An Inventory Strategy with Emergency Order in Cluster Supply Chain

Benhe Gao
Logistics Management Division
Graduate School at Shenzhen, Tsinghua University
Shenzhen, China
gaobh@sz.tsinghua.edu.cn

Yili Jiang
Logistics Management Division
Graduate School at Shenzhen, Tsinghua University
Shenzhen, China
jiangyili1990@126.com

Abstract—The enterprises in the cluster supply chain network not only cooperate in a chain, but also compete and cooperate cross other supply chains. We focus on bidirectional replenishment policy for the enterprises in two chains so that both of the enterprises can reduce cost. Meanwhile, we provide a research method of horizontal cooperation for enterprises in the cluster supply chain network.

Keywords- cluster supply chain; bidirectional emergency replenishment; replenishment cross the chains

I. INTRODUCTION

In some provinces of China like Guangdong and Zhejiang, the regional economy is treated as lump economy. There are many small and medium enterprises gathering in these places and they form a cluster supply chain network. Li Jizi(2004) says cluster supply chain is an organizational integration of industrial cluster and supply chain. Generally, the inventory system of small enterprise is small-scale, while sometimes they are faced with emergency conditions like some large order or delayed normal order, so emergency replenishment appears across supply chain. That is, some enterprises located in different supply chains ask for emergency order or provide replenishment with each other. Similar problem has been studied by many scholars.

In the early time, Barankin(1961), Daniel(1962) and Neuts(1964) do research on a periodic review model for an inventory system with normal and emergency orders. They suppose the normal orders need a lead time, while emergency replenishment appears across supply chain. That is, some enterprises located in different supply chains ask for emergency order or provide replenishment with each other. Similar problem has been studied by many scholars.

In the early time, Barankin(1961), Daniel(1962) and Neuts(1964) do research on a periodic review model for an inventory system with normal and emergency orders. They suppose the normal orders need a lead time, while emergency replenishment appears across supply chain. That is, some enterprises located in different supply chains ask for emergency order or provide replenishment with each other. Similar problem has been studied by many scholars.

Bulinskaya(1964), Fukada(1964) and Veinott(1966) extend the length of lead time. Whittmore(1978) constructs a dynamic model which has multi period and the lead times can be long or short. Blumenfeld et.al(1985) introduces an inventory strategy with emergency order. He assumes emergency replenishment quantity is big enough so that shortage can be avoided. Gross and Soriano(1972),Chiang and Gutierrez(1996) analyze a period review model for an inventory system with normal and emergency orders. They suppose the normal orders need a lead time, while emergency orders can arrive at once.

Bulinskaya(1964), Fukada(1964) and Veinott(1966) extend the length of lead time. Whittmore(1978) constructs a dynamic model which has multi period and the lead times can be long or short. Blumenfeld et.al(1985) introduces an inventory strategy with emergency order. He assumes emergency replenishment quantity is big enough so that shortage can be avoided. Gross and Soriano(1972),Chiang and Gutierrez(1996) analyze a period review model for an inventory system with normal and emergency orders. They suppose the normal orders need a lead time, while emergency orders can arrive at once.

II. COOPERATION FOR INVENTORY SYSTEM IN CLUSTER SUPPLY CHAIN

Li Jizi(2004) regards cluster supply chain as the coupling organizing form between supply chain and industrial cluster. The enterprises in the cluster supply chain network not only cooperate in a chain, but also compete and cooperate cross other supply chains, which can improve the enterprises' work efficiency and service quality, and circumvent the market risk in some extend. There are a variety of ways for inventory coordination in cluster supply chain. We divide it to the following 2 types:

A. Unidirectional /Bidirectional emergency replenishment for Homogeneous Enterprises at Single/Multi stage ,

Figure 1. Replenishment for Homogeneous Enterprises

B. Unidirectional /Bidirectional emergency replenishment for Inhomogeneous Enterprises at Single/Multi stage ,

Figure 2. Replenishment for inhomogeneous Enterprises

III. BASIC ASSUMPTIONS

Take the bidirectional emergency replenishment inventory system between two homogeneous enterprises in the cluster supply chain for example. We assume this cluster supply chain consists of two single chains. Each chain consists of three parts: Supplier(S), Manufacture(R), Customer(C), R₁ can order from upstream S₁, also can ask for emergency replenishment from R₂. Similarly, R₂ can order from upstream S₂ or R₁ in the other supply chain. We assume inventory levels are reviewed
periodically. When it drops to a certain value, it may be out of stock or has been out of stock, then the manufacturer will ask for emergency replenishment.

