Analyzing Brittleness of Complex System Based on The Catastrophe Theory

Jian Guo[1,2] Xinglin Chen[1] Hongzhang Jin[3] Dongjian Wu[4]

[1]School of Astronautics, Harbin Institute of Technology, Harbin 150001, P. R. China
[2]College of Power and Engine Engineering, Harbin Engineering University, Harbin 150001, P. R. China
[3]College of Automation, Harbin Engineering University, Harbin 150001, P. R. China
[4]Foreign Languages Department, Harbin Engineering University, Harbin 150001, P. R. China

Abstract

It is hard to set up catastrophe potential function by using conventional methods and choose control variables of catastrophe potential function. So the method of setting up potential function is given to analyze the brittleness of complex system by means of relations among systems. The feasibility of analyzing complex system by using catastrophe theory is also discussed. Based on the data come from authority department, brittleness potential function is set up through the relation among systems. Then the concrete process of derivation and the results of analysis are given. And the conclusions, which are conformed to reality, are got.

Keywords: Complex system, Brittleness, Catastrophe theory, Potential function

1. Introduction

The establishing systematics has been presented by Chinese scientist since 1980. After more than twenty years, there are some research work of complex system have been done by taking synthesis methods from qualitative analysis to quantitative analysis.

All complexity matters are of emergent characteristics. Here emergence has two-sided implication, one is come forth of the whole complexity matter or a certain hierarchy unity; the other is bursting variation of complexity matter inside or every decomposed section in a certain hierarchy inside [1]. The characteristic of catastrophe comes to light at once, as long as that the property of the object, which is researched, involves in common variance among different material hierarchies. Those properties, which come into existence recently, behave catastrophe in result by examining from systematic higher hierarchy to systematic lower hierarchy.

The method of setting up brittleness potential function by using traditional methods is not easy got unless the parameters in the potential function is easily defined. And in generally, there are many control variables, which have an effect on systems. Furthermore, there is interaction existing among these control variables. We cannot easily choose state variables and control variables, which have no interaction and are key variables. So it is difficult to use this method to solve problems. But sometimes we can analyze the characteristics of systems and change it into catastrophe models to solve problems by using catastrophe theory in [2]-[3].

The feasibility of analyzing complex system by using catastrophe theory is discussed in this paper. In the light of data come from authority department, brittleness potential function is set up by taking the relation of evolution among systems. And the concrete process of derivation and the results of analysis are given.

2. Summarization of complex system brittleness theory

2.1. The definition of brittleness

In order to signify one new property of complex system, the brittleness, which belongs to mechanics of materials, is quoted in complex system and is given another new definition [4]-[6].

Brittleness: some section (subsystem) is collapsed when the whole complex system is affected by inside or outside factors of interference. The behavior caused the whole complex system is collapsed because else sections (subsystems) or the whole complex system are directly or indirectly affected by that behavior. That makes the whole complex system change from one ordered state (normal work state) to the other state (collapse state). This property is named brittleness of complex system.

2.2. The definitions depend on brittleness

In order to easily understand brittleness, some definitions are given as follows.

Brittleness source: the section (subsystem) that causes brittleness is called brittleness source. We can also say that the collapse of this section (subsystem) causes the collapse of else sections (subsystems), this section (subsystem) is called brittleness source.

Brittleness receiver: the collapse of this section (subsystem) that is affected by the collapse of another section (subsystem) is called brittleness receiver.

After the definitions of brittleness source and receiver are given, the definition of brittleness procedure is presented as follows.

Brittleness procedure: when some brittleness source is motivated, at least one brittleness receiver happens. This procedure is called brittleness procedure.

The relation among brittleness procedure, brittleness source and brittleness receiver is showed in Fig.1.

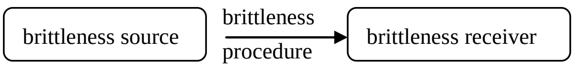

Fig.1: Relation among brittleness procedure, brittleness source and brittleness receiver.

Therefore, the concept of brittleness is also described as follows. When some brittleness source is motivated, the influence that is brought by the brittleness source can make one brittleness receiver take place at least. The process is called that brittleness is motivated. We can also say that brittleness is motivated when one brittleness procedure happens at least.

There is an emphatic point that brittleness source and brittleness receiver are not unique. There are such conditions that one brittleness source and many brittleness receivers, one brittleness source and receiver, many brittleness sources and one brittleness receiver.

