A Practical Method to Calculate the Prestress of Suspendome Structures

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Abstract. A practical method to calculate the prestressing forces of suspendome structures using force method is put forward in this paper. For a suspendome structure having n hoop cables, n control points can be selected. A system of linear homogeneous equations with n unknown cable forces can be obtained through the vertical displacements of n control points under the vertical loads and each hoop cable's unit prestress. The coefficients of the equations can be obtained through FEA software using force method principle. In the end, a numerical example is given to describe the calculating process and verify the validity of the proposed method.

Introduction

Suspendome structure is a structural system combined with prestressed cables and single-layer latticed shell, which is proposed by M.Kawaguchi, a Japanese scholar, for the first time in 1993. At present, almost twenty large engineering projects have adopted suspendome structures in China, such as Badminton Gymnasium for 2008 Beijing Olympic Games [2,3] and Ji'nan Olympic Sports Center Gymnasium [4].

Based upon traditional latticed shells, suspendome structures can be formed by adding struts, hoop cables and radial cables inside (Fig. 1). Usually, there are double radial cables for each strut, it can also be a single one. Suspendome structures with double radial cables are more stable, so they are used in most practical engineering.

(a) Single-layer Latticed Shell   (b) Struts, Hoop Cables and Radial Cables
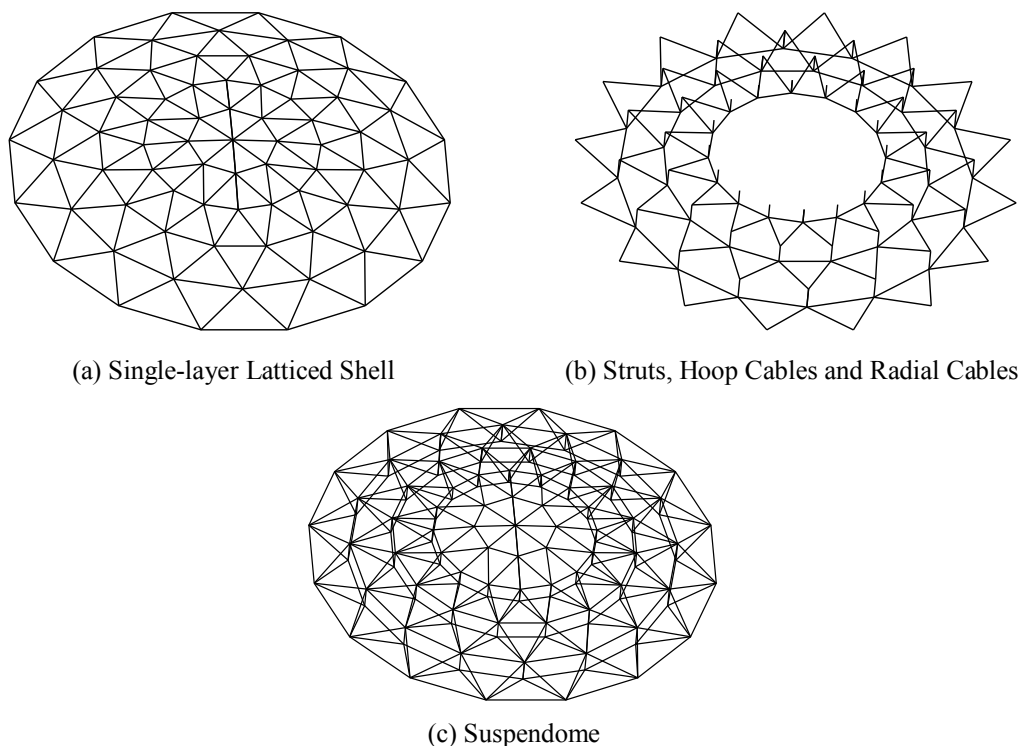
(c) Suspendome

Fig. 1 Suspendome Structure Schematic Diagram

Suspendome structure design is nothing but determining the structural shape and computing the prestressing force in cables, which can also be called shape-state analysis [5]. Shape-state analysis can be classified into force-finding, form-finding, force-finding and form-finding. However, since the process is complex, so shape-state analysis is not practical for structural design and more suitable for the study on structural performance. In fact, for suspendome design, architectural modeling design is usually determined by the architects, while the major work of the structural engineers is to mesh the roof reticulated shell, arrange the struts, hoop cables and radial cables, and calculate the cable tensions based on loads in the end.