Table 1 and Table 2 are the summary of parameters.

Table 1. Decision variables in the model

<table>
<thead>
<tr>
<th>Decision Variables</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S(k)$</td>
<td>Inventory level target under normal order, use (up to $S$) policy, $k=1, 2$</td>
</tr>
<tr>
<td>$S_0(k)$</td>
<td>Inventory level target under emergency order, use (up to $S_0$) policy, $k=1, 2$</td>
</tr>
<tr>
<td>$Q(k)$</td>
<td>Inventory level for deciding whether to provide emergency replenishment or not</td>
</tr>
</tbody>
</table>

Table 2. Other parameter variables in the model

<table>
<thead>
<tr>
<th>Parameter Variables</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>The cycle length for a periodic review.</td>
</tr>
<tr>
<td>$L$</td>
<td>The lead time that orders are placed under normal order</td>
</tr>
<tr>
<td>$D(k)$</td>
<td>Expected demand in a cycle $D^{(i)} \sim E^{(i)}(\mu^{(i)}, \sigma^{(i)})$</td>
</tr>
<tr>
<td>$t$</td>
<td>The point that manufacturer has transverse periodic inventory review.</td>
</tr>
<tr>
<td>$i$</td>
<td>The $i$th day in a cycle.</td>
</tr>
<tr>
<td>$Q_e(k)$</td>
<td>Emergency ordering quantity</td>
</tr>
<tr>
<td>$C$</td>
<td>Normal ordering cost</td>
</tr>
<tr>
<td>$C_h$</td>
<td>Holding cost</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Shortage cost</td>
</tr>
<tr>
<td>$C_e$</td>
<td>The extra cost for emergency replenishment</td>
</tr>
<tr>
<td>$OH_{i}^{(k)}$</td>
<td>On-hand inventory of manufacturer on $i$th day in a cycle</td>
</tr>
<tr>
<td>$OH^{(k)}$</td>
<td>On-hand inventory of manufacturer in $(t+1)$th day</td>
</tr>
<tr>
<td>$BO_{i}^{(k)}$</td>
<td>The number of units backordered on $i$th day in a cycle, $k=1, 2$</td>
</tr>
<tr>
<td>$BO^{(k)}$</td>
<td>The number of units backordered in a cycle</td>
</tr>
<tr>
<td>$NS_{i}^{(k)}$</td>
<td>The inventory level on the $i$th day, $i=1, 2, \ldots, T$, $k=1, 2$</td>
</tr>
</tbody>
</table>

IV. ANALYSIS AND CALCULATION

The model for the inventory cost is (1).

$$C = CR + CH + CP + CE - RE$$ (1)

CR denotes the normal ordering cost, CH denotes the Holding cost, CP denotes shortage cost, CE denotes the extra cost for emergency replenishment, RE denotes the extra profit for emergency replenishment. Since CR is constant, we can write $E(C^{(k)})$ as following:

$$E(C^{(k)}) = E(CH^{(k)} + CP^{(k)} + CE^{(k)} - RE^{(k)})$$

$$= c_h \sum_{i=1}^{T} E(OH_{i}^{(k)}) + c_p E(Q_{0}^{(k)}) + c_e E(OH_0^{(k)})$$

$$+ c_e (E(Q_0^{(k)} I_0^{(k)}) - E(Q_0^{(k)} I_0^{(k)}))$$ (2)

