Rock and Soil Damage-Fracture Space Mechanics：

Damage-Fracture Space

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Abstract

Each of the current analytical methods used in rock and soil CT mechanics is "observation", wasting the precious CT quantitative information. Utilizing the CT number, the perfect degree and the damage-fracture degree of a point inside the rock sample are defined, the λ level perfect field, the λ level damage-fracture field and the λ_1-λ_2 level intercepted section are defined according to the λ level perfect degree and the level damage-fracture degree, the size of the λ level perfect field, the λ level damage-fracture field and the λ_1-λ_2 level intercepted section can be accurately calculated avail of the measure theory and the set theory. Perfect space and damage-fracture space of the rock and soil media are established based on the above conceptions. The paper paves a new way for the quantitative analysis of the rock and soil mechanics by means of the CT datum, simultaneously establishing the foundation of "the rock and soil damage-fracture space mechanics". If the λ level is magnified from the mesomechanics value to the macroscopic one, this method is suitable for the macroscopic geotechnical mechanics, the uniform description of the macroscopic mechanics and the mesomechanics is realizing, the uniform description of the continuum mechanics and porous media or crannied media mechanics comes true, too.

Keywords: The perfect degree, The damage-fracture degree, The λ level perfect field, The λ level damage-fracture field, The λ_1-λ_2 level intercepted section, The measure, The perfect space, The damage-fracture space, The rock and soil damage-fracture space mechanics

1. Introduction

X-radial computerized topography analysis is called CT for short. Computerized tomography can inspect the inner change of the material and the structure without damage, simultaneously having the higher resolving power. The rock CT image reflects the absorptive degree at every points of the rock for X-ray, essentially being a digital image, the value of every pixel represents the CT number. The gray value in the CT image express the size of the CT number. According to the physical principle of CT, the CT number is direct proportion to the density of the rock, the lighting color in the CT image represents the zone of the rock with the higher density, contrarily the dark color represents the part of the rock with the lower density.

The characteristics of the CT number research on the rock and soil mechanics are that the experimental expenditure is large and the quantitative information on each point can be offered. It is obvious that the resource about the CT number not only be rare, but also include the plentiful information, so it should be utilized much cherishingly.

However, the existing problems in the rock and soil CT number mechanics are that the average quantitative information of each point wastes the resource and that doing the direct observation through the CT image generates the analytical error of the visual angle.

The CT datum contains the abundant quantitative information from strain, damage, fracture to failure based on each CT resolving power element of the geotechnical medium. However, the current research on the evolution process of the damage and meso-crack is observed and analyzed only from the view of the size, mean and variance of the CT number, the gray variety of the CT image[1]-[4], the statistic frequency of the CT number[5], the damage variable[5], the damage evolution equation[6], the damage evolution rate[7], the damage contour curve [8], the threshold value of the CT number[9], the structural tensor of the soil mass[9] and the density damaged increment[10] etc., a systematic and characteristic researching method has not formed so far. These means in which the noise existed in the image was not abatement, the tradition researching methods is preserved, the proper mathematic describing means not discovered yet, the quantitative information of CT resolving power elements averaged the mean information of the whole rock sample or certain local part, applying the rare quantitative information of each point to the qualitative description, all these result in wasting the resources of the CT image heavily. On the other hand, because of

the size of the CT resolving power, the effect of gray image's contrast, the different disposal methods which each author applied to the CT image and the analytical error of the visual angle, the different author derived of the distinct CT image under the condition of the same scale crack or the uniform density, the researching error is resulted from the artificial factor. The same kind of the research in papers [11]-[12] begins early, but the research methods are the same to the above, without breakthrough [13-14]. So the CT research of the current geotechnical mechanics actually locates at "the observation stage of the CT image".

From the Figure 1, we can see the results of the same fractured sandstone sample through the different methods disposed, it is hard to believe that they are the same width or area of the break region.

