A Method for Solving Group Linguistic Decision-Making Problems Based on IOWA

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Abstract

In this paper, we will provide a method for solving group decision-making problems based on $IOWA$ operators. In which, linguistic assessments of decision makers could be selected freely, *i.e.*, let $S = \{s_1, s_2, \cdots, s_n\}$ be basic linguistic assessments, decision maker could select his (or her) linguistic assessment from S, or freely give, such as, s_t, where $t \in [1, n]$. Formally, $s_t \in [s_i, s_{i+1}] (i \in \mathcal{N})$ could be explained by linguistic assessment of the decision maker lied between s_i and s_{i+1}. The OWA pairs of $IOWA$ operator are obtain by assessment level of experts and linguistic assessment of experts. Especially, we provide a new method to deal with 'tie' of $IOWA$ operators. Example shows that our method is feasible.

Keywords: OWA operator, $IOWA$ operator, Group decision-making, Aggregation

1. Introduction

The induced aggregation operators are an interesting research topic, which are receiving increasing attention. In [1], Yager, et al introduce a class of induced ordered weighted averaging ($IOWA$) operators which take as their argument pairs, called OWA pairs, in which one component is used to induce an ordering over the second components which are exact numerical values and then aggregated. In most voting system, the decisions are based on the condition that the selection projects are given, in this paper, we allow the decision makers give any semantic expression as long as to according with their desire. In this paper, the assessment level of decision makers based on linguistic assessment matrix are adopted as the order inducing value u_i, which together with linguistic assessments a_i of decision makers to construct OWA pair, $\langle u_i, a_i \rangle$. By aggregating all OWA pairs, we can obtain the final result. In this paper, a new method is proposed to deal with 'tie'. This paper is set out as follows. In Section 2, we summarize the $IOWA$ operators. In Section 3, a new method to deal with 'tie' is proposed. In section 4, we analyze the method of this paper. An example is in Section 5. In Section 6, we draw our conclusions.

2. Preliminaries

In [6], as noted the OWA aggregation, $F_W(a_1, ..., a_n) = W^T B$, makes use the reordering operator $B = Reorder(A)$. The ordering is based upon the value of the arguments, b_j is the value that is the jth largest of the arguments. Inspired by the work in [16], it appears that we can consider a more general policy towards the formulation of the ordered argument vector B. In this more general framework, we shall assume each of the argument values to be aggregated, a_i is a component of a more complex object which we shall for our immediate purpose represent as a two-tuple $\langle u_i, a_i \rangle$ and denote as an OWA pair. In this more general approach to OWA aggregation, we shall order the arguments, form the vector B, based upon the u_i values. In particular, our procedure for calculating the OWA aggregation of these OWA pairs, $F_W(\langle u_1, a_1 \rangle, ..., \langle u_n, a_n \rangle) = W^T B_u$ is as follows. We form the ordered argument vector B_u so that b_j is the a_i value of the OWA pair having the jth largest u value. In discussing these OWA pairs, $\langle u_i, a_i \rangle$, because of its role we shall refer to the u_i as the order inducing variable and a_i as the argument variable. The following simple example illustrates the approach:

Example 1 *Assume we have four OWA pairs* $\langle 3, 0 \rangle, \langle 7, 0.2 \rangle, \langle 2, 0.9 \rangle, \langle 6, 1 \rangle$ *we want to aggregate using the weighting vector* $W^T = [0.4, 0.3, 0.2, 0.1]$

The first step is to order the OWA pairs $\langle u_i, a_i \rangle$ based on the ordering inducing variable u_i.

$$Ordered\ OWA\ Pairs$$

$$\langle 7, 0.2 \rangle$$

$$\langle 6, 1 \rangle$$

$$\langle 3,0\rangle$$

$$\langle 2,0.9\rangle$$

From this order we obtain the components of the vector B by taking the ordered list of the a_i values thus $b_1=0.2$,$b_2=1$,$b_3=0$ and $b_4=0.9$. Using this ordering, we get $F_W(\langle u_i,a_i\rangle)=\sum_{j=1}^{4} w_j b_j = (0.4)(0.2)+(0.3)(1)+(0.2)(0)+(0.1)(0.9)=0.47$

Let us look at the properties associated with these Induced Ordered Weighted Average ($IOWA$) Operators, $F_W(\langle u_i,a_i\rangle)$. These operators are symmetric, each of the objects involved in the aggregation are treated in the same way. These operators exhibit the bounding property characteristic of mean operators, for any order inducing variable and any weighting vector

$$Min_i[a_j] \le F_W(\langle u_i,a_i\rangle) \le Max_j[a_j]$$

These $IOWA$ operators are monotonic, if $a_i \ge \widehat{a_i}$ for all i then $F_W(\langle u_i,a_i\rangle) \ge F_W(\langle u_i,\widehat{a_i}\rangle)$.