When $OH_{i}^{(1)} < S_0^{(1)}$, R1 asks for emergency replenishment from R2, the quantity $Q_0^{(1)} = S_0^{(1)} - OH_0^{(1)}$. While R2 decides whether to provide replenishment by (3):

$$I_0^{(2)} = \begin{cases} 1 & OH_0^{(2)} - Q_0^{(2)} \geq Q_0^{(1)} \\ 0 & OH_0^{(2)} - Q_0^{(2)} < Q_0^{(2)} \end{cases}$$ (3)

The inventory level of R1 and R2 can be wrote as (4):

$$NS_{i}^{(1)} = OH_{i}^{(1)} - D_{i-1}^{(1)}, NS_{i}^{(2)} = OH_{i}^{(2)} - D_{i-1}^{(2)}$$

$$i = t+1, t+2, \ldots, T.$$ (4)

Let $E(C_{0}^{(1)})$ denote the expected cost of R1 under normal order, $E(C_{0}^{(2)})$ denote the expected cost under bidirectional emergency order.

$$E(C^{(1)}) = E(C_{0}^{(1)}) + c_h \sum_{i=1}^{T} \Delta E(OH_{i}^{(1)}) +$$

$$c_p \Delta E(Q_{0}^{(1)}) + c_e E(OH_0^{(1)})$$ (5)

$$E(C^{(2)}) = E(C_{0}^{(2)}) + c_h \sum_{i=1}^{T} \Delta E(OH_{i}^{(2)}) +$$

$$c_p \Delta E(Q_{0}^{(2)}) + (c_e - c_p) (E(Q_0^{(2)} I_0^{(2)}) - E(Q_0^{(2)} I_0^{(2)}))$$

When $i=t+1, t+2, \ldots, T$, the increments of on-hand inventory of R1 and R2 can be wrote as (6):

$$E(Q_0^{(0)} I_0^{(0)}) = \int_{0}^{\infty} \int_{0}^{\infty} (S_0^{(0)} - S^{(0)} + y) f_{1,2}(S^{(2)} + S^{(2)} - S_0^{(0)} - Q^{(2)} - y) f_{1,2}(y) dy$$

$$E(Q_0^{(2)} I_0^{(2)}) =$$

$$\int_{0}^{\infty} \int_{0}^{\infty} (S_0^{(2)} - S^{(2)} + y) f_{1,2}(S^{(2)} + S^{(2)} - S_0^{(0)} - Q^{(2)} - y) f_{1,2}(y) dy$$ (6)

So according to the above equations,

$$E(C^{(1)}) = E(C_{0}^{(1)}) + c_h \sum_{i=1}^{T} \Delta E(OH_{i}^{(1)}) + c_p \Delta E(OH_0^{(1)}) +$$

$$(c_e - c_p) (E(Q_0^{(2)} I_0^{(2)}) - E(Q_0^{(2)} I_0^{(2)}))$$ (7)
Similarly,
\[
E(C^{(2)}) = E(C^{(2)}_0) + c_h \sum_{i=1}^{T} \Delta E(OH_t^{(2)}) + c_p \Delta E(OH_t^{(2)}) \\
+ \left(c_e - c_p\right) E(Q_t^{(2)}_0) - \left(c_e - c_p\right) E(Q_t^{(2)}_0)
\]  
(8)

This problem can be solved as a two-objective optimization problem:
\[
\text{min } E(C^{(1)}), E(C^{(2)}) \\
S.t. \quad S_t^{(1)} \geq 0, S_t^{(2)} \geq 0
\]  
(9)

A. Emergency order analysis

We assume that the emergency ordering quantity of \( R_1 \) is \( \Phi_t^{(1)} \), the inventory level on \( t \) th day is \( m \). \( \Pi_t^{(1)} \) denotes the cost of \( R_1 \) when \( t \) gets emergency replenishment.

