Transfer-probability Matrix of Repairable parallel-series d System

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Abstract

From the fact that a parallel-series system made up of components has a complicated state space and transfer-probability matrix, the element classification and structural characteristic of the transfer-probability matrix of a parallel-series system with only a set of repair equipment were investigated. To describe the change of the transfer-probability matrix, the concept of growing matrix was introduced, and the change regularity of the transfer-probability matrix, resulted from the variation of components in the system, was obtained.

Keywords: Reliability, Transfer-probability Matrix, Repairable Parallel-series System.

1. Introduction

There are a lot of studies on the reliability of repairable system (refer to [1-5]). For example,[2] introduces the basic analysis method of Markov decision method—the decision of system situation transition probability matrix. [3] studies a circular consecutive-2-out-of-n:F repairable system, using the definition of generalized transition probability and the rule with priority to repair the key component, the state transition probability of the system is derived. To a given repairable parallel-series system, its reliability index has something to do with state space and transfer-probability matrix [6,7]. Many methods are introduced to calculate the reliability of repairable parallel-series system. However, when the system has changed the analysis of the system's reliability is relatively few. Specifically, if a part of the system changed, its reliability will change correspondingly. A general method to calculate its value of change is to reanalyze its state space, write out its transfer-probability matrix, and then recalculate its reliability function. This method is complicated and not applicable in practical use. In order to solve this problem, this thesis introduces some new concepts, studies the element classification and structural relationship of system's transfer-probability matrix, especially, summarizes the law of change of transfer-probability matrix when some components of a system have changed. To be specifically, the transfer-probability matrix changes on the basis of the original matrix when a subsystem is connected to a given repairable parallel-series system in parallel or units are added to certain subsystem in parallel. The results and introduced concepts can not only make calculation easier, but also be applied to further analyze the reliability of large and complicated net systems.

2. Model Hypothesis

Hypothesis of studied system in the thesis are as follows:

(1) Components' life distributions of the system and fixing time distributions after repairing, and other relative distributions being index distributions, all the random variables related to these distributions are independent.

(2) Components work and are repaired under the guidance that components will be fixed according to the time they break down and be put to use as soon as they are fixed.

(3) Suppose that at the beginning, every component is new and looks like new even after repairing.

(4) Ignore the "cost" of linkage and failure influence among each unit and phrases.

3. Concepts

We start with the simplest example in order to make the description easier.

The system of the reliability block-diagram (See Fig. 1.) is connected by two subsystems in series, the first subsystem being connected by two units in parallel while the second subsystem by one unit. The failure rate and repair rate of the three units are $\lambda_{11}, \lambda_{12}, \lambda_2$ and $\mu_{11}, \mu_{12}, \mu_2$ respectively. A maintenance worker is responsible for repairing.

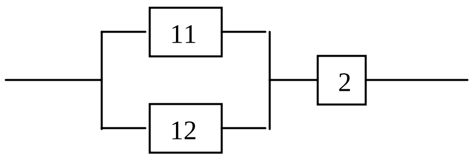

Fig. 1. Reliability block-diagram

According to the concepts of family, growth point and terminal point, and the calculating method of unit [8], we can easily write out the state space, draw the state transfer graph and then write out the transfer-probability matrix.

$$T = \left(\begin{array}{c|c|c|ccccc}
1-(\lambda_{11}+\lambda_{12}+\lambda_2) & \lambda_{11} & \lambda_{12} & \lambda_2 & 0 & 0 & 0 & 0 \\
\hline
\mu_{11} & 1-(\lambda_{12}+\lambda_2+\mu_{11}) & 0 & 0 & \lambda_{12} & \lambda_2 & 0 & 0 \\
\mu_{12} & 0 & 1-(\lambda_{11}+\lambda_2+\mu_{12}) & 0 & 0 & 0 & \lambda_{11} & \lambda_2 \\
\hline
\mu_2 & 0 & 0 & 1-\mu_2 & 0 & 0 & 0 & 0 \\
0 & \mu_{12} & 0 & 0 & 1-\mu_{12} & 0 & 0 & 0 \\
0 & \mu_2 & 0 & 0 & 0 & 1-\mu_2 & 0 & 0 \\
0 & 0 & \mu_{11} & 0 & 0 & 0 & 1-\mu_{11} & 0 \\
0 & 0 & \mu_2 & 0 & 0 & 0 & 0 & 1-\mu_2
\end{array}\right)$$

$$= \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix}$$

Definition 1: Partitioned matrices A_{00} , A_{11} ,…, $A_{D-m+1,D-m+1}$ on the diagonal line form a branch chain, each matrix on which is diagonal matrix. A_{00} is the root matrix block of the first order square matrix. Suppose:

$$\sum_{i=1}^{m}\sum_{j=1}^{n_i}\lambda_{ij} = \lambda_{11} + \cdots + \lambda_{1n_1} + \cdots + \lambda_{m1} + \lambda_{mn_m} \underline{\underline{\Delta}} \lambda$$

Thus, $A_{00} \equiv 1-\lambda$; A_{11} is the first matrix block of growth point, its orders being the sum of all first growth points in the family. A_{22} is the second matrix block of growth point, its orders being the sum of all second growth points in the family; …; $A_{D-m+1,D-m+1}$ is the matrix block of terminal point, its orders being the sum of all terminal points in the family. Among which, each A_{ll} ($l = 1, 2, \cdots, D-m$) is formed by a series of diagonal matrices— A_{ll}^{i} ($i = 1, 2, \cdots, m$) along the diagonal line. A_{ll}^{i} is the l-th matrix block of growth point in the family i , its orders being B_{il} .