Under the assumption that grids dividing and cable arrangement have been completed, it is aimed at considering ways to compute the prestress in order to make reticulated shell structures more mechanically reasonable. The above can be classified as force-finding according to Ref. [5]. Force-finding is to calculate the prestressing force of hoop cables and radial cables, it is the key to design of the prestressed steel structure and suspendome structure.

In this paper, a simple, targeted method is put forward to calculate the lower chord cable prestress of suspendome structures. It aims to specify the vertical displacements of the control points of roof reticulated shell to a specific value under the vertical loads (dead loads and partial snow load) and hoop cable prestress. The specified displacement value can be zero, in other words, there is no vertical deformations over the control points under the vertical loads and prestressing force. Since this is the control objective of most design of the beam string structures, it has a certain rationality.

The method referred in this paper is clear and easy to calculate. The equations to determine the tension force of each hoop cable can be obtained by using the existing FEA software. According to the formula, each prestressing force value can be computed.

Computation Principle of Force Method

Cable Force Determination Principle

Suspendome composes of the upper chord dome, the lower chord cables and the struts connecting them. The upward equilibrium load can be generated by the lower chord cables towards the upper chord dome through the struts. At the same time, the radial pressures of the upper chord struts are produced by exerting the lower chord cable tension on the upper chord. Through the arch effect, they will generate upward nodal loads. The equilibrium load produced by the lower chord cables is easy to compute, while the vertical equilibrium load generated by the upper chord radial pressure is difficult to calculate. Therefore, it is the core issues to determine the prestressing force and to figure out how to determine the lower chord cable tensions so that the upper dome can be essentially in axial compression state or the vertical displacement of the upper dome under the vertical loads and cable tensions is zero. It can also be used as a fundamental principle of cable tension computation.

(a) Suspendome profile   (b) Reticulated shell under vertical loads

(c) The first hoop cable   (d) The second hoop cable
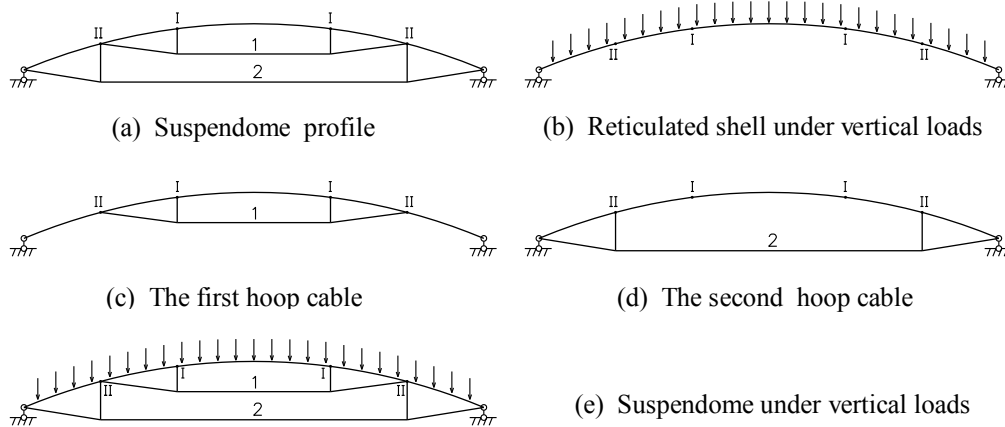
(e) Suspendome under vertical loads

Fig. 2 Computation Principle of Force Method Diagram

Computation principle of force method diagram is shown in Fig. 2. Under the external loads and cable prestresses, the displacements of the lap of nodes including node I are approximately zero, and the same is the case with the lap of nodes including node II. If there are n hoop cables, n laps of control points can be selected. What is mentioned above is the computation principle.