\( \Pi_t^{(1)} \) denotes the cost of \( R_1 \) when \( t \) does not get emergency replenishment.

\[
\Pi_t^{(1)} = c_h \sum_{i=1}^{T} \left[ \int_0^{m} F_{i-1}^{(1)}(x)dx \right] + c_p \left[ \int_0^{m} F_{i-1}^{(1)}(x)dx + \mu_t^{(1)}(T-t) - m \right]
\]  
(10)

\[
\Pi_t^{(0)} = c_h \sum_{i=1}^{T} E(OH_t^{(1)}) + c_p \times E(BO_t^{(1)}) + c_e \times Q_t^{(1)}
\]  

\[
= c_h \sum_{i=1}^{T} \left[ \int_0^{m+Q_t^{(1)}} F_{i-1}^{(1)}(x)dx \right] + c_p \left[ \int_0^{m+Q_t^{(1)}} F_{i-1}^{(1)}(x)dx + \mu_t^{(1)}(T-t) - m - Q_t^{(1)} \right] + c_e Q_t^{(1)}
\]  

(11)

So the inventory cost of \( R_1 \) drops to \( H_t^{(1)}(Q_t^{(1)}) \).

\[
H_t^{(1)}(Q_t^{(1)}) = \left(c_e - c_p\right)Q_t^{(1)} - c_h \sum_{i=1}^{T} \left[ \int_0^{m+Q_t^{(1)}} F_{i-1}^{(1)}(x)dx \right] - c_p \left[ \int_0^{m+Q_t^{(1)}} F_{i-1}^{(1)}(x)dx \right]
\]  
(12)

Corollary 1: we can prove, there is a value \( M_t^{(2)} \), greater than 0,\( \Pi_t^{(2)} \) when \( m \geq M_t^{(1)} \), \( R_1 \) doesn’t need replenishment; \( \Pi_t^{(2)} \) when \( m < M_t^{(1)} \), the emergency ordering quantity \( Q_t^{(1)} = M_t^{(1)} - m \). \( H_t^{(1)} \) peaks.

And \( Q_t^{(1)} = S_t^{(0)} - OH_t^{(1)} \).

We can obtain \( S_t^{(0)} \) by the equation:

\[
c_e - c_p - c_h \sum_{i=t+1}^{T} F_{i-1}^{(1)}(S_t^{(0)}) - c_p F_{T-t}^{(1)}(S_t^{(0)}) = 0
\]  
(13)

B. Emergency replenishment analysis

\( t \) days earlier, the holding cost of \( R_2 \) does not matter to emergency replenishment. Let \( \Pi_t^{(2)} \) denote the cost of \( R_2 \) that gets emergency replenishment, \( \Pi_t^{(2)} \) denote the cost of \( R_2 \) that does not get emergency replenishment.

When \( NS_t^{(1)} < S_t^{(0)} \), \( Q_t^{(2)} = S_t^{(0)} - NS_t^{(1)} \).

\[
\Pi_t^{(2)} = c_h \sum_{i=1}^{T} \left[ \int_0^{m+Q_t^{(2)}} F_{i-1}^{(2)}(x)dx \right] + c_p \left[ \int_0^{m+Q_t^{(2)}} F_{i-1}^{(2)}(x)dx + \mu_t^{(2)}(T-t) - m \right]
\]  
(14)

\[
= c_e \times E(BO_t^{(2)}) + c_e \times Q_t^{(2)}
\]  

(15)

\[
= c_e \sum_{i=1}^{T} \left[ \int_0^{m+Q_t^{(2)}} F_{i-1}^{(2)}(x)dx \right] + c_p \left[ \int_0^{m+Q_t^{(2)}} F_{i-1}^{(2)}(x)dx + \mu_t^{(2)}(T-t) - m - Q_t^{(2)} \right] - c_e Q_t^{(2)}
\]

When \( \Pi_t^{(2)} > \Pi_t^{(2)} \), which means

\[
H_t^{(2)}(Q_t^{(2)}) = (c_e - c_p)Q_t^{(2)} + c_h \sum_{i=1}^{T} \left[ \int_0^{m+Q_t^{(2)}} F_{i-1}^{(2)}(x)dx \right] + c_p \left[ \int_0^{m+Q_t^{(2)}} F_{i-1}^{(2)}(x)dx \right] > 0
\]  
(16)

Then, \( R_2 \) will provide emergency replenishment.