There are a number of ways in which the aggregation of OWA pairs is different from the aggregation of OWA singletons. For example, if W is the Max aggregation operator, $w_1=1$ and $w_j=0$ for $j\neq 1$. In the ordinary case this returns the largest of the a_i in the $IOWA$, it returns the argument value of the pair having the largest u value.

An important issue that must be addressed when using these $IOWA$ operators arises when there is a tie in the ordering operation. In the following section we will give the method to deal with the problem.

3. The new method to deal with the 'tie'

In [1], Yager introduce a method to manage the 'tie'. Consider aggregation of the objects $\langle 5,1\rangle$, $\langle 3,0.5\rangle$, $\langle 8,0.6\rangle$, $\langle 5,0.4\rangle$ under the weighting vector $w_t=[0.3,0.25,0.25,0.2]$

$$Ordered\ OWA\ Pairs$$

$$\langle 8,0.6\rangle$$

$$tie \quad \langle 5,1\rangle \quad \langle 5,0.4\rangle$$

$$\langle 3,0.5\rangle$$

We see that there is a tie between $\langle 5,1\rangle$ and $\langle 5,0.4\rangle$ with respect to order inducing variable. It can be easily shown that if we brake this tie by selecting $\langle 5,1\rangle$ ahead of $\langle 5,0.4\rangle$ giving us the ordered argument vector

$$B=\begin{bmatrix}0.6\\1\\0.4\\0.5\end{bmatrix}$$

We would get a different aggregated value then by selecting $\langle 5,0.4\rangle$ ahead of $\langle 5,1\rangle$, which would get the ordered argument vector

$$B=\begin{bmatrix}0.6\\0.4\\1\\0.5\end{bmatrix}$$

That is $(0.4)(0.6)+(0.3)(1)+(0.2)(0.4)+(0.1)(0.5) \neq (0.4)(0.6)+(0.3)(0.4)+(0.2)(1)+(0.1)(0.5)$.

In [1] the policy we shall follow in the case of ties in the order inducing process is to replace the arguments of the tied OWA pairs by the average of the arguments of the tied pairs in forming the B vector. Thus in the preceding illustration, when forming the B matrix we replace the argument component of each of $\langle 5,1\rangle$ and $\langle 5,0.4\rangle$ by their average 0.7, $(1+0.4)/2$.

This substitution gives us an ordered argument vector

$$B=\begin{bmatrix}0.6\\0.7\\0.7\\0.5\end{bmatrix}$$

Following this process can be essentially shown to be equivalent to calculating the aggregated value as $W^TB=\frac{1}{2}[W^TB_1+W^TB_2]$. We note if q items are tied, we replace these by q replica's of their average.

Although Yager's method can manage the tie, but the original objects are changed. Here we introduce a new method to deal with the tie without the change of the objects.

Let $Z=\{\langle u_i,a_i\rangle | i=1,2,\cdots n\}$ be all OWA pairs. $\langle u_i,a_i\rangle$, $\langle u_j,a_j\rangle$ is a tie, i.e., $u_i=u_j$

For $\{a_i|i=1,2,\cdots n\}$, the weight of a_i in OWA operator is denoted as $w_i=\frac{b_i^\alpha}{\sum_{i=1}^{n} b_i^\alpha}$, where b_i is the ith largest element of the collection of the aggregated objects $a_1,a_2,\ldots a_n$, $\alpha\in(-\infty,\infty)$ here we let $\alpha=0.5$, $a=w_ia_i+w_ja_j$, we use $\langle u_i,a\rangle$ instead of $\langle u_i,a_i\rangle$ and $\langle u_j,\ a_j\rangle$ in $\{\langle u_i,a_i\ \rangle | i=1,2,\cdots n\}$,i.e., $Z'=\{\langle u_1,a_1\rangle,\cdots,\ \langle u_{i-1},a_{i-1}\rangle,\ \langle u_i,\ a\rangle,\cdots,\ \langle u_{j-1},\ a_{j-1}\rangle,\ \langle u_{j+1},\ a_{j+1}\ \rangle,\cdots,\ \langle u_n,\ a_n\rangle\}$

with the value of w we have introduced, we will have $IOWA_w\{\langle u_i,a_i\ \ \rangle | i=1,2,\cdots n\} = IOWA'_w\{\langle u_1,\ \ a_1\rangle,\cdots,\ \ \langle u_{i-1},\ \ a_{i-1}\rangle,\ \ \langle u_i,\ \ a\rangle,\ \cdots,\langle u_{j-1},\ a_{j-1}\rangle,\ \langle u_{j+1},\ a_{j+1}\rangle,\ \cdots,\langle u_n,\ a_n\rangle\}$

