A Method to Solve Multi-attribute Group Linguistic Decision-making Problems

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Abstract

This paper presents a method to solve multi-attribute group decision-making with linguistic assessment. The method is based on the use of fuzzy linguistic approach, and on the use of fuzzy majority of consensus, represented by means of a linguistic quantifier. The individual consensus preference degrees and the individual consensus importance degrees are defined. The primary focus of this paper is to obtain an available scheme array over the set of alternatives.

Keywords: Multi-attribute Group decision-making ($MAGDM$), Fuzzy linguistic approach, The consensus importance degree

1. Introduction

Based on fuzzy logic and fuzzy linguistic approach, several of authors have presented a lot of linguistic decision-making methods and related applications in order to solve some practical problems in our real-world. For example, group decision making [1], multi-criteria decision making [2], multistage decision making, etc.

Many aspects of different activities in the real world cannot be assessed in a quantitative form, but rather in a qualitative one, *i.e.*, with vague or imprecise knowledge. In that case, a better approach may be to use linguistic assessments instead of numerical values. The variables which participate in these problems are assessed by means of linguistic terms [3, 17]. As we have known, especial in decision making, the qualitative forms of the information may be unavailable or unnecessary. Under the circumstances, it is very suitable and necessary for human to introduce the fuzzy linguistic approach to deal with some practical problems. The fuzzy linguistic approach is an approximate technique that represents qualitative aspects as linguistic values by means of linguistic variables [4]. According to this approach, we are allowed to use some linguistic variables rather than numerical values to express our opinions. For example, when evaluating how many work experiences an expert has, we usually use some linguistic terms like "many" or "extremely many" instead of number values like "1" or "2". Besides, in fact, human are more inclined to take advantage of natural language to demonstrate, analyze, generalize, even estimate.

A model of Multi-attribute Group Decision-Making is established in an environment where there is a question to solve, a set of possible alternatives, a set of attributes and a set of individuals (experts, judges, etc), who have different importance labels. They present their opinions or preferences over the set of possible alternatives according to different attributes. The priority of this model is to find an array of the available alternatives. The result will mostly be determined by some individuals who can reach an agreement under several of criteria/attributes and wield considerable influence in the whole course of the decision making. Generally, there are a finite set of alternatives $X = \{x_1, \cdots, x_k\}$, a finite set of attributes $A = \{a_1, \cdots, a_n\}$, and a finite set of individuals ($e.g$, experts or decision makers) $D = \{d_1, \cdots, d_m\}$ with their respective importance label defined as a linguistic value, such that, $\mu_L(k) \in S$ denotes the importance label of individual "d_k". At the same time, these linguistic values have their respective relevance degree defined as a real number, that is, $\mu_G(k) \in [0, 1]$ denotes the relevance degree of linguistic value of the individual "d_k".

In practice, individuals could have linguistic assessments about the preference degree of the alternative x_k over x_l [7]-[10]. A scale of certainty expressions (linguistically assessed) would be presented to the individuals, who could then use it to describe their degree of certainty in a preference. In this environment, we have linguistic preference relations to provide individuals' opinions [1].

On the other hand, a group of individuals have not identical opinions even under a single attribute/criterion. So, in a complex situation of $MAGDM$, the opinions are impractical and difficult to reach a complete agreement. The consensus mentioned in this article can be interpreted the opinions of most of individuals. In addition, each individual has own importance label, that is, every individual play a different part during the process of decision making.

This article is structured as follows: In Section 2, we make a brief review of the fuzzy linguistic approach. In Section 3, we present the model. In Section 4, in order to catch on this model more directly, we provide an easy example. Finally, in Section 5, some conclusions are pointed out.

2. Fuzzy linguistic approach and linguistic quantifiers

2.1. Fuzzy linguistic approach

The fuzzy linguistic approach represents qualitative aspects as linguistic values by means of linguistic variables [3]. We have to choose the appropriate linguistic descriptors for the term set and their semantics. In order to accomplish this objective, an important aspect is the "granularity of uncertainty", *i.e.*, the level of discrimination among different counts of uncertainty. Typical values of cardinality used in the linguistic models are odd ones, such as 7 or 9, where the mid term represents an assessment of middle case, and with the rest of the terms being placed symmetrically around it [5, 6].