Vertical Displacements of the Upper Chord Control Points under Loads

First, take no account of the influence of the lower chord cables, then calculate the nodal displacements of the upper suspendome under the vertical loads. The calculation model is shown in Fig. 2(b).

The displacements of the control point I, II, III…under the vertical loads can be respectively recorded as: $\Delta_{I\,F}$, $\Delta_{II\,F}$, $\Delta_{III\,F}$…
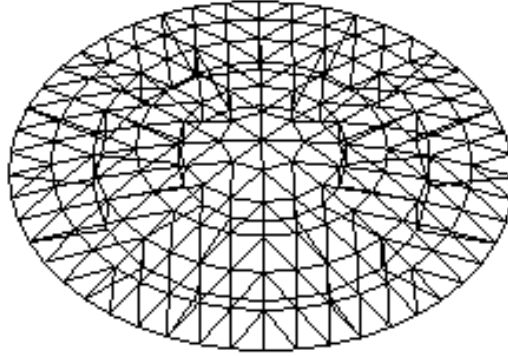

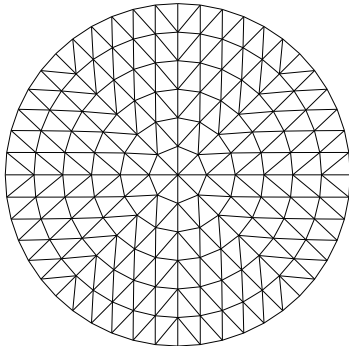Fig. 3 Axonometric Drawing of Suspendome Example
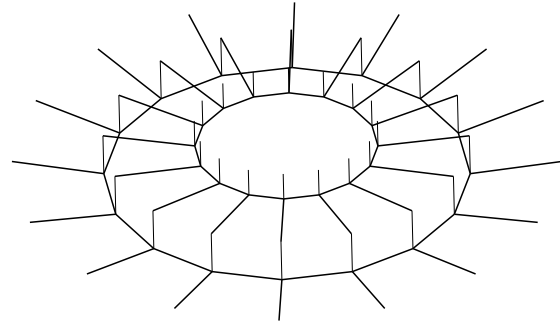

Fig. 4 The Upper Chord Struts Layout

Fig. 5 The Lower Chord Cables and Struts Layout

Vertical Displacements of Control Points under Hoop Cable Tension

Remove all the loads, including the structural gravity. Give an unit prestress on the first hoop cable, the second hoop cable, the third hoop cable…then compute the vertical displacements of the control points.

Take no account of the influence of all the other hoop cables except the first one, exert an unit prestress on the first hoop cable, then calculate the vertical displacements of the control points, these can be respectively recorded as: $\Delta_{I\,1}$, $\Delta_{II\,1}$, $\Delta_{III\,1}$…

Take no account of the influence of all the other hoop cables except the second one, exert an unit prestress on the second hoop cable, then calculate the vertical displacements of the control points, these can be respectively recorded as: $\Delta_{I\,2}$, $\Delta_{II\,2}$, $\Delta_{III\,2}$…

Take no account of the influence of all the other hoop cables except the third one, exert an unit prestress on the third hoop cable, then calculate the vertical displacement of the control points, these can be respectively recorded as: $\Delta_{I\,3}$, $\Delta_{II\,3}$, $\Delta_{III\,3}$…

Equations of Cable Tension Calculation

According to force method principle and prestress determination principle, the equations can be obtained as follows:

$$\begin{cases} \delta_{\mathrm{I}\,1}T_1 + \delta_{\mathrm{I}\,2}T_2 + \delta_{\mathrm{I}\,3}T_3 = -\Delta_{\mathrm{I}\,\mathrm{F}} \\ \delta_{\mathrm{II}\,1}T_1 + \delta_{\mathrm{II}\,2}T_2 + \delta_{\mathrm{II}\,3}T_3 = -\Delta_{\mathrm{IIF}} \\ \delta_{\mathrm{III1}}T_1 + \delta_{\mathrm{III2}}T_2 + \delta_{\mathrm{III3}}T_3 = -\Delta_{\mathrm{IIIF}} \\ \text{.......} \end{cases} \tag{1}$$

Calculating the first, the second, the third…hoop cable tension by solving the equations. The results can be respectively recorded as T_1, T_2, T_3.