Utilize the same illustration we have the aggregation of the objects $< 5, 1 >, < 3, 0.5 >, < 8, 0.6 >$ $, < 5, 0.4 >$. Performing the ordering of the objects we get

$$Ordered\ OWA\ Pairs$$

$$\langle 8, 0.6\rangle$$

$$tie \quad \langle 5, 1\rangle \quad \langle 5, 0.4\rangle$$

$$\langle 3, 0.5\rangle$$

Here we divide the objects into three parts $\langle 8, 0.6\rangle, (\langle 5, 1\rangle, \langle 5, 0.4\rangle), \langle 3, 0.5\rangle$ so we can see that in the second part we can consider the $IOWA$ operator as the OWA operator. Here we need three w to weight the three parts. Utilize the value of the w in [5] $w_i = \frac{b_i^\alpha}{\sum_{i=1}^n b_i^\alpha}$, so we can have the $w = [1, (0.6126, 0.3874), 1]$. It should be note that here we consider the u as the b_i, while in the OWA operator, the b_i should be the object themselves. We consider the $0.6126 \times 1 + 0.3874 \times 0.4 = 0.76756$ as the second parts of the new $IOWA$ operator, so we have the new $IOWA$ $\langle 8, 0.6\rangle, \langle 5, 0.76756\rangle, \langle 3, 0.5\rangle$. Use the same method we get the $w = [0.41, 0.32, 0.26]$, so $IOWA = 0.6 \times 0.42 + 0.76756 \times 0.32 + 0.5 \times 0.26 = 0.6276$. With the same w, use Yager's method, we obtain the $IOWA = 0.3 \times 0.6 + 0.25 \times 0.7 + 0.25 \times 0.7 + 0.2 \times 0.5 = 0.63$.

We can express our new method as we divide the objects into several parts depend upon how much ties in the ordering operation. We put the object have the same u together and consider them as the OWA operator, aggregate the objects, we can obtain a new $IOWA$ operator. Use the same method to obtain w to aggregate the new $IOWA$ base on the u, then we can get the final result. If $k(k{>}2)$ items are tied, we do this for k times.

It should be clear that in the usual OWA aggregation ties don't present a problem, the reason for this is that the ordering variable is the same as the argument variable and however we place the tied objects leads to the same result.

4. Obtaining linguistic assessment and assessment level of experts

The linguistic approach is an approximate technique, which represents qualitative aspects as linguistic values by means of linguistic variables [7]-[14]. Suppose that $S = \{s_i | i = -t, ...t\}$ is a finite and totally ordered discrete term set, where s_i represents a possible value for a linguistic variable. For example, a set of nine terms S could be $S = \{s_0 =$ extremely poor, $s_1 =$ very poor, $s_2 =$ poor, $s_3 =$ slightly poor, $s_4 =$ fair, $s_5 =$ slightly good, $s_6 =$ good, $s_7 =$ very good, $s_8 =$ extremely good $\}$ in which $s_i < s_j$ if $i < j$

To preserve all the given information, we extend the discrete term set S to a continuous term set $S = \{s_\alpha | \alpha \in [-t, t]\}$. If $s_\alpha \in S$, then we call s_α an original linguistic term, otherwise, we call s_α a virtual linguistic term [15]. In general, the decision makers use the original linguistic terms to evaluate alternatives, and the virtual linguistic terms can only appear in operation [4].

In general, we can't give a fuzzy linguist information a value, although we know which semantic interval it lies. But if we can change our thinking, videlicet we don't give the fuzzy linguist information a certain semantic value, while we utilize the number axis, let the decision maker provide the certain position of the fuzzy linguist information in the number axis, through the compute of the distance we can just use it as a certain semantic to deal with.