The semantic of the linguistic term set is defined by assuming that the meaning of each linguistic term is given by means of a fuzzy subset defined in the $[0, 1]$ interval, which are usually described by membership functions [5, 9][11]-[14]. A computationally efficient way to characterize a fuzzy number is to use a representation based on parameters of its membership function [15]. Because the linguistic assessments given by the users are just approximate ones, some authors consider that liner trapezoidal membership functions are good enough to capture the vagueness of those linguistic assessments, since it may be impossible and unnecessary to obtain more accurate values [5, 7, 9, 16]. This parametric representation is achieved by the 4-tuple (a, b, d, c), where b and d indicate the internal in which the membership value is 1, with a and c indicating the left and right limits of the definition domain of the trapezoidal membership function [5].

One of case of this type of representation is the linguistic assessments whose membership functions are trapezoidal is presented as following:

$P = Perfect = (0.16, 1, 1, 0)$,
$VH = Very - High = (0.18, 0.84, 0.84, 0.16)$,
$H = High = (0.16, 0.66, 0.66, 0.18)$,
$M = Medium = (0.16, 0.5, 0.5, 0.16)$,
$L = Low = (0.18, 0.34, 0.34, 0.16)$,
$VL = Very - Low = (0.16, 0.16, 0.16, 0.18)$,
$N = None = (0, 0, 0, 0.16)$

Accordingly, to establish what kind of label set to use ought to be the first priority. Then, let $S = \{s_i\} \quad i \in H = \{0, \cdots, T\}$, be a finite and totally ordered term set on $[0, 1]$ in the usual sense [4, 5, 7]. Any label s_i represents a possible value for a linguistic real variable, that is, a vague property of constraint on $[0, 1]$. We consider a term set with odd cardinal, where the middle label represents an uncertainty of "approximately 0.5" and the rest terms are placed symmetrically around it. Moreover, the term set must have the following characteristics:

(a) The set is ordered: $s_j \leq s_i$ if $j \leq i$;

(b) There is the negation operator: $Neg(s_i) = s_j$ such that $j = T - i$;

(c) Maximization operator: $Max(s_i, s_j) = s_i$ if $s_j \leq s_i$;

(d) Minimization operator: $Min(s_i, s_j) = s_i$ if $s_j \geq s_i$.

2.2. Linguistic quantifiers

The fuzzy linguistic quantifiers were introduced by $Zadeh$ in 1983 [18]. Linguistic quantifiers are typified by terms such as "most" , "at least half", "all", "as many as possible" and assumed a quantifier Q to be a fuzzy set in [0,1] . In this paper, we will use the relative quantifier to represent proportion type statements. Then, if Q is a relative quantifier, then Q can be represented as a fuzzy subset of $[0, 1]$ such that for each $r \in [0, 1]$, $Q(r)$ indicates the degree to which r portion of objects satisfies the concept denoted by Q [1].

We assume a label set $L = \{l_i\}, i \in J = \{0, \cdots, U\}$, denoted Q ,

$Q : [0, 1] \rightarrow L$

The relative quantifier we will use is defined as follows:

$$Q(r) = \begin{cases} l_O, & if r < a, \\ l_i, & if a \leq r \leq b, \\ l_U, & if r > b. \end{cases}$$

l_O and l_U are the minimum and maximum labels in L, respectively, and $l_i = Sup_{(\ l_q \in M)}\{l_q\}$ with $M = \{l_q \in L : \mu_{l_q}(r) = Sup_{(t \in J)}\{\mu_{l_t}(\frac{r-a}{b-a})\}\}$ with $a, b, r \in [0, 1]$.