This paper firstly defines the concepts of the perfect degree and the damage-fracture degree of a certain point inside the rock and soil media space, furthers to define the concepts such as the λ level perfect degree, the λ level damage-fracture degree, the λ level perfect field, the λ level damage-fracture field and the λ_1-λ_2 level intercepted section etc., presents the measures of the λ level perfect field, the λ level damage-fracture field and the λ_1-λ_2 level intercepted section and so on. Finally, the paper shows the concepts of the rock and soil perfect space or damage-fracture space. The latter article will deliver the new indexes of the physical state based on the CT number such as the λ level damage-fracture ratio, the λ level damage-fracture rate, the λ level damage-fracture content, the λ_1-λ_2 level intercepted section ratio and rate etc., and do the primary research on the physical property and mechanics meaning of these indexes, and do some research on the relation between these new indexes and the routine physical indexes. Utilizing the knowledge of the set theory in another paper studies the position and criterion of the damage-fracture produced based on the CT number, presents the concepts of the safety zone, the damaging zone and the fractured zone, gives the rules of compressing and expanding, constructs the divisional description of the rock and soil stress-strain law and the constitutive theory of the rock and soil damage-fracture space. At last the author also researches the problems such as the lower and upper boundary surfaces of the damaging zone, the relative length of the fractured zone, the number of the fissure or crack in the fractured zone and the space size of the crack etc..

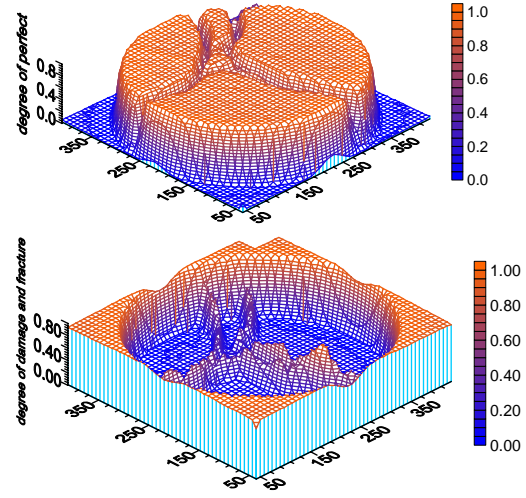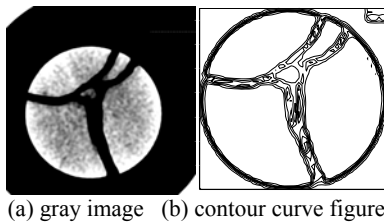(a) gray image   (b) contour curve figure

Fig.1: The same CT image through different methods disposed.

Note: the sampling of this figure begins from row 105 to row 408, from column 42 to column 430 with the alternation, so the number of the resolving power elements is 73320, the following figures use the same sampling method.

Fig. 2: the distribution figure of the perfect degree and the damage-fracture degree (the resolving power element counted:73320)

All these theory based on the damage-fracture degree (or the perfect degree) of the rock and soil space points, synthetically using the mathematical knowledge of the set theory, the measure theory and the Lebesgue integral investigates the theory of divisional description of the rock and soil stress-strain law, initiates a new stage when the CT numbers research is comprehensively, systemically and independently applied to the quantitative analysis of the rock mechanics. Simultaneously this method leads to a new subject produced, it is named "the rock and soil damage-fracture space mechanics". The fracture means the crack, presenting the destroy of the material mechanics property, moreover the damage means the material mechanics property weakened, this series of the article regard the damage-fracture degree as the key point, the sets of the damage-fracture field as the target, the research of the rock and soil material mechanics property is developed on the damage-fracture space, so named "the damage-fracture space mechanics".

In addition, the definitions of the perfect degree and the damage-fracture degree in the context are based on the CT numbers, consequently the theories in the articles are all built around the CT number. According to the relation between the CT number and the density, ditto, the density can be used to define the perfect degree and the damage-fracture degree, and the other

macroscopic mechanics parameters can also be selected to define them, consequently many theories in these papers are still tenable.