Numerical Example

To verify the validity of the developed method, a numerical example is given to detail the calculating process. This case is a preliminary design scheme of a project. The suspendome is 60m in diameter and 6m high. According to the spherical layout, the radial direction is averaged to six, the upper is a Kiewitt-type latticed shell [6,7]. Two hoop cables are arranged, each one has 16 struts. 16 radial single cables can be arranged radially, 16 double cables may also be arranged. This example adopted the radial single cables, as shown in Fig. 3. In the preliminary design, the steel pipes with 200mm in diameter and 10 mm thick are selected for all the upper chord struts.

1) The vertical nodal forces of the roof latticed shell under the structural gravity and the roof dead loads are 20kN. Regardless of the influence of the lower cables, the displacements of the upper latticed shell can be obtained, as shown in Fig. 6. The displacements of the control points respectively are:

the average displacement of the control points of round I is －71.208mm;

the average displacement of the control points of round II is －78.982mm;

from which

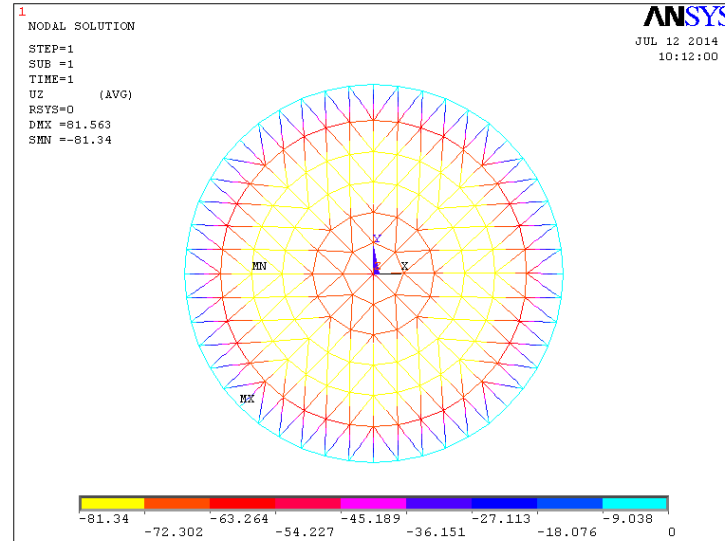$\Delta_{\mathrm{I\,F}} = -71.208$mm, $\Delta_{\mathrm{II\,F}} = -78.982$mm.

Fig. 6 Vertical Displacements of Upper Dome under Vertical Loads (mm)

2) Regardless of the influence of the second hoop cable and exert an initial strain on the first hoop cable from the inside out, then the hoop cable tensions can be calculated. The first hoop cable tension is = 20.026kN, the second is =0. Based on the results above, the displacements of the control points can be obtained as follows:

the average displacement of the control points of round I is 1.127mm;

the average displacement of the control points of round II is －1.113mm;

from which

$\delta_{I\,1} = 1.127/20.026 = 0.0563\text{mm/kN}; \quad \delta_{II\,1} = -1.113/20.026 = -0.0556\text{mm/kN}.$

3) Regardless of the influence of the first hoop cable and exert an initial strain on the second hoop cable from the inside out, then the hoop cable tensions can be calculated. The first hoop cable tension is = 0, the second is =20.250kN. Based on the results above, the displacements of the control points can be obtained as follows:

the average displacement of the control points of round I is 1.715mm;
the average displacement of the control points of round II is 2.337mm;
from which