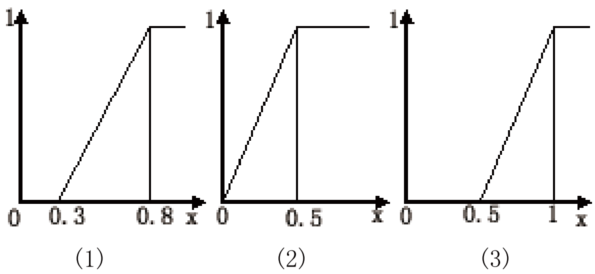

Fig. 1: Linguistic quantifiers

Some examples of relative quantifiers are shown in [1], see Fig.1, where the parameters (a, b) are $(0.3, 0.8)$, $(0, 0.5)$ and $(0.5, 1)$,respectively. In Fig.1, (1) is "Most",(2) is "At least half", (3) is "As many as possible"

In this paper, Q will be used to obtain the individual consensus preference degree and individual consensus importance degree. In the following, for simplicity and without loss of generality, we still assume that the term set for the preferences among options and for evaluating the consensus degrees by means of the quantifier Q are the same and denoted S [1].

3. The method of solving Multi-attribute Group decision making problems

We assume a set of alternatives $X = \{x_1, \cdots, x_k\}$, a set of attributes $A = \{a_1, \cdots, a_n\}$ and a set of individuals $D = \{d_1, \cdots, d_m\}$. Each individual has an importance label represented by linguistic variable. These linguistic variables stem from the linguistic term set S. As we have mentioned at the beginning, define $\mu_L(k) \in S$ as the importance label of the individual "d_k". And for each $\mu_L(i)(d_i \in D)$, we suppose to define a relevance degree $\mu_G(k) \in [0, 1]$ in order to distinguish different labels. Then, the described model considers that each individual $d_k \in D$ provides his or her opinions on X as preference relation linguistically assessed into the term set, S

$\phi_{p_j^i} : XxX \rightarrow S$

Under different attributes $j \in A$,each individual $i \in D$ presents own perspectives on X as a fuzzy preference relation $P_j^i \subset XxX$, with $P_j^i(k, l) \in S$ denoting the linguistically assessed preference degree of the alternative x_k over x_l .we assume that P_j^i is reciprocal in the sense, $P_j^i(k, l) = NegP_j^i(l, k)$, and by definition $P_j^i(k, k) = " - "$.

In order to obtain an array of all alternatives, we have to make a comparison between any two alternatives. As we all know, for any an alternative pair (x_k, x_l), if there are n attributes and m individuals, then there are $n \times m$ preference labels to represent the compared result of x_k and x_l. These preference labels stem from linguistic term set S. $R(i, j)$ is a preference relation that each individual under all different attributes provides preference degrees of the alternative x_i over x_j. $R(i, j)_{k,\ l} = s_t$ $(s_t \in S)$ denotes the individual d_k takes s_t as preference label of the alternative x_i over x_j under the attribute l. In addition, S is denoted the linguistic label set, is defined as

$$S = \{s_t \mid t = 1, \cdots, T\}.$$

3.1. The individual consensus preference degrees

The aim of this process is to find some linguistic preference labels that the most individuals can reach an agreement under some attributes from $R(i, j)$.

At first, in order to calculate conveniently, we turn the $R(i, j)$ into n column vectors, that is, $R(i, j) = \{R_1, \cdots, R_n\}$.

Then, $R_{ij}^{cl}[s_t]$ is the counting number of s_t in l column vector, $R_{ij}[s_t]$ is a set of the index of the column vector, $R_{ij}[s_t] = \{l | R_{ij}^{cl}[s_t] > 1,\ l = 1, \cdots, n\}$.

$I_{ij}^C[s_t]$ is the individual consensus preference degree of the alternative pair (x_i, x_j) , is defined as following:

$$I_{ij}^C[s_t] = \begin{cases} \frac{\sum_{l \in R_{ij}[s_t]} (R_{ij}^{cl}[s_t]/m)}{\sharp(R_{ij}[s_t])} & R_{ij}^{cl}[s_t] > 1 \\ 0, & \text{otherwise} \end{cases}$$

Where $\sharp$ stands for the cardinal of the set $R_{ij}[s_t]$.

At last, we choose an appropriate linguistic quantifier Q_1 to represent the concept of the individual consensus linguistic preference fuzzy majority. $PC_{ij}^{Q_1}$ is represented the portion of the individuals, who have take s_t as their unanimous opinions in the $R(i, j)$, and,

$$PC_{ij}^{Q_1}[s_t] = Q_1(I_{ij}^C[s_t]) \in S$$

3.2. The individual consensus importance degrees

From the former phase, we can get the preference labels that most individuals have chosen. In this part, our primary goal is to calculate the importance degrees of the most parts of the individuals who have reached an agreement.