Definition 2: $A_{ij}\,(i < j)$ is a matrix block of failure rate, its elements being 0 or the failure rate of the unit. $A_{ij}\,(i > j)$ is a matrix block of repair rate, its elements being 0 or the repair rate of the unit.

Definition 3: Elements' positions of the matrix block of failure rate and the matrix block of repair rate are symmetrical on the branch chain, however, the element— λ_{st} in the matrix block of failure rate is μ_{st} in the matrix block of repair rate, which is called symmetry-like.

Besides, other definitions are given.

Definition 4: Due to the change of orders, matrix $m_1 \times n_1$ changes to matrix $m_2 \times n_2$, and $m_1 \le m_2$, $n_1 \le n_2$ (or $m_1 \ge m_2$, $n_1 \ge n_2$).Such change is named the growth (or degeneration) of the matrix.

Definition 5: Inserting a matrix between row i , $i+1$ and line j , $j+1$ is called the insertion of matrix.

If the number of subsystem or the units in certain subsystem changed, the transfer-probability matrix will grow (or degenerate) while $A_{ij}\,(i \ne j)$ grows (or degenerate) according to the growth (or degeneration) of A_{ii} . We can deduce form the given block T, after determining the growth of A_{ii} , the numbers of rows and lines of $A_{ij}\,(i \ne j)$ change correspondingly. Next, the growth and degeneration of diagonal matrix is discussed below.

We have to insert line $\delta = |m-n|$ after Line l Row l in square matrix A_m to form square matrix A_n. Suppose:

$$P(n\times m, l, \delta) = P^{l}_{n\times m}(\delta)$$

$$\triangleq \begin{pmatrix} & & l & & & \\ 1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & & \vdots \\ 0 & \cdots & 1 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 1 & \cdots & 0 \\ \vdots & & \vdots & \vdots & & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 1 \end{pmatrix}$$

$$= \begin{pmatrix} I_l & 0 \\ 0 & 0 \\ 0 & I_{m-l} \end{pmatrix}$$

Definition 6: In the equality listed below:

$$A_n = P(n\times m, l, \delta)\cdot A_m \cdot P'(n\times m, l, \delta)$$

$P(n\times m, l, \delta)$ is called growth matrix, while $P'(n\times m, l, \delta)$ degenerate matrix.

When $m<n$,

$$A_n = P(n\times m, l, \delta)\cdot A_m \cdot P'(n\times m, l, \delta) \qquad (1)$$

When $m>n$,

$$A_n = P'(n\times m, l, \delta)\cdot A_m \cdot P(n\times m, l, \delta) \qquad (2)$$

The way to use two matrices — A_{ik} and B_{il} to write out matrix $C_{i,k+l}$ is called the chaining of matrix.

$$C_{i,k+l} = \begin{pmatrix} a_{11} & \cdots & a_{1k} & b_{11} & \cdots & b_{1l} \\ \vdots & & \vdots & \vdots & & \vdots \\ a_{i1} & \cdots & a_{ik} & b_{i1} & \cdots & b_{il} \end{pmatrix} = \left(c_{ij}\right)_{i\times(k+l)}$$

4. Main Results

Next, let's see how T changes as the subsystem of the system or units' number of certain subsystem changed.

Example 1 Add a unit in series on the second subsystem of the reliability block diagram (See Fig.2.). Then we get the reliability block diagram of new system (See Fig.3.).

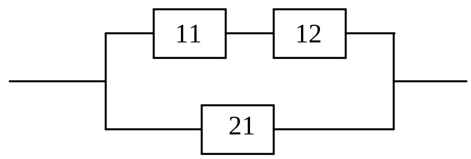

Fig.2. Reliability block-diagram of original System.