In the future, the characteristic variables of the rock and soil mass such as the fissure, stratum, section, pore, the structure parameter etc. will be transformed into the stochastic variable, utilizing the mathematic method establishes the three-dimensional random distribution mechanics model of the rock and soil mass, using the method in these papers analyses the rock and soil mechanics, serving for the engineering construction. Thus, these papers introduce a general mechanics researching method, not confining to the disposal of the CT image, which will be discussed in another articles.

The method used in these articles can also be applied to the CT analysis of the medicine and the industrial process and to the image analysis of the remote sensing technology, has an extensive applied prospect. Simultaneously, if the λ level is amplified to the macroscopic value, the researching method in the paper adapts to the macroscopic geotechnical mechanics, so this series of the articles realize the gradual transition and the uniform description of the meso-mechanics and the macro-mechanics, achieve the uniform description of the continuum mechanics and porous media or fissured media mechanics very naturally.

The article suites not only for the rock material but also for the soil material, but the following examples regard rock material as the object described.

All the new conceptions not explained the derivation in this paper are originated by the author independently, all belongs to the original job.

2. The basic precondition

The paper is discussed under the precondition as following,

(a) The discussion is within the Newton mechanics.

(b) The experiment is real-time CT testing.

(c) The density of the air is the minimum.

(d) The density of the rock cement or the soil grain without damage is the maximum.

(e) The volume strain is not produced from the rock cement, the soil grain and cement when the rock or soil samples bearing force, but derived from the opening and the closing of the meso-crack.

(f) As the definition of the perfect degree or the damage-fracture degree based on the CT number, the mesomechanics scale in this paper is the scale of the CT resolving power element. If the perfect degree or the damage-fracture degree are defined through other mechanics parameters, the mesomechanics scale should be altered accordingly.

3. The perfect degree and the damage-fracture degree of the rock

The total points of the target of the rock or rock mass researched are defined as the totally set, representing by the Ω.

$$\Omega = \left\{ (x, y, z) \middle| \begin{array}{l} (x, y, z) \text{ is the arbitrary point of the target} \\ \text{of the rock or rock mass researched} \end{array} \right\}$$

Where, the bracket represents set, the formula before the vertical line denotes the object of the set, the formula after the vertical line expresses the condition that the objects should satisfaction. This kind of representation is often used in the following parts of the paper.

The CT number equals to the computerized tomography of the X-ray, namely using the value of a pixel represents the synthetic physical information such as density of the rock sample's corresponding point or part. CT number of a certain point in the CT image can reflects the relative density at the point of the rock sample, there is the relation between them, but that is not the showing relation, using $H(x, y, z)$ presents the CT number of the space point (x, y, z), denotes as H for short. Its definition shows as following.

$$H = \text{CT number} = \frac{\mu_t - \mu_w}{\mu_w} \times 1000$$

Where, the μ_t and μ_w denote the linear decay coefficient of the X-ray of the mineral and water corresponding. The CT number of air is the minimum, its value defines -1000, the CT number of pure water is 0, the CT number of pure ice equals to -100.

It can be predicted that the CT number of the same rock sample from the perfect zone, the damaging zone to the fractured zone should distribute within a certain scope, simultaneously exist the maximum and the minimum numerical value. The definitions of the perfect zone, the damaging zone and the fractured zone will give in the following paper.

The mark max H denotes the maximum CT number of a certain kind of the rock sample, the minimum CT number of all the rock samples is the air's, the numerical value is -1000. The CT number at a point of a certain kind of rock sample is standardization by the maximum and the minimum numerical value, these standard CT numbers are the function which is defined on Ω with the value field [0 1]. Defines

$$p(x, y, z) = \frac{H(x, y, z) + 1000}{\max H + 1000}$$

as the perfect degree of the rock sample at the point of (x, y, z). The so-called CT number of a certain point (x, y, z) presents the CT number of the

CT resolving power element where this coordinate point locate, with the improvement of the CT resolving power this little element will be continuously reduced.

The reasons why the conception of the perfect degree is introduced are as following, any part of the rock sample is considered to be perfect, the zone without damage is perfect, the zone where the crack has been produced is also perfect, however, their perfect degree are different. The perfect degree of the rock cement is 1, the perfect degree of the region broken completely is 0, the perfect degree of other region is a certain number between 0 and 1.