At first, to make calculate easily, we take the relation $R(i,j)$ apart m row vectors, that is,

$$R(i,j) = \begin{pmatrix} V_1 \\ \cdot \\ \cdot \\ \cdot \\ V_m \end{pmatrix}$$

Then, the $V_{ij}[s_t]$, $(i,j \in X, t = 0,\cdots,T)$, is a set of row vectors' index, in which there is a s_t label at least. $V_{ij}[s_t]$ is defined as following:

$V_{ij}[s_t] = \{k | s_t \in V_k, k = 1,\cdots,m\}$

The counting number of the s_t label in the V_k row vector of the $R(i,j)$ is represented $V_{ij}^{ck}[s_t]$.

So, the individual consensus importance degree of the alternative pair (x_i, x_j) is calculated according to the expression:

$$I_{ij}^G[s_t] = \begin{cases} \sum_{k \in V_{ij}[s_t]} (V_{ij}^{ck}[s_t]/n) \times \mu_G(k), & V_{ij}^{ck}[s_t] \neq 0 \\ 0, & \text{otherwise} \end{cases}$$

In the same way, we assume a suitable quantifier Q_2 to show the concept of the individual consensus importance fuzzy majority. $PG_{ij}^{Q_2}$ is denoted the importance degree of the partial individuals , who have take s_t as their coincident ideas in the $R(i,j)$, and ,

$PG_{ij}^{Q_2}[s_t] = Q_2(I_{ij}^G[s_t]) \in S$

3.3. The comparison of the alternative pairs

Now, we can make a comparison between the any two alternatives about the feasibility through the above process. Then an array of the available alternative will be obtained.

4. Example

In this Section, we will take an example to demonstrate the model how to solve problem under the circumstance of a $MAGDM$. Let us suppose an investment company, which wants to invest a sum of money in the best option.There is a panel with five possible options in which to invest the money:

x_1 is a car company,
x_2 is an arms company,
x_3 is a tool company,
x_4 is a food company,
x_5 is a computer company,

The investment company must make a decision according to five attributes:

a_1 is the risk analysis,
a_2 is the profit analysis,
a_3 is the social-political impact analysis,
a_4 is the environmental impact analysis,
a_5 is the growth analysis.

There are a work group which compose of five experts who have different work experiences.

$D = \{d_1, d_2, d_3, d_4, d_5\}$.

We consider the following nine linguistic label set with their respective associated semantic [1, 5]:

$C = Certain = (0,1,1,0)$
$EL = Extremely\ likely = (0.05, 0.98, 0.99, 0.01)$
$ML = Most\ likely = (0.06, 0.78, 0.92, 0.05)$
$MC = Meaning\ full\ chance$
$= (0.05, 0.6, 0.8, 0.06)$
$IM = It\ may = (0.09, 0.41, 0.58, 0.07)$
$SC = Small\ chance = (0.05, 0.22, 0.36, 0.06)$
$VLC = Very\ low\ chance = (0.06, 0.1, 0.18, 0.05)$
$EU = Extremely\ unlikely$
$= (0.01, 0.01, 0.02, 0.05)$
$I = Impossible = (0,0,0,0)$

The linguistic labels set S is defined as following:

$S = \{s_8, s_7, s_6, s_5, s_4, s_3, s_2, s_1, s_0\}$
and,
$C = s_8,\ EL = s_7,\ ML = s_6,$
$MC = s_5,\ IM = s_4,\ SC = s_3,$
$VLC = s_2,\ EU = s_1,\ I = s_0$

At first, the important labels of the individuals and the relevances of the labels are:

$\mu_L(1) = s_8,\ \mu_L(2) = s_6,\ \mu_L(3) = s_4,$
$\mu_L(4) = s_2,\ \mu_L(5) = s_0$