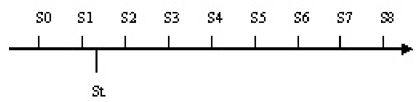

Fig. 1: The number axis

With the number axis, we let the decision maker provide the evaluation, with the computer of the distance, we can have the certain value, such as from the number axis we will know s_t which lies between S_1 *and* S_2, its value is $S_{1.2}$. So we can obtain the linguist assessment matrix.

We should note that there exist different background, knowledge and so on among the decision makers, so it is difficult to ensure the value of the evaluate to consistent, so it is very important to analyze assessment level of experts. When we get the linguistic assessment matrix, we can use the method in [2] to analyze assessment level of the decision maker, we can consider the evaluation as the u for the $IOWA$ operator, so we will finally deal with the whole problem.

The follows definitions is appeared in [2], with them we can get the assessment level of the decision makers.

Definition 1 *[6] suppose* $(r_1, a_1), (r_2, a_2), ..., (r_m,$

a_m) *is a group of evaluation of the individual decision maker which will be aggregated, so LWD operator be defined as*

$$(r_E, a_E) = LWD((r_1, a_1), ..., (r_m, a_m)) \quad (1)$$

thereinto, a_E *is the evaluation of the group,* r_E *is the important degree of the group evaluations. They can be obtained as follow:*

$$a_E = max\{min(r_1, a_1), ..., min(r_m, a_m)\} \quad (2)$$

$$r_E = \Phi_L(r_1, r_2, ..., r_m) \quad (3)$$

In [16] the author give the definition and property of declination between two linguist.

Definition 2 a_i^k *and* a_i^l *is the lingual evaluation of export* e_k *and* e_i*, thereinto,* $a_i^k = s_u$*,* $a_i^l = s_v$*,* $s_u, s_v \in S$*, then* $\rho(a_i^k, a_i^l)$ *or* ρ_{kl}^i *is the coherence index of alternative aggregation* x_i *by export* e_k *and* e_l

$$\rho(s_u, s_v) = 1 - \frac{|u - v|}{T} \quad (4)$$

Definition 3 $X^k = [a_1^k, a_2^k, ...a_n^k]$ *is the project sorting representation vector,* $\forall k \in J$*, thereinto,* a_i^k *is the language evaluation of* x_i *by export* e_k*,* $\rho(a_i^k, a_i^l)$ *or* ρ_{kl}^i *is the coherence index of alternative aggregation* x_i *by export* e_k *and* e_l*,* C_i *is the coherence vector of* x_i *by the group of exports,* μ_i *is the coherence index of* x_i *by the group of exports.*

$$C_i = [\rho_{12}^i \rho_{13}^i ... \rho_{1m}^i \rho_{23}^i \rho_{24}^i ... \rho_{2m}^i \rho_{m-1,m}^i] \quad (5)$$

$$\mu_i = \phi(\rho_{12}^i, ..., \rho_{1m}^i, \rho_{23}^i, \rho_{24}^i, ..., \rho_{2m}^i, \rho_{m-1,m}^i) \quad (6)$$

Definition 4 *Suppose* $\rho(a_i^k, a_i^l)$ *or* ρ_{kl}^i *is the coherence index of alternative aggregation* x_i *by export* e_k *and* e_l *thereinto,* $\rho_{kl}^i = \rho_{lk}^i$*,* $\forall l, k \in J$*, then* C^{kl} *is the coherence vector of* x_i *by export* e_k *and* e_l*,* μ^{kl} *is the coherence index of* x_i *by export* e_k *and* e_l

$$C^{kl} = [\rho_{kl}^1 \rho_{kl}^2 ... \rho_{kl}^n] \quad (7)$$

$$\mu^{kl} = \phi(\rho_{kl}^1, ..., \rho_{kl}^n) \quad (8)$$

Definition 5 *suppose* μ^{kl} *is the coherence index by export* e_k *and* e_l*, then* C^k *is the coherence vector of by the export* e_k *and the group of exports,* μ^k *is the coherence index by the export* e_k *and the group of exports.*

$$C^k = [\mu^{k1} \mu^{k2} \mu^{k(k-1)} \mu^{k(k+1)} ... \mu^{km}] \quad (9)$$

$$\mu^k = \phi(\mu^{k1}, ..., \mu^{k(k-1)}, \mu^{k(k+1)}, ..., \mu^{km}) \quad (10)$$