Brittleness is one base attribute of complex system and exists following with complex system all the while. It doesn't disappear on the strength of the evolution of system and variation of external environment.

3. Based on the relation of evolution among systems to set up brittleness potential function

In a general way, catastrophe phenomena are of the following basic characteristics. There are many stable states in the state space of systematic evolution. When variables are changed, the state of system may jump from one stable state to another stable state. In the procedure catastrophe happens. There is unstable stationary state existing among these different stable states. When system jumps from one stable state to another state, it directly strides over the unstable stationary state. The unstable stationary state is not reached in fact. The variant value of control parameters can make system finish the change of one stable state to another stable state suddenly. The occurrence of catastrophe has relationship with the direction of change of control variables. When control variables change from one direction or another direction, the state of catastrophe is not the same and has hysteretic phenomena. Nearby the branching curve, the minute difference of change of control variable path causes the complete unlike characteristic state of system in [2]-[3] and [9].

In the next process, relation of evolution between two single-systems is derived in detail. In the process of deriving relation of evolution, we take one system as state variable and take the other system as mapping of state variable. In order to illustrate easily, we assume these two single-systems as system A and system B. So as to research relation of evolution between system A and system B, we assume system A as state variable x and system B as mapping y of system A

As the space-time we lived in is four-dimensional, we only think about adding four control variables at most. u, v, w, t are the added control variables. And assuming coefficient of the highest power term in the relation of evolution equals one.

3.1. One state variable x and one control variable u

There are two kinds of relations of evolution under this condition and showed as follows.

$$y = x^2 + u \qquad (1)$$

$$y = x^2 + ux \qquad (2)$$

Equation (2) can be changed into another form, which is showed as follows.

$$y = (x + \frac{u}{2})^2 - \frac{u^2}{4} \qquad (3)$$

From the result of changing (2), the form of (3) is same as the form of (1) materially. When there is only one state variable and control variable, the relation of evolution between them is the form of (1). Equation (1) is the equilibrium surface of fold catastrophe.

So when there is only one state variable and control variable, relation of evolution between system A and system B is analyzed based on the characteristics of fold catastrophe.

3.2. One state variable x and two control variables u,v

There are three kinds of relations of evolution under this condition and showed as follows.

$$y = x^2 + ux + v \quad (4)$$

$$y = x^3 + ux + v \quad (5)$$

$$y = x^3 + ux^2 + v \quad (6)$$

Equation (4) can be changed into another form, which is showed as follows.

$$y = (x + \frac{u}{2})^2 + v - \frac{u^2}{4} \quad (7)$$

From the result of changing (4), it is the first condition. So this condition is cancelled.

Equation (6) also can be changed into another form, which is showed as follows.

$$y = x(x + \frac{u}{2})^2 - \frac{u^2}{4}x + v \quad (8)$$

If $x_1 = x + \frac{u}{2}$, then the result is showed as follows.

$$y = x_1{}^3 - \frac{u}{2}x_1{}^2 - \frac{u^2}{4}x_1 + \frac{u^3}{8} + v \quad (9)$$

If $x_2 = x_1 - \frac{u}{6}$, then the result is showed as follows.

$$y = x_2{}^3 - \frac{u^2}{3}x_2 + \frac{2}{27}u^3 + v \quad (10)$$

From the result of changing (6), the form of (10) is same as the form of (5) materially. Equation (5) is same as the equilibrium surface of cusp catastrophe. Therefore under this condition, relation of evolution between system A and system B is the form of (5) and analyzed based on the characteristics of cusp catastrophe.

In like manner, the relation of evolution between system A and system B can be got under the below two conditions.

3.3. One state variable x and three control variables u,v,w

$$y = x^4 + ux^2 + vx + w \quad (11)$$

Equation (11) is same as the equilibrium surface of swallowtail catastrophe. Therefore under this condition, relation of evolution between system A and system B is the form of (11) and analyzed based on the characteristics of swallowtail catastrophe.

3.4. One state variable x and four control variables u,v,w,t

$$y = x^5 + tx^3 + ux^2 + vx + w \quad (12)$$

Equation (12) is same as the equilibrium surface of butterfly catastrophe. Therefore under this condition, relation of evolution between system A and system B is the form of (12) and analyzed based on the characteristics of butterfly catastrophe.

