Effects of Cable Flexural Rigidity on the Free Vibrations of Suspension Bridges

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Abstract. The natural frequencies of vibration and the corresponding mode shapes of a suspension bridge with varying bending stiffness of cables are determined, and the changes that occur in these characteristics with respect to parametric changes in the bridge are examined. Single span L=64,0 m suspension steel footbridge with hangers and flexible cables is selected as an initial model used to study the dynamic characteristics of a suspension system. Based on FEA parameter studies are presented in which the vibration characteristics are studied as a function of structural and material parameters, such as cable flexural stiffness, modulus of elasticity and mass density of structural components. It is generally found that the main cable bending stiffness contributes a considerable effect on the natural frequencies of the suspension system.

Introduction

The suspension bridges are inherently the flexible structures and in the majority of cases special measures of stiffening must be incorporated in the design. Some suspension bridges have in the past suffered from structural failure [e.g. 1, 2]. It is well known that the suspension footbridges that are long, light and slender structures are particularly susceptible to large vibrations induced by pedestrians and wind actions leading sometimes to serviceability and safety problems [3, 4, 5]. The structural measures commonly adopted to reduce vibrations of bridges consist of local or total structural modifications by changing mainly the stiffness or the mass of a structure as well as through the installation of tuned mass dampers. As is shown in our previous investigations a possible strategy for modification of structural stiffness of a suspension system is the variation of main cable stiffness [6]. By changing of the cable to stiffening girder bending stiffness ratio it is possible to reduce considerably the vertical displacements of the loaded suspension system and to retain its original geometric form. The stabilization of the deformability of the suspension bridges by giving certain bending stiffness to the suspension cables are also known in practice. The famous London Tower Bridge can be mentioned. It should be mentioned also that in the long-span suspension bridges the rigidity of main cables becomes dominant with respect to that of the deck [7]. So far as the authors' know, the investigation on the suspension bridge dynamic response taken into consideration bending stiffness of cables until now was not carried out. This paper treats the free vibration response of suspension steel footbridge with varying flexural rigidity of main cables using FE method.

Footbridge Description FE Model

The effect of main cable bending stiffness on the free vibration characteristics of a suspension bridge was analysed on an example of the real footbridge designed as a suspension steel structure. The footbridge consists of a single span 64 m long and 4 m width superstructure, which includes two main suspension cables, the suspenders and the stiffening deck girder. For computational purposes suspension system was slightly simplified. The suspension bridge model considered in this study is shown in Fig.1.

The following data were used in the numerical simulation: L = 64,0 m, cable sag f_0=6,5 m, width of the deck 4,0 m, Steel suspender rods, of 20 mm diameter equally spaced every 3,0 m. Flexible cables are of 0,05 m diameter. The rigidity of flexible cables is EA_c=41,2x10^4 kN and that of stiffening deck

$EI_{dx}=10{,}7 \times 10^4$ kNm^2 and $EI_{dy}=3870$ kNm^2. For all structural members the basic material is steel with E=210 GPa, ν=0,3 and γ=7850 kg/m^3.
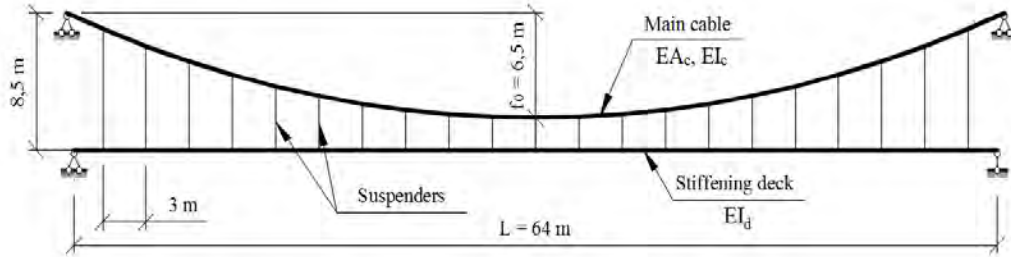