5. Example

Let us suppose a vote problem, which want the customer give the evaluation about the home appliances market. Suppose alternative aggregation $X = x_1, x_2, x_3, x_4$, index aggregation $P = p_1, p_2, p_3, p_4$, and there are five experts in this problem. There is a panel with four possible alternatives.
(1) x_1 is the television
(2) x_2 is the refrigeratory
(3) x_3 is the roller washing machine
(4) x_4 is the air-condition
There are four attributes as follow:
(1) The price
(2) The quality
(3) The after service
(4) The exterior
The four possible alternatives x_i $(i = 1, 2, 3, 4)$ are evaluated using the linguistic term set
$S = \{s_0 =$ extremely poor, $s_1 =$ very poor, $s_2 =$ poor,$s_3 =$ slightly poor, $s_4 =$ fair,$s_5 =$ slightly good, $s_6 =$ good, $s_7 =$ very good, $s_8 =$ extremely good $\}$

Use the method we have introduce in part 3, utilize the number axis and the method in [2], we can get the value of the evaluation as following

$$B = \begin{bmatrix} 0.6 \\ 1 \\ 0.4 \\ 0.5 \end{bmatrix}$$

$R^1 = [s_{6.6}, s_7, s_{6.5}, s_6]$
$R^2 = [s_6, s_7, s_{5.5}, s_{5.5}]$
$R^3 = [s_6, s_6, s_{5.5}, s_{5.5}]$
$R^4 = [s_8, s_{7.5}, s_7, s_7]$
$R^5 = [s_6, s_6, s_{5.5}, s_{7.5}]$

$$A_1 = \begin{pmatrix} s_{1.2} & s_{1.7} & s_{1.6} & s_{2.1} \\ s_{6.5} & s_7 & s_6 & s_6 \\ s_5 & s_{6.2} & s_{5.8} & s_{5.5} \\ s_6 & s_{5.4} & s_{6.3} & s_{4.9} \end{pmatrix}$$

$$A_2 = \begin{pmatrix} s_{3.9} & s_{4.1} & s_{4.5} & s_{4.7} \\ s_6 & s_{6.2} & s_{5.5} & s_{5.4} \\ s_{5.2} & s_5 & s_{4.6} & s_{4.4} \\ s_{4.9} & s_5 & s_{4.6} & s_4 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} s_{4.4} & s_{4.9} & s_5 & s_{5.5} \\ s_{5.9} & s_{5.7} & s_{5.5} & s_4 \\ s_{4.4} & s_{4.7} & s_{3.8} & s_{3.9} \\ s_4 & s_{3.3} & s_{2.9} & s_{4.4} \end{pmatrix}$$

$$A_4 = \begin{pmatrix} s_{0.9} & s_1 & s_{0.8} & s_{1.1} \\ s_{7.5} & s_{7.1} & s_{6.6} & s_6 \\ s_{6.9} & s_{7.3} & s_7 & s_{6.6} \\ s_7 & s_{5.6} & s_{6.7} & s_{5.5} \end{pmatrix}$$

$$A_5 = \begin{pmatrix} s_{5.2} & s_{5.7} & s_{5.5} & s_{7.5} \\ s_{5.5} & s_{5.7} & s_5 & s_{3.9} \\ s_4 & s_{4.2} & s_{3.7} & s_{3.5} \\ s_{3.7} & s_2 & s_{3.5} & s_4 \end{pmatrix}$$

Utilize (1)-(3), we can obtain the projects' sequencing of every customer
$a_i^1 = (s_{2.1}, s_7, s_{6.2}, s_{6.3})$
$a_i^2 = (s_{4.7}, s_{6.2}, s_{5.2}, s_5)$
$a_i^3 = (s_{5.5}, s_{5.9}, s_{4.7}, s_{4.4})$
$a_i^4 = (s_{1.1}, s_{7.5}, s_{7.3}, s_7)$
$a_i^5 = (s_{5.7}, s_7, s_{4.2}, s_4)$

We use the proportional fuzzy quantify "the most", with the formulation 4-10, we have $\mu^{12} = \phi(\rho_{12}^1, \rho_{12}^2, \rho_{12}^3, \rho_{12}^4) = (0.675, 0.9, 0.875, 0, 8375) = 0.88125$. With the same method we will have $\mu^{13} = 0.8275$, $\mu^{14} = 0.885$, $\mu^{15} = 0.78125$, $\mu^{23} = 0.94625$, $\mu^{24} = 0.77875$, $\mu^{25} = 0.9$, $\mu^{34} = 0.725$, $\mu^{35} = 0.95375$, $\mu^{45} = 0.67875$. $\mu^1 = \phi(\mu^{11}, \mu^{12}, \mu^{13}, \mu^{14}, \mu^{15}) = (1, 0.88125, 0.8275, 0.885, 0.78125) = 0.86125$, with the same method we get $\mu^2 = 0.89$, $\mu^3 = 0.87925$, $\mu^4 = 0.84575$, $\mu^5 = 0.833$