$\mu_G(1) = 0.6,\ \mu_G(2) = 0.4,\ \mu_G(3) = 0.3,$
$\mu_G(4) = 0.2,\ \mu_G(5) = 0.1$

Let five individuals be, whose linguistic preference relations under five different attributes using the above label set are,

the linguistic preference relations of the individual d_1:

$$P_1^1 = \begin{pmatrix} - & s_3 & s_5 & s_2 & s_4 \\ s_5 & - & s_4 & s_4 & s_3 \\ s_3 & s_4 & - & s_2 & s_5 \\ s_6 & s_4 & s_6 & - & s_1 \\ s_4 & s_5 & s_3 & s_7 & - \end{pmatrix} \quad P_2^1 = \begin{pmatrix} - & s_6 & s_4 & s_2 & s_6 \\ s_2 & - & s_5 & s_4 & s_1 \\ s_4 & s_3 & - & s_2 & s_3 \\ s_6 & s_4 & s_6 & - & s_7 \\ s_2 & s_7 & s_5 & s_1 & - \end{pmatrix}$$

$$P_3^1 = \begin{pmatrix} - & s_2 & s_5 & s_0 & s_3 \\ s_6 & - & s_6 & s_4 & s_2 \\ s_3 & s_2 & - & s_2 & s_6 \\ s_8 & s_4 & s_6 & - & s_7 \\ s_5 & s_6 & s_2 & s_1 & - \end{pmatrix} P_4^1 = \begin{pmatrix} - & s_3 & s_5 & s_3 & s_6 \\ s_5 & - & s_2 & s_3 & s_1 \\ s_3 & s_6 & - & s_2 & s_4 \\ s_5 & s_5 & s_6 & - & s_0 \\ s_2 & s_7 & s_4 & s_8 & - \end{pmatrix}$$

$$P_5^1 = \begin{pmatrix} - & s_3 & s_5 & s_2 & s_4 \\ s_5 & - & s_6 & s_1 & s_7 \\ s_3 & s_2 & - & s_2 & s_3 \\ s_6 & s_7 & s_6 & - & s_4 \\ s_4 & s_1 & s_5 & s_4 & - \end{pmatrix}$$

the linguistic preference relations of the individual d_2:

$$P_1^2 = \begin{pmatrix} - & s_3 & s_4 & s_6 & s_4 \\ s_5 & - & s_3 & s_5 & s_7 \\ s_4 & s_5 & - & s_2 & s_1 \\ s_2 & s_3 & s_6 & - & s_4 \\ s_4 & s_1 & s_7 & s_4 & - \end{pmatrix} P_2^2 = \begin{pmatrix} - & s_2 & s_3 & s_6 & s_7 \\ s_6 & - & s_5 & s_6 & s_3 \\ s_5 & s_3 & - & s_1 & s_4 \\ s_2 & s_2 & s_7 & - & s_5 \\ s_1 & s_5 & s_4 & s_3 & - \end{pmatrix}$$

$$P_3^2 = \begin{pmatrix} - & s_2 & s_1 & s_0 & s_6 \\ s_6 & - & s_5 & s_7 & s_2 \\ s_7 & s_3 & - & s_4 & s_3 \\ s_8 & s_1 & s_4 & - & s_1 \\ s_2 & s_6 & s_5 & s_7 & - \end{pmatrix} P_4^2 = \begin{pmatrix} - & s_2 & s_1 & s_7 & s_3 \\ s_6 & - & s_4 & s_5 & s_6 \\ s_7 & s_4 & - & s_6 & s_3 \\ s_1 & s_3 & s_2 & - & s_2 \\ s_5 & s_2 & s_5 & s_6 & - \end{pmatrix}$$

$$P_5^2 = \begin{pmatrix} - & s_2 & s_4 & s_1 & s_4 \\ s_6 & - & s_6 & s_1 & s_0 \\ s_4 & s_2 & - & s_3 & s_4 \\ s_7 & s_7 & s_5 & - & s_5 \\ s_4 & s_8 & s_4 & s_3 & - \end{pmatrix}$$

the linguistic preference relations of the individual d_3:

$$P_1^3 = \begin{pmatrix} - & s_4 & s_6 & s_7 & s_4 \\ s_4 & - & s_4 & s_0 & s_3 \\ s_2 & s_4 & - & s_2 & s_1 \\ s_1 & s_8 & s_6 & - & s_0 \\ s_4 & s_5 & s_7 & s_8 & - \end{pmatrix} P_2^3 = \begin{pmatrix} - & s_2 & s_6 & s_7 & s_3 \\ s_6 & - & s_4 & s_6 & s_3 \\ s_2 & s_4 & - & s_0 & s_2 \\ s_1 & s_2 & s_8 & - & s_1 \\ s_5 & s_5 & s_6 & s_7 & - \end{pmatrix}$$

$$P_3^3 = \begin{pmatrix} - & s_4 & s_5 & s_1 & s_4 \\ s_4 & - & s_7 & s_3 & s_5 \\ s_3 & s_1 & - & s_4 & s_3 \\ s_7 & s_5 & s_4 & - & s_2 \\ s_4 & s_3 & s_5 & s_6 & - \end{pmatrix} P_4^3 = \begin{pmatrix} - & s_3 & s_5 & s_0 & s_2 \\ s_5 & - & s_7 & s_1 & s_3 \\ s_3 & s_1 & - & s_2 & s_4 \\ s_8 & s_7 & s_6 & - & s_1 \\ s_6 & s_5 & s_4 & s_7 & - \end{pmatrix}$$

$$P_5^3 = \begin{pmatrix} - & s_3 & s_5 & s_0 & s_6 \\ s_5 & - & s_7 & s_1 & s_2 \\ s_3 & s_1 & - & s_2 & s_3 \\ s_8 & s_7 & s_6 & - & s_4 \\ s_2 & s_6 & s_5 & s_4 & - \end{pmatrix}$$

the linguistic preference relations of the individual d_4:

$$P_1^4 = \begin{pmatrix} - & s_3 & s_2 & s_6 & s_7 \\ s_5 & - & s_1 & s_7 & s_4 \\ s_6 & s_7 & - & s_4 & s_5 \\ s_2 & s_1 & s_4 & - & s_2 \\ s_1 & s_4 & s_3 & s_6 & - \end{pmatrix} P_2^4 = \begin{pmatrix} - & s_6 & s_6 & s_2 & s_4 \\ s_2 & - & s_6 & s_0 & s_3 \\ s_2 & s_2 & - & s_1 & s_4 \\ s_6 & s_8 & s_7 & - & s_5 \\ s_4 & s_5 & s_4 & s_3 & - \end{pmatrix}$$

$$P_3^4 = \begin{pmatrix} - & s_4 & s_2 & s_3 & s_6 \\ s_4 & - & s_6 & s_5 & s_4 \\ s_6 & s_2 & - & s_4 & s_3 \\ s_5 & s_3 & s_4 & - & s_2 \\ s_2 & s_4 & s_5 & s_6 & - \end{pmatrix} P_4^4 = \begin{pmatrix} - & s_5 & s_5 & s_7 & s_6 \\ s_3 & - & s_6 & s_4 & s_2 \\ s_3 & s_2 & - & s_3 & s_1 \\ s_1 & s_4 & s_5 & - & s_3 \\ s_2 & s_6 & s_7 & s_5 & - \end{pmatrix}$$

$$P_5^4 = \begin{pmatrix} - & s_3 & s_5 & s_7 & s_4 \\ s_5 & - & s_6 & s_4 & s_3 \\ s_3 & s_2 & - & s_3 & s_2 \\ s_1 & s_4 & s_5 & - & s_1 \\ s_4 & s_5 & s_6 & s_7 & - \end{pmatrix}$$

the linguistic preference relations of the individual d_5:

$$P_1^5 = \begin{pmatrix} - & s_5 & s_7 & s_0 & s_3 \\ s_3 & - & s_1 & s_4 & s_2 \\ s_1 & s_7 & - & s_2 & s_4 \\ s_8 & s_4 & s_6 & - & s_3 \\ s_5 & s_6 & s_4 & s_5 & - \end{pmatrix} P_2^5 = \begin{pmatrix} - & s_2 & s_6 & s_7 & s_5 \\ s_6 & - & s_4 & s_0 & s_3 \\ s_2 & s_4 & - & s_2 & s_2 \\ s_1 & s_8 & s_6 & - & s_1 \\ s_3 & s_5 & s_6 & s_7 & - \end{pmatrix}$$