The opposition of the perfect degree defines as following,

$$d(x,y,z)=1-p(x,y,z)=\frac{\max H-H(x,y,z)}{\max H+1000}$$

$d(x,y,z)$ represents the damage-fracture degree at the point (x,y,z) of the rock samples. Any part of the rock sample can be considered to have been damaged, the zone without damage is also damaged, the region where the crack has been produced is damaged, too, however, their damage-fracture degree is different. The damage-fracture degree of the rock cement is 0, the damage-fracture degree of the region broken completely is 1, the damage-fracture degree of the other regions is a certain number between 0 and 1.

From figure 2, which are the distribution figures of the perfect degree and damage-fracture degree of the sandstone's in the figure 1, it is regarded that these figures are different from the contour curve figure, the maximum and the minimum numerical value in these figures are 1 and 0 respectively.

The concepts of the perfect degree and the damage-fracture degree at a certain point reflect the relative stiffness degree and the relative dilapidation degree of the point in this kind of rock sample. Obviously,

$$p(x,y,z)+d(x,y,z)=1$$

4. The λ level perfect field and the λ level damage-fracture field of the rock

The perfect degree and the damage-fracture degree make the concepts fuzzy that the rock is perfect, damaged, crack or broken, but the professional prefer using the accustomed physical conception to the fuzzy one, so the concept of the perfect level is introduced.

Assume $0 \le \lambda \le 1$. The rock sample is called the weak λ level perfect at the point (x_0, y_0, z_0) when $p(x_0, y_0, z_0) \ge \lambda$. The rock sample is called the strong λ level perfect at the point (x_0, y_0, z_0) when $p(x_0, y_0, z_0) > \lambda$. In order to simple the conception,

the weak λ level perfect is named the λ level perfect. Likewise, the rock sample is called the λ level damage-fracture at the point (x_0, y_0, z_0) when $d(x_0, y_0, z_0) \ge \lambda$.

The meaning of the rock sample with the weak λ level perfect at the point (x_0, y_0, z_0) is that its perfect degree is greater than or equals to 100λ%, simultaneously the meaning of the rock sample with the λ level damage-fracture at the point (x_0, y_0, z_0) is that its damage-fracture degree is greater than or equals to 100λ%.

The rock sample is called λ level non-perfect at the point (x_0, y_0, z_0) means that its damage-fracture degree is greater than or equals to 1-λ, simultaneously the rock sample is called λ level non-damage-fracture at the point (x_0, y_0, z_0) means that its perfect degree is greater than or equals to 1-λ.

The set

$$\{(x,y,z)|\ (x,y,z)\in\Omega \text{ and } p(x,y,x)\ge\lambda\}$$

is called the weak λ level perfect field of the rock sample.

The set

$$\{(x,y,z)|\ (x,y,z)\in\Omega \text{ and } p(x,y,x)>\lambda\}$$

is called the strong λ level perfect field of the rock sample.

Hereafter, the weak λ level perfect field is simple as the λ level perfect field, showing by p_λ.

p_λ represents the set of the points whose perfect degree are larger than or equal to λ in the rock sample, i.e. λ level the perfect part of the rock. Ditto, the strong λ level perfect field shortens to the strong p_λ.

Obviously, $p_\lambda \supseteq$ strong p_λ.

Simultaneously, the set

$$\{(x,y,z)|\ (x,y,z)\in\Omega \text{ and } d(x,y,x)\ge\lambda\}$$

is called the weak λ level damage-fracture field of the rock sample. It is also shortened to be the λ level damage-fracture field, presenting by d_λ. d_λ means the set of the points whose damage-fracture degree are larger than or equal to λ in the rock sample, i.e. λ level the damage-fracture part of the rock. Ditto, the strong λ level damage-fracture field is defined, also shortened to be the strong d_λ.

Obviously, $d_\lambda \supseteq$ strong d_λ.