When the concrete form of catastrophe potential function of relation among systems can be defined, its equilibrium surface and bifurcation are easily got. Relation among systems is analyzed based on the equilibrium surface and bifurcation. Control factors added in relation of evolution among systems is conformed to bifurcation, where system jumps. All critical points of potential function or all systematic equilibrium points (stable or else) constitute equilibrium surface. Singularity set is one subset of all degenerate critical points of potential function. Bifurcation is got by projecting singularity set to control space and is set of points that all can make forms of catastrophe potential function change in control space. Namely it is the place where system jumps and brittleness is motivated easily.

4. Analyzing brittleness of complex system based on catastrophe theory

4.1. Feasibility of analyzing brittleness of complex system based on catastrophe theory

In the process of brittleness of complex system developed, there are a few subsystems collapsed. The effect caused by collapsed subsystems is not big enough to the whole complex system. In case of that, people did not take any preventive measures to prevent the state of the whole complex system developing towards 'bad' direction. As it developing to a certain degree, people begin to notice it and take some effective measures to control its extension. Otherwise brittleness of the whole complex system will be motivated. It caused directly result is that the whole complex system collapsed. The effect caused by the collapsed complex system is not estimated to other complex system.

The brittleness of complex system with cusp catastrophe model is analyzed directly. The degree of controlling force and time are controlling factors, the number of collapsed subsystems is state variable. When the number of collapsed subsystems equals N, this point of this time is critical point. When the number of collapsed subsystems is few, we do not pay full attention to it and take any measures to control its extension. The degree of controlling force equals zero or negative. At this condition, that motivates the state of collapsed subsystems numbers increasing quickly. When the number increases with the time going on to get the critical value N, it carries out one kick. At this time, it gives rise to the recurrence of specialists and specialists take effective measures. Otherwise, after some time, the state of complex system gets the state of no way controlling and the influence of this case to others is going to be inestimable even collapse. At the instant of carrying out kick, it kicks from one state of recurrence to the other state of no recurrence. At this time, as the degree of controlling force increasing, the number of collapsed subsystems begins to decrease. At last it carries out kick of another direction.

Through above disserting, it is reasonable and workable to describe the developing conditions of brittleness of complex system by using cusp catastrophe model. In Fig.2, the top is state of not recurrence, the under part is state of recurrence.

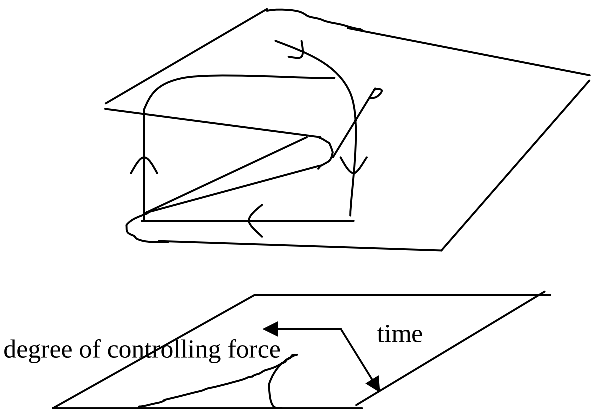

Fig. 2: Cusp catastrophe model.

4.2. Based on the relation of evolution among systems to analyze brittleness of complex system

4.2.1. The process of derivation

Taking a group of data come from authority department, data is showed in Table 1.

106	105	89	103	113	126	96	152
21	27	17	17	7	20	20	50

continuing

101	122	96	114	69	98	70	97
18	20	15	20	8	9	14	12

continuing

94	48	54	42	48	48	39	27
12	3	4	4	5	4	3	2

continuing

28	19	17	7	12	8	15	15	26	13
2	2	1	2	1	0	1	1	1	1

Table1: Original data.

The data of the first row is considered as one system A . And the data of the second row is considered as the other system B .To establish the relation of evolution between A and B , The data of the first row is considered as state variable x and the data of the second row is considered as mapping y of x . Based on the data come from authority department, polynomial is matched. Rank of model is defined by χ^2 and coefficient of polynomial is defined by least two-multiply estimate. The result of matching is showed as follows.

$$y(x) = 0.0061x^3 - 0.4981x^2 + 12.6669x + 4.2210 \quad (13)$$

To maintain the universality, the coefficient of (13) is replaced by a,b,c,d . The result is showed as follows.