$$P_3^5 = \begin{pmatrix} - & s_2 & s_1 & s_1 & s_2 \\ s_6 & - & s_5 & s_4 & s_3 \\ s_7 & s_3 & - & s_2 & s_3 \\ s_7 & s_4 & s_6 & - & s_2 \\ s_6 & s_5 & s_5 & s_6 & - \end{pmatrix} P_4^5 = \begin{pmatrix} - & s_7 & s_0 & s_2 & s_5 \\ s_1 & - & s_5 & s_4 & s_3 \\ s_8 & s_3 & - & s_7 & s_6 \\ s_6 & s_4 & s_1 & - & s_1 \\ s_3 & s_5 & s_2 & s_7 & - \end{pmatrix}$$

$$P_5^5 = \begin{pmatrix} - & s_3 & s_4 & s_6 & s_7 \\ s_5 & - & s_2 & s_1 & s_0 \\ s_4 & s_6 & - & s_4 & s_3 \\ s_2 & s_7 & s_4 & - & s_0 \\ s_1 & s_8 & s_5 & s_8 & - \end{pmatrix}$$

Then, we get the preference relation $R(1,2)$ of the alternative pair (x_1, x_2), is represented following:

$$R_{(1,2)} = \begin{pmatrix} S_3 & S_6 & S_2 & S_3 & S_3 \\ S_3 & S_2 & S_2 & S_2 & S_2 \\ S_4 & S_2 & S_4 & S_3 & S_3 \\ S_3 & S_6 & S_4 & S_5 & S_3 \\ S_5 & S_2 & S_2 & S_7 & S_3 \end{pmatrix}$$

According to the process of the model, we can get some important data at the first step.

$\langle A \rangle$. The individual consensus preference degree

To be more easy to get the result, we can calculate the total number of the preference labels in $R(1,2)$.

The $R(1,2)$ can be expressed the five column vectors,

$R(1,2) = \{R_1, R_2, R_3, R_4, R_5\}$

$\sum_{l=1}^{5} R_{12}^{cl}[s_3] = 9, \quad \sum_{l=1}^{5} R_{12}^{cl}[s_2] = 8,$

$\sum_{l=1}^{5} R_{12}^{cl}[s_4] = 3, \quad \sum_{l=1}^{5} R_{12}^{cl}[s_5] = 2,$

$\sum_{l=1}^{5} R_{12}^{cl}[s_6] = 2, \quad \sum_{l=1}^{5} R_{12}^{cl}[s_7] = 1$

then,

$R_{12}[s_3] = \{1,4,5\}, R_{12}[s_2] = \{2,3\},$

$R_{12}[s_4] = \{3\}, \qquad R_{12}[s_6] = \{2\}$

$R_{12}^{c1}[s_3] = 3, R_{12}^{c4}[s_3] = 2, R_{12}^{c5}[s_3] = 4$

$R_{12}^{c2}[s_2] = 3, R_{12}^{c3}[s_2] = 3 \; R_{12}^{c3}[s_4] = 2$

$R_{12}^{c2}[s_6] = 2$

Now we can get the consensus degree of the individual.

$I_{12}^{C}[s_3] = (\frac{3}{5} + \frac{2}{5} + \frac{4}{5})/3 = 0.6$

$I_{12}^{C}[s_2] = (\frac{3}{5} + \frac{3}{5})/2 = 0.6$

$I_{12}^{C}[s_4] = \frac{2}{5} = 0.4$

$I_{12}^{C}[s_6] = \frac{2}{5} = 0.4$

We shall use the linguistic quantifier $Q_1 =$ "*At least half*" with the pair $(0, 0.5)$ to measure the consensus preference degree of the five individuals under five different attributes.

$PC_{12}^{Q_1}[s_3] = Q_1(0.6) = C$

$PC_{12}^{Q_1}[s_2] = Q_1(0.6) = C$

$PC_{12}^{Q_1}[s_4] = Q_1(0.4) = ML$

$PC_{12}^{Q_1}[s_6] = Q_1(0.4) = ML$

Remark 1:

1) According to some attributes, there are *at least half* experts reach an agreement that the alternative x_1 has small chance available over x_2.