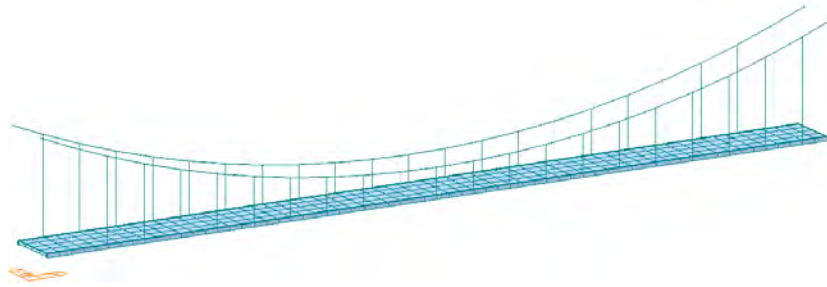Fig. 1 Single Span Suspension Bridge Model

Fig. 2 Finite Element Space Model of the Footbridge

Dynamic analysis of the footbridge was carried out using 3D FE model developed in the program MIDAS Civil, as shown in Fig. 2. The elements TRUSS3D were used to represent the flexible cable and the suspenders. Beam elements BEAM3D were used for the rigid cables and the stiffening girder. The girder deck and the cables are hinged to the tower footings and tower saddles, respectively. The cables are fixed against translation in any direction at the towers.

Modal Parameters of Footbridge with Flexible Cables

Dynamic analysis of a suspension structure was carried out in terms of natural frequencies and mode shapes which are always very important characteristics for serviceability behavior of the footbridges. It is decided to select 9 modes in the frequency range 0-5 Hz to be important for pedestrian induced excitations. These include five vertical-dominant, one lateral-dominant and four tension-dominant modes of the deck (Table 1). The fundamental mode is a bending mode with one node. Note, that during numerical simulations the vertical V1 and torsional T1 modes with one half waves were not identified. All torsional modes are coupled with lateral vibrations. The frequency ratio between the 1st torsional and 1st vertical modes (fT2/fV2=1,812/0,616=2,94) suggested that footbridge has relatively high structural torsional stiffness [8].

The mode shapes of two lowest vertical and two torsional modes are shown in Fig. 3.

Tab.1 Computed Eigen-Frequencies and Eigen-Modes

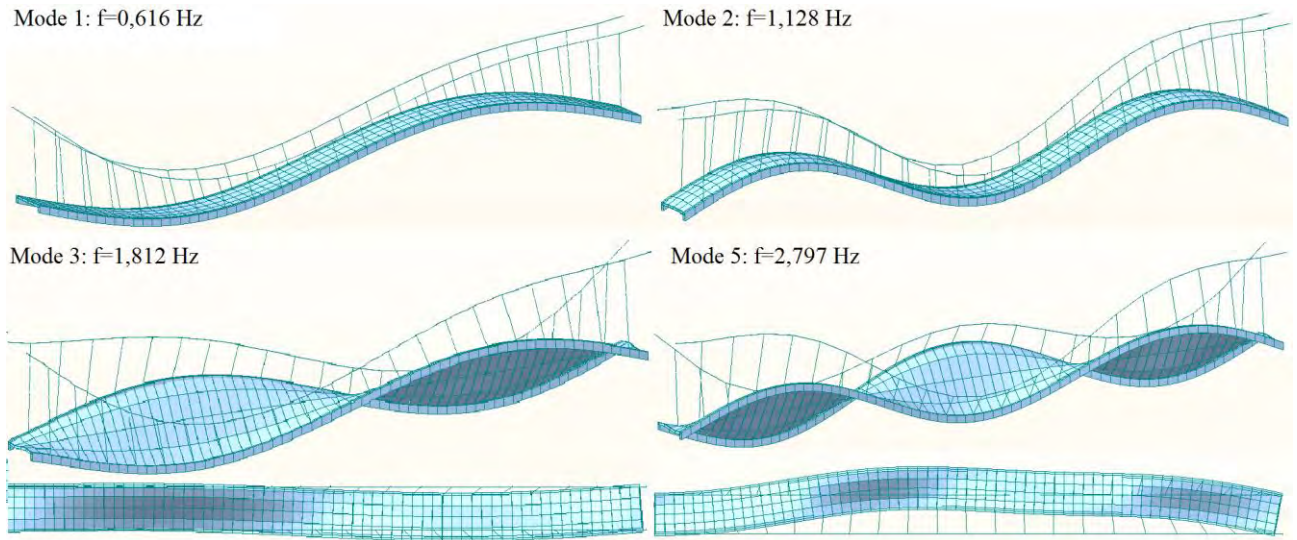| Mode No	Frequency [Hz]	Character of eigen-mode
1 (V2)	0,616	2nd vertical
2 (V3)	1,128	3rd vertical
3 (T2-L)	1,812	2nd torsional-lateral
4 (V4)	2,392	4th vertical
5 (L-T3)	2,797	3rd lateral-torsional
6 (V5)	3,632	5th vertical
7 (T4-L)	3,668	4th torsional-lateral
8 (T5-L)	4,744	5th torsional-lateral
9 (V6)	4,952	6th vertical