$$y(x) = ax^3 + bx^2 + cx + d \quad (14)$$

Now considering (14), do primary transformation $X = x + \frac{b}{3a}$, so the result is showed as below.

$$y(X) = aX^3 + (c - \frac{b^2}{3a})X + (\frac{2b^3}{27a^2} - \frac{bc}{3a} + d) \quad (15)$$

So its brittleness potential function can be got and is showed as follows.

$$Y(X) = \frac{a}{4}X^4 + (\frac{c}{2} - \frac{b^2}{6a})X^2 + (\frac{2b^3}{27a^2} - \frac{bc}{3a} + d)X \quad (16)$$

If coefficient of tiptop rank is one, so (16) is changed as follows.

$$V(x) = x^4 + (\frac{2c}{a} - \frac{2b^2}{3a^2})x^2 + (\frac{8b^3}{27a^3} - \frac{4bc}{3a^2} + \frac{4d}{a})x \quad (17)$$

Equation (17) is compared with standard model of cusp catastrophe and the result is showed as follows.

$$u = \frac{2c}{a} - \frac{2b^2}{3a^2}, v = \frac{8b^3}{27a^3} - \frac{4bc}{3a^2} + \frac{4d}{a} \quad (18)$$

In (17) the coefficients of x and x^2 are separately named normal factor and splitting factor. It reflects the following reality: when $u>0$, the change of v only causes continuous change of x; otherwise it will cause discontinuous change of x. It also says that v is main force to determine systematic some inhere character when u is not very large (compare with v). Then u develops continually to become main force, once u and v have relation of bifurcation set, state of x happens catastrophe.

When u>0, so there is such condition showed as below.

$$\frac{2c}{a}-\frac{2b^2}{3a^2}>0\Rightarrow 3ac>b^2 \qquad (19)$$

When the relation among a, b and c is satisfied with the relation showed in (19), the relation between system A and system B is continuity and smooth. Otherwise the relation between them is going to be discontinuity. Once the relation among a, b, c and d is satisfied with bifurcation set, qualitative change will be brought about.

The relation among a, b and c is only considered in (19). What does d influence on the relation between system A and system B? Considering bifurcation set of system, in the light of (18), there is such condition showed as below.

$$8\left(\frac{2c}{a}-\frac{2b^2}{3a^2}\right)^3+27\left(\frac{8b^3}{27a^3}-\frac{4bc}{3a^2}+\frac{4d}{a}\right)^2=0 \qquad (20)$$

Simplify (20), so the result is showed follows.

$$a'd^2+b'd+c'=0 \qquad (21)$$

In (21), a', b', c' are separately showed as follows.

$$\begin{cases} a'=243a^3 \\ b'=162a^2bc+36ab^3 \\ c'=36a^2c^3+15ab^2c^2-8b^4c \end{cases} \qquad (22)$$

In (21), assume $a'\neq 0$, namely $a\neq 0$, so the result is showed as below.

$$\left(d+\frac{b'}{2a'}\right)^2+\frac{c'}{a'}-\frac{b'^2}{4a'^2}=0 \qquad (23)$$

In (23), the form of this balance curved surface is similar as that of fold catastrophe.

The set of singularities is conformed to subset of balance curved surface of the following equation.

$$2\left(d+\frac{b'}{2a'}\right)=0 \qquad (24)$$

Namely (24) is one point (0, 0). Bifurcation set B is that the set of singularities project control space.

$$\frac{c'}{a'}-\frac{b'^2}{4a'^2}=0 \qquad (25)$$

Control space is divided into two parts by bifurcation set.

$$\begin{cases} \frac{c'}{a'}-\frac{b'^2}{4a'^2}>0 \\ \frac{c'}{a'}-\frac{b'^2}{4a'^2}<0 \end{cases} \qquad (26)$$

If $\frac{c'}{a'}-\frac{b'^2}{4a'^2}>0$, (23) has no real number solution. Namely it has no critical points. Otherwise, it has two critical points. One is minimal point, the other is maximum point. And it also has two equilibrium points. One is stable equilibrium point. The other is unstable equilibrium point. In the condition of $\frac{c'}{a'}-\frac{b'^2}{4a'^2}=0$, they merge into a inflection point.