If the proper value of λ is given, d_λ can presents the damage zone of the classical damage mechanics or the rock crack of the classical fracture mechanics. Figure 3 shows some λ level perfect fields and λ level

damage-fracture fields of the above sandstone sample in different level.

Avail of the concepts of the λ level perfect field and λ level damage-fracture field, the natural transition from the rock meso-mechanics to macro-mechanics can be realized smoothly, simultaneously the damage zone of the meso-mechanics and the fracture zone of the macroscopic mechanics can be harmoniously unified in the follow papers. The concept of the λ level damage-fracture field has the more abundant connotation than the traditional damage-fracture field's, its operation are very concise.

Obviously,

$$p_\lambda \cup \text{strong } d_{1-\lambda} = \Omega$$

There-into, $\cup$ represents the union of the two sets.
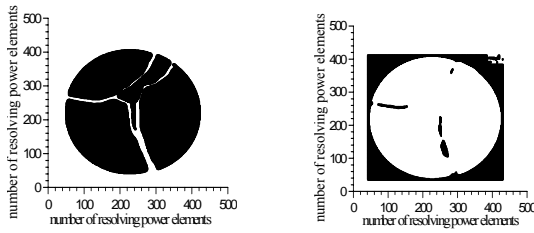
(a)The 0.7 level perfect field. (b) The 0.7 level damage-fracture field.
(resolving power element number counted: 52671)
(resolving power element number counted:16949, containing air's)
Fig. 3: Some λ level perfect fields and λ level damage-fracture fields of the sandstone sample

The set $\Omega - p_\lambda$ is called the λ level non-perfect field of the rock sample, marking $\overline{p_\lambda}$, which presents the complementary set of the λ level perfect field. The set $\Omega - d_\lambda$ is called the λ level non-damage-fracture field of the rock sample, marking $\overline{d_\lambda}$, which presents the complementary set of the λ level damage-fracture field. Where, the over-line is the operator sign of evaluating the complementary set. Because the complementary set of the complementary set equals to itself, so

$$p_\lambda = \overline{\overline{p_\lambda}} ; \ d_\lambda = \overline{\overline{d_\lambda}}$$

With the λ decreasing, the reviewed crack scale gradually decreases, the points merged into the damage-fracture field increases accordingly, therefore the λ level damage-fracture field increases gradually with the decreasing of the λ. You can sure from the Figure 3 that the λ level perfect field also increases gradually with the decreasing of the λ, it is reviewed from the angle of the perfect degree. These two conclusion don't contradict with each other, the problem is reviewed on the contradictory positions. Using the formulas show as the following,

$d_{\lambda_2} \supseteq d_{\lambda_1}$ and $p_{\lambda_2} \supseteq p_{\lambda_1}$ come into existence at the same time when $\lambda_2 < \lambda_1$. Where, $\supseteq$ presents the inclusion relation of the sets.

5. The measure of the λ level perfect field and The λ level damage-fracture field of the rock

To describe the sizes of the λ level perfect field and the λ level damage-fracture field, the concept of the measure based on the mathematics is introduced. About the problem what is the measure, please referring to the measure theory[15]. The measure of the set $(*)$ presents by $m(*)$. If the geotechnical operators are strange to the definition of the measure, you can think it as the product of the number and the length, area or volume of the CT resolving power element, for example $m(d_\lambda)$ can be considered to be the area of the λ level damage-fracture field.

Why the measure is adopted while the length, area or volume are not to adopted directly, the reason is that the target reviewed is the set, whose elements are points, the length, area or volume of the set sometimes don't exist. For example, the set constitutes by all of the rational number is a numerable set, all of the numerable sets are measurable, furthermore their measure equal 0, but we can not measure the length of this set.

