepts of the λ level damage-fracture ratio and the λ level damage-fracture rate, the λ_1-λ_2 level intercepted section ratio and rate, the λ level damage-fracture content based on the CT number, simultaneously furthering to research the relation among the λ level damage-fracture rate, the CT number and the density, the relation among the bulk strain, the density damaged increment and the CT number, the relation among stress, strain and the density damaged increment. The article [15] and this paper through utilizing modern mathematics quantitatively analyze the property of the rock and soil CT mechanics, synchronously establish a stable foundation for “the rock and soil damage-fracture space mechanics”. In the future, the constitutive theory of the divisional damage-fracture will be further studied.

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