$\delta_{I\,2} = 1.715/20.250 = 0.0847\text{mm/kN}; \quad \delta_{II\,2} = 2.337/20.250 = 0.1154\text{mm/kN}.$

Eq.(1) may then be written:

$$\begin{cases} 0.0563T_1+0.0847T_2=71.208\text{mm/kN} \\ -0.0556T_1+0.1154T_2=78.982\text{mm/kN} \end{cases} \tag{2}$$

solve the equations above can get the following results:

$T_1= 136.318\text{kN}, \qquad T_2=750.098\text{KN}$

Exert all the nodal loads on the suspendome structure, adjust the first hoop cable tension to 136.4kN and the second hoop cable tension to 750.1kN, then calculate the structural displacements. The results are shown in Fig. 7.
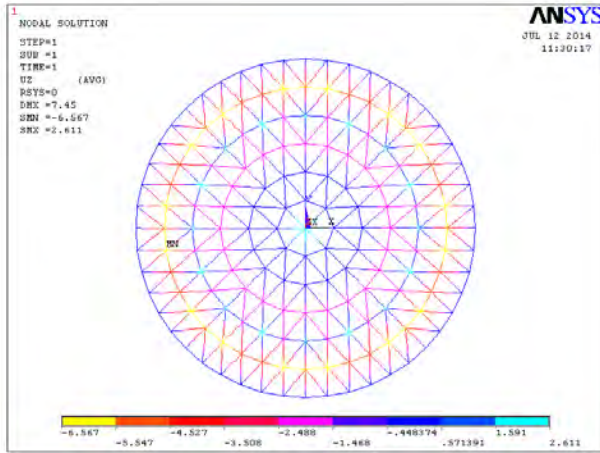

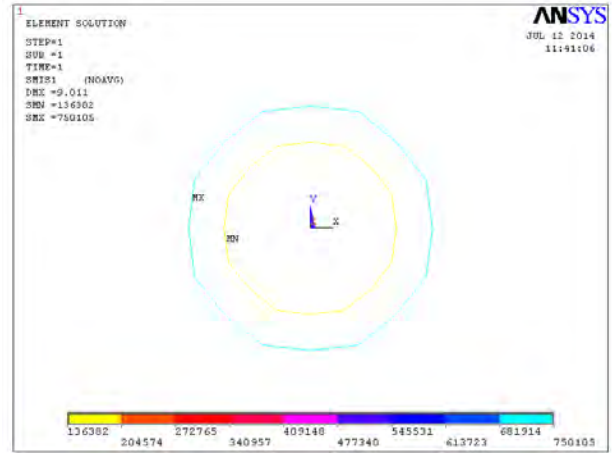Fig. 7 Displacement of the Upper Dome (mm)　　　　Fig. 8 Hoop Cable Tension Diagram (N)

The results are as follows: the vertical displacement of round I is 0.013mm. The vertical displacements of the control points of round II respectively are 2.36mm, 1.73mm, -1.79mm, -1.79mm. The average value is 0.13mm. From Fig. 8 we can see that the first cable tension is 136.382kN and the second is 750.105kN. The results show that displacement control has essentially achieved the goal.

Base on the above results, each cable section can be determined, and design for the prestressed cables can also be completed.

Conclusion

In this paper, a practical method to calculate the lower chord cables prestressing forces of suspendome structures using force method principle is put forward:

1) If there are n hoop cables, n laps of control points can be selected. Equations of force method can be produced by controlling the displacements of the control points of n laps. Generally, the displacements of the control points under prestresses and vertical loads can be basically controlled to zero.

2) Once only letting a hoop cable in tension, then the coefficients of the equations can be obtained by calculating according to the hoop cable tensions and the corresponding displacements.

3) The numerical example given in the paper has verified the validity of the mentioned method. Since the displacements of the control points of each lap did not quite gee, so the results could not quite be controlled to zero. In this example, the average displacement of the control points of the first lap is 0.013mm, the average displacement of the control points of the second lap is 0.13mm, and the results meet the needs of the engineering.

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