The law that d_λ alters with the change of λ has been studied in the last paragraph, hereby the law that the mechanics property and the physical state of the rock sample alter with the change of $m(d_\lambda)$. Certainly, the CT number $H(x, y, z)$, the perfect degree $p(x, y, z)$ or the damage-fracture degree $d(x, y, z)$ also can be directly utilized to study the mechanics property and the physical state of the rock sample, but it should be noticed that $H(x, y, z)$ is a ternary function defined on the 3-dimensional space with the variable value field, $p(x, y, z)$ and $d(x, y, z)$ are the functions defined on the 3-dimensional space with the value field [0 1], while the measure of d_λ or p_λ is an unary function of λ. Therefore, the measure of the λ level damage-fracture field is applied to researching, whose superiority is obvious, which makes the problem concise very much.

As the so-called points are the CT resolving power element here, the λ level perfect field p_λ and the λ level damage-fracture field d_λ are all measurable.

For example, the measure $\mathrm{m}(d_\lambda)$ of the λ level damage-fracture field of the sandstone changed with λ, the variable curve can be showed in the figure 4, the measure is the product of the accumulative number and the area of one resolving power element.
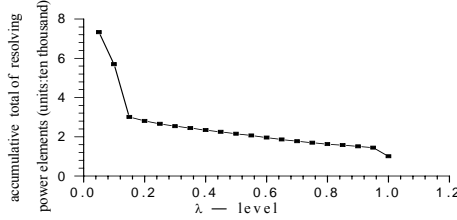

Fig. 4: The measure of the λ level damage-fracture field.
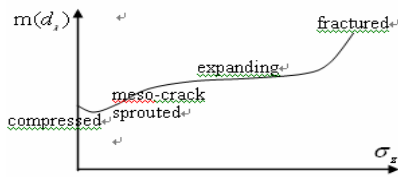

Fig. 5: The relation between $\mathrm{m}(d_\lambda)$ and the stress.

According to the research of the paper [2], the rock materials endured variable loads undergo the stages of being compressed, meso-crack sprouted, branching, expanding, fracture produced and failure, contemporary, the λ level damage-fracture field of the same section also change with the enhancement of stress, the process of the field suffers the phases of descending, enhancing, mildly increasing and sharply increasing. As a whole, $d_\lambda\big|_{\sigma_2} \supseteq d_\lambda\big|_{\sigma_1}$ when $\sigma_2 > \sigma_1$, which increasing with the outside loads. The rock sample's $\mathrm{m}(d_\lambda)$ suffers the phases of descending, enhancing, mildly increasing and sharply gaining, which correspond to the stages of the rock sample being compressed, expanding, the CT scale crack evolving, failure. Showed by figure 5, which is actually a relation curve between stress and distortion.

According to the research of the paper [2], the λ level damage-fracture field's size, shape and position of the different section should be different corresponding to a certain loading state, which is derived of the non-homogeneous of the rock sample.

6. The λ_1-λ_2 level intercepted section of the rock

Supposed $0 \leq \lambda_1 \leq \lambda_2 \leq 1$, the difference set of the λ_1 level damage-fracture field and the strong λ_2 level damage-fracture field is defined to be the λ_1-λ_2 level intercepted section of rock, denoting $d_{\lambda_1-\lambda_2}$. It can be seen from its definition as following

$$d_{\lambda_1-\lambda_2} = d_{\lambda_1} - \mathrm{strong}\, d_{\lambda_2}$$
$$= \{(x,y,z)\,|\,d(x,y,z) \geq \lambda_1\} - \{(x,y,z)\,|\,d(x,y,z) > \lambda_2\}$$

The two formulas in the following is equivalence to the definition of the λ_1-λ_2 level intercepted section.

$$d_{\lambda_1-\lambda_2} = \{(x,y,z)\,|\,\lambda_1 \leq d(x,y,z) \leq \lambda_2, 0 \leq \lambda_1 \leq \lambda_2 \leq 1\}$$
$$d_{\lambda_1-\lambda_2} = \{(x,y,z)\,|\,(x,y,z) \in d_{\lambda_1} and (x,y,z) \notin \mathrm{strong}\, d_{\lambda_2}\}$$

The physical meaning of the λ_1-λ_2 level intercepted section is the set of all points of the rock sample whose density in a certain range.