Fig. 3 Mode Shapes of the First Two Vertical and Two Coupled Torsional Modes of the Footbridge

Modal Parameters of Footbridge with Varying Rigidity of Cables

The effect of cable flexural stiffness on the modal properties of the footbridge is studied by changing the cable to beam bending stiffness ratio $\xi = EI_c/EI_d = 0{,}2;\ 0{,}5;\ 1{,}0;\ 2{,}0$ and $5{,}0$. The main cables are steel tubular members with varying cross-section's diameter. The vertical and lateral bending stiffness of the deck is maintained constant.

Vertical frequencies for all vertical resonant modes for different cable stiffness ratio ξ are presented in Fig 4,a and Table 2. The stiffening effect is clearly observed. The natural frequencies of the footbridge with stiffened cables were up to 60% higher for asymmetric modes and up to 80% higher for symmetric modes than those of the footbridge with flexible cables (Table 2).
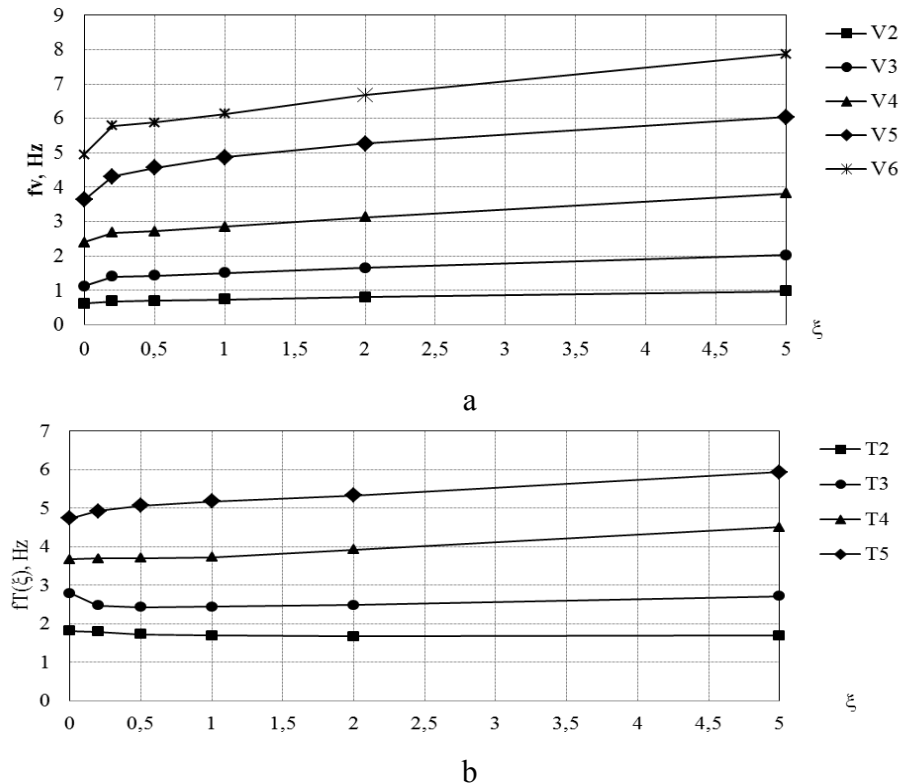

a

b

Fig. 4 Effect of the Cable Stiffness Ratio ξ on the Vertical (a) and Torsional (b) Frequencies of Vibration