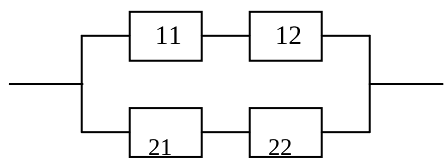

Fig. 3. Reliability block-diagram with a unit added

Suppose.

$$\lambda_i = \sum_{j=1}^{n_i} \lambda_{ij}$$

Then in the original system $\lambda_1 = \lambda_{11} + \lambda_{12}$, $\lambda_2 = \lambda_{21}$

And in the new system

$$T_o = \left[\begin{array}{c|ccc|cccc} 1-\lambda & \lambda_{11} & \lambda_{12} & \lambda_{21} & 0 & 0 & 0 & 0 \\ \hline \mu_{11} & 1-(\lambda_2+\mu_{11}) & 0 & 0 & \lambda_{21} & 0 & 0 & 0 \\ \mu_{12} & 0 & 1-(\lambda_2+\mu_{12}) & 0 & 0 & \lambda_{21} & 0 & 0 \\ \mu_{21} & 0 & 0 & 1-(\lambda_1+\mu_{21}) & 0 & 0 & \lambda_{11} & \lambda_{12} \\ \hline 0 & \mu_{21} & 0 & 0 & 1-\mu_{21} & 0 & 0 & 0 \\ 0 & 0 & \mu_{21} & 0 & 0 & 1-\mu_{21} & 0 & 0 \\ 0 & 0 & 0 & \mu_{11} & 0 & 0 & 1-\mu_{11} & 0 \\ 0 & 0 & 0 & \mu_{12} & 0 & 0 & 0 & 1-\mu_{12} \end{array}\right]$$

T_n =

$$\left(\begin{array}{c|cccc|cccccccc}
1-\lambda & \lambda_{11} & \lambda_{12} & \lambda_{21} & \lambda_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
\mu_{11} & 1-(\lambda_2+\mu_{11}) & 0 & 0 & 0 & \lambda_{21} & \lambda_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\
\mu_{12} & 0 & 1-(\lambda_2+\mu_{12}) & 0 & 0 & 0 & 0 & \lambda_{21} & \lambda_{22} & 0 & 0 & 0 & 0 \\
\mu_{21} & 0 & 0 & 1-(\lambda_1+\mu_{21}) & 0 & 0 & 0 & 0 & 0 & \lambda_{11} & \lambda_{12} & 0 & 0 \\
\mu_{22} & 0 & 0 & 0 & 1-(\lambda_1+\mu_{22}) & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{11} & \lambda_{12} \\
\hline
0 & \mu_{21} & 0 & 0 & 0 & 1-\mu_{21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \mu_{22} & 0 & 0 & 0 & 0 & 1-\mu_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \mu_{21} & 0 & 0 & 0 & 0 & 1-\mu_{21} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \mu_{22} & 0 & 0 & 0 & 0 & 0 & 1-\mu_{22} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \mu_{11} & 0 & 0 & 0 & 0 & 0 & 1-\mu_{11} & 0 & 0 & 0 \\
0 & 0 & 0 & \mu_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 1-\mu_{12} & 0 & 0 \\
0 & 0 & 0 & 0 & \mu_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 1-\mu_{11} & 0 \\
0 & 0 & 0 & 0 & \mu_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1-\mu_{12}
\end{array}\right)$$

We can find out that the form of A_{00} is the same on T_n and T_o, only the value of λ changed. A_{11} in T_n is grown from A_{11} in T_o . The position and form of growth can be deduced from the growth matrix, to be specific:

$$B_{11} = A_{11} - \lambda_{22} \cdot I$$

$$C_{11} = P(4\times 3,3,1)\cdot B_{11}\cdot P'(4\times 3,3,1)$$

Then in the new system

$$A_{11} = C_{11} + [1-(\lambda_1+\mu_{22})]\cdot \delta_{44}$$

If the system contains $A_{22}, \ldots, A_{m-1,m-1}$, the growth method is similar to that of A_{11}.We can use the way applied in the series-parallel system to determine $A_{m,m}$ and $A_{ij}\,(i \neq j)$, or use another way. After determining A_{11} , A_{22} ,…, $A_{m-1,m-1}$, we fix A_{ij} $(i < j)$ first, then each row in the matrix should meet two requirements: First, the sum of all the row elements in T is 1, i.e., all the failure rates on A_{ii} should appear. Second, the line on which the first non-zero element of this row exists should be correspond to the line on which the first zero element of the upper line exists. Having $A_{ij}\,(i < j)$ and according to the similarly symmetry between $A_{ij}\,(i > j)$ and $A_{ij}\,(i < j)$, we can write out the matrix block of repair rate. Then according to the rule that the sum of all the elements in each row of T is 1, we can obtain a diagonal matrix $A_{m,m}$ — a terminal matrix block.

5. Conclusions

For a system with the states transferring having Markov quality the transfer-probability matrix is the basis of analyzing system reliability after verifying the state space. State-transferring diagram and transfer-probability matrix are the cruxes of solving the repairable compound system reliability.

In the article, according to the fact that the parallel-series systems made up of many components have complicated state space and transfer-probability matrix, some new concepts are introduced. On the basis of these concepts, classifies the complicated state space well-organized, describes the structure of the state space and gets the quantity relation of each state. Thus, a ration description for compound system state-transferring diagram is obtained which greatly reduces the analyses and operation complexity.

At the same time, to the discussed systems, the element classification and the structure relation of systems' transfer -probability matrix under the circumstance of repairable equipment has been studied. Especially, brings forward a method—matrix growing to describe the change of transfer- probability matrix and gets the change law of transfer-probability matrix when some components of a system have changed.

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