In the geotechnical engineering practice, the soil crust is a λ_1-λ_2 level intercepted section, the weak inter-bedded strata is also a intercepted section, the rock section is a intercepted section, the bedding, lay, unilateralism, crack and the fracture band etc. are all the intercepted sections, too. Obviously the extension of the intercepted section's concept is abundant, but its connotation is very concise, i.e. the whole points whose density are all nearly equivalent.

In fact, for the same kind of homogeneous material, the magnitude of the density determines the strength of the rock and soil mass, so the λ_1-λ_2 level intercepted section is actually the set of points with the equivalent strength.

Certainly, the difference set of two perfect fields can also be used to define the λ_1-λ_2 level intercepted section of rock, besides signing it by $p_{\lambda_1-\lambda_2}$.

The size of the λ_1-λ_2 level intercepted section in the perfect field or in the damage-fracture field can also be surveyed by measure, the definition of the intercepted section's measures equals to the λ level perfect field's or λ level damage-fracture field's. Figure 6 shows the intercepted section of the perfect field or the damage-fracture field on the above sandstone. Obviously, the conception of the intercepted section not only has a abundant extension but also is very useful. Utilizing the intercepted section, the perfect zone and the fractured zone, the crack zone and the air, the compressed zone and the cement zone etc. can be easily differentiated, moreover the volume, area and width of the zones such as the perfect zone, fractured zone, air, crack zone, compressed zone, cement zone can be accurately surveyed by their measures. The survey method is as easy as the pupils count number, which need to work out the resolving power elements number of the intercepted section, the measure is the product of the number and the volume, area or width of the resolving power element. These functions of the intercepted section figure cann't be achieved on the gray image.

The figure7(a) shows the statistic elements numbers of the 0.0-0.1, 0.1-0.2, …, 0.9-1.0 level intercepted sections of the rock sample, which is multiplied by the area of the resolving power element is namely the measure of each intercepted sections. The figure7(b) and figure7 (c) show the statistic elements

numbers of the intercepted sections on other rock samples. The homogeneous degree of the rock sample can be judged by figure of the statistic elements number of the intercepted section, too. The second point count from back in the figure7(a) goes up, which illuminates the crack has been produced in the homogeneous rock sample. The second point count from back and the point in the mid part of the figure7(b) both ascend, which explains the crack has been produced in the hard rock with weak inter-bedded strata. The figure7(c) shows the existing hard block in soft rock, but the crack has not been generated, it is a perfect sample at that time. The last point ascends which shows the statistic zone of these three rock samples have a large amount of air.

(a)

(b)
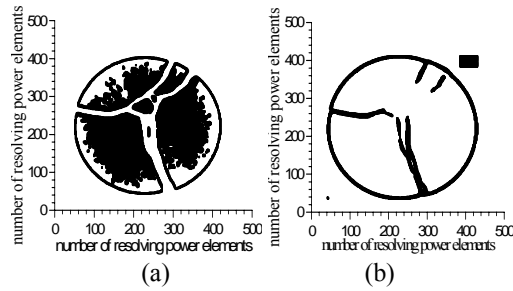
(a) 0.85-0.95 level intercepted section of the perfect field, (the resolving power element counted:28839)

(b) 0.5-0.9 level intercepted section of the damage-fracture field, (the resolving power element counted:6214)

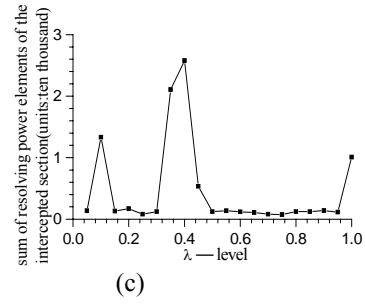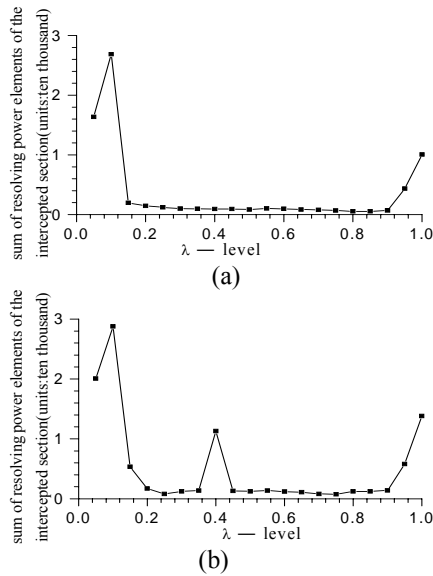Fig. 6: The intercepted sections of the perfect field and the damage-fracture field of the sandstone sample.