Corollary 2: we can prove, when \( OH_t^{(2)} > M_t^{(2)} \) and \( Q_t^{(1)} \leq OH_t^{(2)} - M_t^{(2)} \), then \( H_t^{(2)} > 0 \).

It follows that
\[ I_{i}^{(2)}(Q_{i}^{(1)}, OH_{i}^{(2)}) = \begin{cases} 1 & \text{if } OH_{i}^{(2)} - Q_{i}^{(1)} \geq Q_{i}^{(2)} \\ 0 & \text{otherwise} \end{cases} \] (17)

\[
(c_e - c_p) + c_h \sum_{i=1}^{T} F_{i}^{(1)}(Q_{i}^{(2)}) + c_p F_{i}^{(2)}(Q_{i}^{(2)}) = 0
\]

V. HEURISTIC POLICY

According to the above equations,

\[ E(C) = E(C^{(1)} + C^{(2)}) = E(C_i^{(1)}) + E(C_i^{(2)}) + \]

\[ c_h \sum_{i=1}^{T} \left[ \Delta E(OH_{i}^{(1)}) + \Delta E(OH_{i}^{(2)}) \right] + c_p \left[ \Delta E(OH_{i}^{(1)}) + \Delta E(OH_{i}^{(2)}) \right] \] (18)

Since the relationship between \(R_1\) and \(R_2\) is mutual, we can regard them as two independent systems. And the cost does not matter to \(c_e\). First, we calculate the inventory policy under normal order, then we obtain \(S^{(1)}\), \(S^{(2)}\) by the following equations:

\[ \sum_{i=1}^{T} F_{i}^{(1)}(S^{(1)}) + c_p \left[ F_{T-L}^{(1)}(S^{(1)}) - 1 \right] = 0 \]

\[ \sum_{i=1}^{T} F_{i}^{(2)}(S^{(2)}) + c_p \left[ F_{T-L}^{(2)}(S^{(2)}) - 1 \right] = 0 \] (19)

VI. EXAMPLE ANALYSIS

The parameter settings used were \(T=7\), \(L=3\), \(t=5\), \(c_h = 1\), \(c_p = 50\), \(c_e = 20\), \(\mu^{(1)} = 40\), \(\mu^{(2)} = 60\), \(\sigma^{(1)} = 20\sqrt{0.4}\), \(\sigma^{(2)} = 20\sqrt{0.6}\).

By using the MATLAB, we obtain

\[ Q^{(1)} = 83, Q^{(2)} = 123 \]

The results under normal order, unidirectional order and bidirectional order are as following:

<table>
<thead>
<tr>
<th></th>
<th>(S^{(1)})</th>
<th>(S_0^{(1)})</th>
<th>(S^{(2)})</th>
<th>(S_0^{(2)})</th>
<th>(E(C^{(1)}))</th>
<th>(E(C^{(2)}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>normal</td>
<td>444</td>
<td>--</td>
<td>654</td>
<td>--</td>
<td>1288.2</td>
<td>1808.8</td>
</tr>
<tr>
<td>unidirectional</td>
<td>440</td>
<td>84</td>
<td>654</td>
<td>124</td>
<td>1266.0</td>
<td>1802.8</td>
</tr>
<tr>
<td>bidirectional</td>
<td>444</td>
<td>84</td>
<td>659</td>
<td>124</td>
<td>1247.78</td>
<td>1773.23</td>
</tr>
<tr>
<td>heuristic policy</td>
<td>444</td>
<td>84</td>
<td>654</td>
<td>124</td>
<td>1250.0</td>
<td>1773.98</td>
</tr>
</tbody>
</table>

VII. SUMMARY

1. By using this policy will bring down costs for the two companies that cooperate with each other.
2. Under bidirectional order and replenishment, the sum of the cost of \(R_1\) and \(R_2\) drops to minimize. Meanwhile, they both cost less.
3. The heuristic algorithm is a feasible solution which can reduce costs.

REFERENCES