Based on the above analysis, when $\frac{c'}{a'}-\frac{b'^2}{4a'^2}>0$, the system is not stable. This condition is also described that empty state is the only possible state. When $\frac{c'}{a'}-\frac{b'^2}{4a'^2}=0$, there is a kick in the system. This discontinues condition of system is showed as below.

$$\begin{cases} d=-\frac{b'}{2a'} \\ c'=\frac{b'^2}{4a'} \end{cases} \qquad (27)$$

Namely it is

$$\begin{cases} d=-\frac{9abc+2b^3}{27a^2} \\ 9ac(3a^2c^2-b^4)=(3abc+b^3)^2 \end{cases} \qquad (28)$$

When the relation of a, b and c is conformed to (28), the catastrophe of d is caused. So the relation of a, b, c and d is defined through (28). Through (19), (20) and (28), the relation of evolution between system A and system B is defined.

4.2.2. Based on the result of match polynomial to analyze brittleness of complex system

In mathematic, the relation between system A and system B is mapping relation. But in brittleness theory, system A is brittleness source of system B and system B is brittleness receiver of system A. And in brittleness theory, in the light of evolution of system A and B, the following conditions are considered. If A collapses, then B collapses. So A is main brittleness source of B. If A collapses, then B does not collapse. So A is non-brittleness source of B. When A collapses, the probability of B's collapse has two conditions and which condition will happen is of uncertainty. So A is sub-brittleness source of B.

Equation (13) is the last matching result. Through the matching result, there is

$$a=0.0061, b=-0.4981, c=12.6669, d=4.2210 \qquad (29)$$

Equation (29) is combined with (18), then

$$u = -292, v = 67530 \quad (30)$$

When u<0, x will be discontinuous. But the relation of u and v are not conformed to bifurcation set.

The result is showed that there is some relation between the data of the first row and the data of the second row. And the data of the first row is sub-brittleness source of the data of the second row.

5. Conclusions

It is difficult to set up catastrophe potential function by using conventional means. With the development of scientific technology, the scales of system are larger and larger gradually. Meanwhile, people pursuit comfortable living can make the scales of system large and the functions of system complexity. As relations among systems are more and more tight, it is hard to choose independent control variables from these interrelation variables. Based on above-said reasons, it is easy, convenient to analyze brittleness of complex system based on relation of evolution among systems in [10].

On the basic of data come from authority department, brittleness of it is analyzed by using relation of evolution among systems. The conclusions are showed as follows:

(1) It is feasible to analyze brittleness of complex system based on catastrophe theory.

(2) Brittleness link among systems can be defined through the relation of evolution among systems.

(3) The result of analysis of the data is conformed to reality of the data.

Acknowledgment

This work is partially supported by the Fund of Harbin Engineering University (Grant No. HEUFT05061) and Postdoctoral Fund of Heilongjiang Province (Grant No. LBH-Z06094).

References

[1] F.H. Ling, *Catastrophe theory and its application*, Shanghai Communication University Publishing House, 1987.

[2] L. Jiang, L.Y. Yu, The application of primary catastrophe theory to social science. *Systematic Engineering Theory and Practice*, 10:113-117, 2002.

[3] Z.Y. Zhao, C.L. Xiang, The application of catastrophe theory in the environmental forecast. *Environmental Monitoring in China*, 4:59-60, 2002.

[4] H.Z. Jin, J. Guo, Q. Wei, Research on brittleness of complex system based on cusp catastrophe model. *Ship Electronic Engineering*, 2:1-3, 8, 2004.

[5] H.Z. Jin, J. Guo, Q. Wei, Brittleness analysis of SARS based on slide t detected method. *Journal of Harbin Engineering University*, 6:640-645, 2003.

[6] J. Guo, H.Z. Jin, Q. Wei, Analyzing brittleness of complex system based on brittleness potential function. *Techniques of Automation and Applications*, 7:3-7, 2003.

[7] B. Li, S.Y. Zhang, Y.H. Li, The qualitative representation and inference of the complex system. *Systematic Engineering Theory and Practice*, 12:15-21, 2002.

[8] Q. Wei, H.Z. Jin, X.L. Yao and J. Guo, Research on the collapse of complex system based on brittleness. *Journal of Harbin Engineering University*, 2:161-165, 2003.

[9] L.H. Chai, M. Shoji, Boiling curves – bifurcation and catastrophe. *International Journal of Heat and Mass Transfer*, 44:4175-4179, 2001.

[10] J. Guo, The application of catastrophe theory in the research of complex system brittleness theory. *Doctoral Dissertation of Harbin Engineering University*, 2004.