(a)

(b)

(c)

Fig. 7: The statistic elements numbers of the intercepted sections of three kinds of rocks.

7. The perfect space and the damage-fracture space of the rock

The conception of the topology space refers to paper[16]. If the subsets family $\{\Re\}$ of a certain nonempty set Ω is contented with the three conditions: The null set and the complete set all belong to it, the intersection of finite subsets belongs to it, and the union set of infinite subset belongs to it. The subsets family $\{\Re\}$ is called the topology of this nonempty set Ω, simultaneously the couple $(\Omega, \Re)$ names the topology space.

The set of all λ level or strong λ level perfect fields is represented by $\{p_\lambda\}$, the set of all λ level or strong λ level damage-fracture fields is represented by $\{d_\lambda\}$, the sets of all $\lambda_1 - \lambda_2$ level intercepted sections about perfect and damage-fracture fields are respectively denoted by $\{p_{\lambda_1-\lambda_2}\}$ and $\{d_{\lambda_1-\lambda_2}\}$. Obviously, $\{p_{\lambda_1-\lambda_2}\}$, $\{d_{\lambda_1-\lambda_2}\}$, $\{p_\lambda\}$ and $\{d_\lambda\}$ considered as the subsets families of the set Ω all meet the definition of the topology, so all they are the topology of the set Ω. Certainly, all the couples of $(\Omega, p_{\lambda_1-\lambda_2})$, $(\Omega, d_{\lambda_1-\lambda_2})$, (Ω, p_λ) and (Ω, d_λ) are the topology spaces.

The couples of $(\Omega, p_{\lambda_1-\lambda_2})$ and $(\Omega, d_{\lambda_1-\lambda_2})$ are called the perfect space and the damage-fracture space respectively. As the special examples, the couples of (Ω, p_λ) and (Ω, d_λ) are also the perfect space and the damage-fracture space of the rock sample respectively.

The following papers are discussed sometimes on the perfect space, sometimes on the damage-fracture space, these two kinds of spaces with different characteristics should be noticeably separated from each other.

Because the main concerned problems in the rock mechanics are the strength and the damaged and destroyed process of the material, the following is inclined to utilize the conceptions of the damage-

fracture degree and the λ level damage-fracture field of the rock, namely discussing the problems on the damage-fracture space, the problems are discussed on the perfect space none but at the most necessary time.

Additionally, it is worthy of notice that the λ level perfect field is different from the λ level non-damage-fracture field in the concept. The λ level perfect field and the λ level non-perfect field are defined on the perfect space (Ω, p_λ), while the λ level damage-fracture field and the λ level non-damage-fracture field are defined on the damage-fracture space (Ω, d_λ). Here the "non" should be understood to be the complement operating of the set.

8. Conclusions

The paper points out that the current CT research of the rock and soil mechanics locate at "the observation stage of the CT image", it firstly defines the concepts of the perfect degree, the damage-fracture degree, the λ level perfect, the λ level damage-fracture, the λ level perfect field, the λ level damage-fracture field and the λ_1-λ_2 level intercepted section of a certain points inside the rock and soil media, also defines the measure of the λ level perfect field, the λ level damage-fracture field and the λ_1-λ_2 level intercepted section, establishing the concepts of perfect space and damage-fracture space of the rock and soil, too, simultaneously studies some basic properties of them. Utilizing modern mathematics quantitatively study the property of the rock and soil CT mechanics, synchronously building a stable foundation for the rock and soil damage-fracture space mechanics.

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