2) According to some attributes, there are *at least half* experts reach an agreement that the alternative x_1 has *very low chance* available over x_2.

3) According to some attributes, there are *at most half* experts reach an agreement that the alternative x_1 *may* available over x_2.

4) According to some attributes, there are *at most half* experts reach an agreement that the alternative x_1 has *mostly likely* available over x_2.

$\langle B \rangle$. The individual consensus importance degree

As we have mentioned in Section 3, there is a representation as following:

$$R(1,2) = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{pmatrix}$$

By calculating ,we get the following equations easily:

$V_{12}[s_3] = \{1,2,3,4,5\}, V_{12}[s_2] = \{1,2,3,5\},$

$V_{12}[s_4] = \{3,4\}$

$V_{12}^{c1}[s_3] = 3, V_{12}^{c2}[s_3] = 1, V_{12}^{c3}[s_3] = 2,$

$V_{12}^{c4}[s_3] = 2, V_{12}^{c5}[s_3] = 1$

$V_{12}^{c1}[s_2] = 1, V_{12}^{c2}[s_2] = 4,$

$V_{12}^{c3}[s_2] = 1, V_{12}^{c5}[s_2] = 2$

$V_{12}^{c3}[s_4] = 2, \; V_{12}^{c4}[s_4] = 1$

Now,we can get the individual important degree I_{12}^{G} ,

$$I_{12}^{G}[s_3] = \frac{6}{10} \times \frac{3}{5} + \frac{4}{10} \times \frac{1}{5} + \frac{3}{10} \times \frac{2}{5} + \frac{2}{10} \times \frac{2}{5} + \frac{1}{10} \times \frac{1}{5} = 0.66$$

$$I_{12}^{G}[s_2] = \frac{6}{10} \times \frac{1}{5} + \frac{4}{10} \times \frac{4}{5} + \frac{3}{10} \times \frac{1}{5} + \frac{1}{10} \times \frac{2}{5} = 0.54$$

$$I_{12}^{G}[s_4] = \frac{3}{10} \times \frac{2}{5} + \frac{2}{10} \times \frac{1}{5} = 0.16$$

We shall use the linguistic quantifier $Q_2 =$ "*Most likely*" with the pair $(0.3, 0.6)$ to measure the importance degree of the individuals who play an important role during the decision making.

$PG_{12}^{Q_2}[s_3] = Q_2(0.66) = C$

$PG_{12}^{Q_2}[s_2] = Q_2(0.54) = ML$

$PG_{12}^{Q_2}[s_4] = Q_2(0.16) = I$

Remark 2:

1) According to some attributes , there are *at least half* experts who have *certainly* work experiences reach an agreement that the alternative x_1 has *small chance* available over x_2.

2) According to some attributes , there are *at least half* experts who have *most likely* work experiences reach an agreement that the alternative x_1 has *very low chance* available over x_2.

3) According to some attributes , there are *at most half* experts who *impossible* have work experiences reach an agreement that the alternative x_1 has *may* available over x_2.

$\langle C \rangle$. The comparison between the alternative pair (x_1, x_2)

After comparison, we find the alternative x_1 has low available than the alternative x_2. It is expressed by $x_1 < x_2$. Similarly, we can get the compared result of the other any two alternatives through the above model. They are $x_3 < x_5$, $x_4 < x_5$, $x_4 < x_1$, $x_3 < x_4$, $x_3 < x_2$, $x_2 < x_5$. Obviously, a nondecreasing permutation of the available alternatives is showed as following:

$x_3 < x_4 < x_1 < x_2 < x_5$

5. Conclusion

In this paper, we have presented a model to solve some problems under Multi-attributes Group decision making. By using fuzzy linguistic approach, all the relevant results are expressed by means of the natural language. We are allowed to use some linguistic variables to express our opinions and preferences, it more fits the habits of the human being and makes the communication among individuals more conveniently. The model mentioned in this paper has a very wide applied areas.

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