It should be noted that a variation of cable stiffness causes to reordering of some modes. The mode V4 and V5 occurred at order 4 an 6 at $\xi=0$, become the fifth and seven, respectively at $\xi=0,2$. Further, the mode V5 changes his position to number 8 at $\xi=2,0$. The second mode V3 being in position 2 becomes mode number 3 only at $\xi=5,0$.

Tab.2 Natural Vibration Frequencies and Frequencies Ratio for the First Four Vertical Modes

ξ	f_{V2} [Hz]	$f_{V2}/f(0)$	f_{V3} [Hz]	$f_{V3}/f(0)$	f_{V4} [Hz]	$f_{V4}/f(0)$	f_{V5} [Hz]	$f_{V5}/f(0)$
0	0,616	1,0	1,128	1,0	2,392	1,0	3,632	1,0
0,2	0,683	1,11	1,404	1,25	2,672	1,12	4,613	1,27
0,5	0,695	1,13	1,433	1,27	2,720	1,14	4,685	1,29
1,0	0,730	1,18	1,503	1,33	2,851	1,19	4,903	1,35
2,0	0,801	1,30	1,652	1,47	3,131	1,31	5,412	1,49
5,0	0,982	1,60	2,031	1,80	3,824	1,60	6,538	1,80

Tab.3 Natural Vibration Frequencies and Frequencies Ratio for the First Four Torsional Modes

ξ	f_{T2} [Hz]	$f_{T2}/f(0)$	f_{T3} [Hz]	$f_{T3}/f(0)$	f_{T4} [Hz]	$f_{T4}/f(0)$	f_{T5} [Hz]	$f_{T5}/f(0)$
0	1,812	1,0	2,797	1,0	3,668	1,0	4,744	1,0
0,2	1,789	0,99	2,473	0,88	3,697	1,01	4,928	1,04
0,5	1,730	0,95	2,436	0,87	3,702	1,01	5,066	1,07
1,0	1,690	0,93	2,443	0,87	3,728	1,02	5,175	1,09
2,0	1,669	0,92	2,486	0,89	3,928	1,07	5,334	1,12
5,0	1,697	0,94	2,545	0,91	4,509	1,23	5,934	1,25

Torsional frequencies for all torsional resonant modes for different ratio ξ are presented in Fig. 4,b and Table 3. It is evident that cable stiffness has different effect on the torsional resonant modes. The stiffened cables increase by about 25% only the frequencies of the higher modes (T4 and T5). The first two lower mode frequencies slightly decrease when the cable stiffness increases and the rate of decrease is the largest for the 1st mode T2 (Table 3). This is, probably, due to increasing the eccentric mass of the cables [8, 9] and reordering of all torsional modes. Torsional and vertical modes change positions between, leading to decrease the values of torsional mode frequencies. The frequency ratio between the 1st torsional T2 and 1st vertical V2 modes is gradually decreasing from 2,94 at $\xi=0$ to 1,73 at $\xi=5,0$ suggesting that footbridge structural torsional stiffness is reducing. On the other hand, it seems that cable stiffness has less influence on torsional modes than on vertical modes. But, with increasing in the cable stiffness the cross section and the weight of cables increases which may result in reducing the frequencies.

The Effect of Material Properties

Main cable properties that would affect the suspension systems modal behavior are geometry and material properties, such as elastic modulus (E and G) and mass density (γ). Due to limited space of present paper, the results of the effect of the first two parameters are not presented here. The effect of these two parameters can be indirectly observed through the dimensionless cable stiffness ratio ξ just described in section 4 of the present paper. In this section only the effect of cable material density on the modal behavior of the footbridge is briefly analysed.