The μ^i is the induced variable in the new $IOWA$ operator. Utilize the value of the $w[5]$, $w_i = \frac{b_i^\alpha}{\sum_{i=1}^n b_i^\alpha}$ we have the
$w^T = [0.2033, 0.202, 0.1999, 0.1981, 0.1967]$

Now we have five group OWA pairs.

Ordered OWA Pairs

$\langle 0.89, s_{4.7}\rangle$

$\langle 0.87925, s_{5.5}\rangle$

$\langle 0.86125, s_{2.1}\rangle$

$\langle 0.84575, s_{1.1}\rangle$

$\langle 0.833, s_{7.5}\rangle$

Ordered OWA Pairs

$\langle 0.89, s_{6.2}\rangle$

$\langle 0.87925, s_{5.9}\rangle$

$\langle 0.86125, s_7\rangle$

$\langle 0.84575, s_{7.5}\rangle$

$\langle 0.833, s_{5.7}\rangle$

Ordered OWA Pairs

$\langle 0.89, s_{5.2}\rangle$

$\langle 0.87925, s_{4.7}\rangle$

$\langle 0.86125, s_{6.2}\rangle$

$\langle 0.84575, s_{7.3}\rangle$

$\langle 0.833, s_{4.2}\rangle$

Ordered OWA Pairs

$\langle 0.89, s_5\rangle$

$\langle 0.87925, s_{4.4}\rangle$

$\langle 0.86125, s_{6.3}\rangle$

$\langle 0.84575, s_7\rangle$

$\langle 0.833, s_4\rangle$

so the final aggregation result is

$IOWA_w^1 = (\langle 0.89, s_{4.7}\rangle, \langle 0.87925, s_{5.5}\rangle, \langle 0.86125, s_{2.1}\rangle, \langle 0.84575, s_{1.1}\rangle, \langle 0.833, s_{7.5}\rangle) = 0.2033 \times 4.7 + 0.202 \times 5.5 + 0.1999 \times 2.1 + 0.1981 \times 1.1 + 0.1967 \times 7.5 = 4.17942$

$IOWA_w^2 = (\langle 0.89, s_{6.2}\rangle, \langle 0.87925, s_{5.9}\rangle, \langle 0.86125, s_7\rangle, \langle 0.84575, s_{7.5}\rangle, \langle 0.833, s_{5.7}\rangle) = 0.2033 \times 6.2 + 0.202 \times 5.9 + 0.1999 \times 7 + 0.1981 \times 7.5 + 0.1967 \times 5.7 = 6.4585$

$IOWA_w^3 = (\langle 0.89, s_{5.2}\rangle, \langle 0.87925, s_{4.7}\rangle, \langle 0.86125, s_{6.2}\rangle, \langle 0.84575, s_{7.3}\rangle, \langle 0.833, s_{4.2}\rangle) = 0.2033 \times 5.2 + 0.202 \times 4.7 + 0.1999 \times 6.2 + 0.1981 \times 7.3 + 0.1967 \times 4.2 = 5.51821$

$IOWA_w^4 = (\langle 0.89, s_5\rangle, \langle 0.87925, s_{4.4}\rangle, \langle 0.86125, s_{6.3}\rangle, \langle 0.84575, s_7\rangle, \langle 0.833, s_4\rangle) = 0.2033 \times 5 + 0.202 \times 4.4 + 0.1999 \times 2.1 + 0.1981 \times 7 + 0.1967 \times 4 = 4.49859$

Rank all the alternatives $x_i (i = 1, 2, 3, 4)$ in accordance we have $x_2 > x_3 > x_4 > x_1$, thus the best alternative is x_2

6. Conclusion

In this paper, we provide a method for solving group decision-making problems based on $IOWA$ operators. In which, linguistic assessments of decision makers could be selected freely. The OWA pairs of $IOWA$ operator are obtain by assessment level of experts and linguistic assessment of experts. Especially, we provide a new method to deal with 'tie' of $IOWA$ operators. Example shows that our method is feasible.

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