The effect self weight of cable is represented by the relative mass density γ/γ_0 where γ_0 is the initial value of mass density used in the current model. The numerical simulations were for ratio γ/γ_0 0,25; 0,50; 1,75; 1,0; 1,25 approximately covering all practical cases. The variation of the relative frequencies of vertical and torsional modes as a function of dimensionless cable mass parameter γ/γ_0 is shown in Fig. 5.

Firstly, it was found that factor γ/γ_0 has very little effect (1-2%) on the values of both vertical and torsional natural frequencies of the system with flexible cables. Comparison of natural frequencies of the system with rigid main cables reveals the tendency for increasing frequencies with decreasing of cable mass parameter γ/γ_0. The bending and rotational responses are linear. It seems that application sof light-weight FRP cables comparing to traditional materials can be reasonable and advantageous.
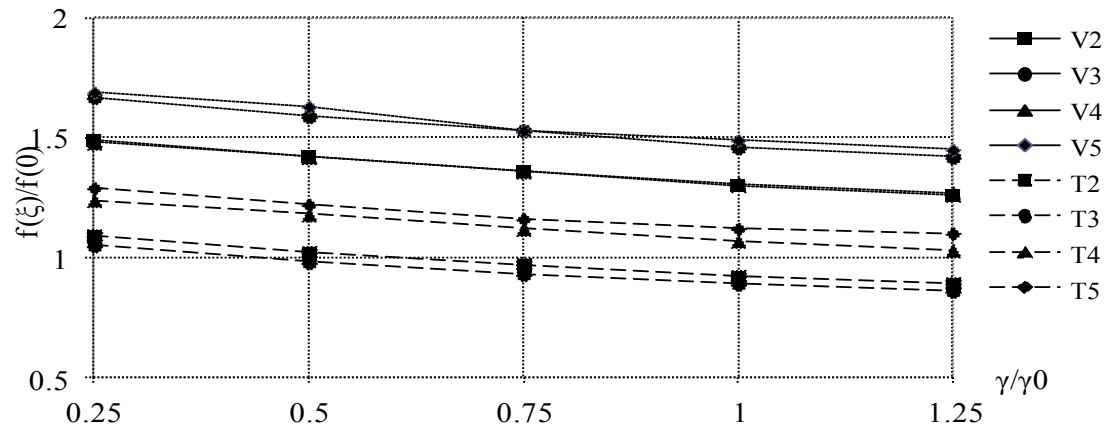
Fig.5 Free Vibration Relative Frequencies as a Function of Cable Mass Ratio γ/γ_0

Conclusions

In this study, a suspension bridge model taken into account varying bending stiffness of cables is proposed to investigate. A single span steel suspension footbridge composed of variable bending stiffness cables and relatively flexible deck as an example of a suspension system was investigated subject to free vibration response.

A three-dimensional finite-element model was used to extract the natural frequencies and obtain their corresponding mode shapes as well as simulating the footbridge response to variation of elastic properties and material densities of bridge cable. Within the frequency range 0–5 Hz to be important for pedestrian induced excitations the lowest nine vibration modes were identified and analysed.

The numerical simulations show that cable bending stiffness contributes a considerable effect on the natural frequencies of the suspension system, and affects vertical modes more significantly than torsional ones. Increasing the cable bending stiffness, the values of frequencies are also increased by maximum of about 80% for vertical modes and of about 25% only for the higher torsional modes. It is also observed that the first lower torsional mode frequencies can slightly decrease when the cable stiffness increases, probably, due to increasing the eccentric mass of the cables and reordering of modes. The higher cable stiffness results in delay of torsional modes and higher vertical modes. Cable mass factor γ/γ_0 does not make considerable changes to the values of frequencies of the system with flexible cables. The both vertical and torsional frequencies of the system with rigid cables increase with decreasing in the cable self-weight.

Main cable geometry and material properties, such as modulus of elasticity (E and G) and density (γ) values would affect the flexural (EI) and torsional (GJ) rigidities of the cross sections as well as the modal mass activated in a specific mode of vibration and as a result the various values of the dynamic stiffness of the whole system and its dynamic response should be obtained.

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