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Hierarchy Based on Configuration-Reader about *k*-Neighborhood Template $\mathcal{A}$-Type Three-Dimensional Bounded Cellular Acceptor

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Abstract

Blum and Hewitt first proposed two-dimensional automata as computational models of two-dimensional pattern processing—two-dimensional finite automata and marker automata, and investigated their pattern recognition abilities in 1967. Since then, many researchers in this field have investigated the properties of automata on two- or three-dimensional tapes. On the other hand, the question of whether or not processing four-dimensional digital patterns is more difficult than processing two- or three-dimensional ones is of great interest from both theoretical and practical standpoints. Thus, the study of four-dimensional automata as the computational models of four-dimensional pattern processing has been meaningful. From this point of view, we are interested in four-dimensional computational models, In this paper, we introduce a new four-dimensional computational model, *k-neighborhood template $\mathcal{A}$-type three-dimensional bounded cellular acceptor* on four-dimensional input tapes, and investigate about hierarchy based on configuration-reader about this model.

Keywords: cellular acceptor, configuration-reader, converter, finite automaton, four-dimension, on-line tessellation acceptor, parallel/sequential array acceptor, Turing machine

1. Introduction and Preliminaries

In 2002, we first introduced a four-dimensional automaton, and investigated some properties [4]. In general, in the multi-dimensional pattern processing, designers often use a strategy whereby features are extracted by projecting high-dimensional space on low-dimensional space. In this paper, from this viewpoint, we introduce a new computational model, *k-neighborhood template $\mathcal{A}$-type three-dimensional bounded cellular acceptor* (abbreviated as $\mathcal{A}$-3$BCA(k)$) on four-dimensional tapes, and discuss some basic properties. An $\mathcal{A}$-3$BCA(k)$ consists of a pair of a *converter* and a *configuration-reader*. The former converts the given four-dimensional tape to the three-dimensional configuration and the latter determines the acceptance or nonacceptance of given four-dimensional tape whether or not the derived three-dimensional configuration is accepted. When a four-dimensional input tape is presented to the $\mathcal{A}$-3*BCA*(k), a three-dimensional cellular automaton as the converter first reads it to the future direction at unit speed (i.e., one three-dimensional rectangular array per unit time). From this process, the four-dimensional tape is converted to a configuration of the converter which is a state matrix of a three-dimensional cellular automaton. Second, three-dimensional automaton as the configuration-reader, reads the configuration and determines its acceptance. We say that a four-dimensional input tape is accepted by the $\mathcal{A}$-3*BCA*(k) if and only if the configuration is accepted by the configuration-reader. Therefore, the accepting power of the $\mathcal{A}$-3*BCA*(k) depends on how to combine the converter and the configuration-reader. An $\mathcal{A}$-3*DBCA*(k) ($\mathcal{A}$-3*NBCA*(k)) is called a k-neighborhood template $\mathcal{A}$-type three-dimensional deterministic bounded cellular acceptor (k-neighborhood template $\mathcal{A}$-type three-dimensional nondeterministic bounded cellular acceptor). A *DA*[1] (*NA*, *DB*[5], *NB*, *DO*[2], *NO*, *DOP*[3], *NOP*, *DP*[3], *NP*, *DTM*[4], *NTM*) is called a three-dimensional deterministic finite automaton (three-dimensional nondeterministic finite automaton, deterministic three-dimensional bounded cellular acceptor, nondeterministic three-dimensional bounded cellular acceptor, three-dimensional deterministic on-line tessellation acceptor, three-dimensional nondeterministic online tessellation acceptor, deterministic three-way parallel/sequential array acceptor, nondeterministic three-way parallel/sequential array acceptor, deterministic four-way parallel/sequential array acceptor, nondeterministic four-way parallel/sequential array acceptor, three-dimensional deterministic Turing machine, three-dimensional nondeterministic Turing machine). Let $T(M)$ be the set of four-dimensional tapes accepted by a machine M, and let $\mathcal{L}$ [$\mathcal{A}$-3*DBCA*(k)] $=\{T|T=T(M)$ for some $\mathcal{A}$-3*DBCA*(k) $M\}$. $\mathcal{L}$ [$\mathcal{A}$-3*NBCA*(k)], etc. are defined in the same way as $\mathcal{L}$ [$\mathcal{A}$-3*DBCA*(k)].

Let Σ be a finite set of symbols. A *four-dimensional tape* over Σ is a four-dimensional rectangular array of elements of Σ. The set of all four-dimensional tapes over is denoted by $\Sigma^{(4)}$. Given a tape $x \in \Sigma^{(4)}$, for each integer $j(1 \le j \le 4)$, we let $l_j(x)$ be the length of x along the jth axis. The set of all $x \in \Sigma^{(4)}$ with $l_1(x) = n_1$, $l_2(x) = n_2$, $l_3(x) = n_3$, and $l_4(x) = n_4$ is denoted by $\Sigma^{(n_1,n_2,n_3,n_4)}$. When $1 \le i_j \le l_j(x)$ for each $j(1 \le j \le 4)$, let $x(i_1, i_2, i_3, i_4)$ denote the symbol in x with coordinates (i_1, i_2, i_3, i_4), as shown in Fig.1. Furthermore, we define $x[(i_1, i_2, i_3, i_4), (i'_1, i'_2, i'_3, i'_4)]$, when$1 \le i_j \le i'_j \le l_j(x)$ for each integer $j(1 \le j \le 4)$, as the four-dimensional input tape y satisfying the following conditions:

(i) for each $j(1 \le j \le 4)$, $l_j(y) = i'_j - i_j + 1$;

(ii) for each $r_1, r_2, r_3, r_4(1 \le r_1 \le l_1(y), 1 \le r_2 \le l_2(y), 1 \le r_3 \le l_3(y), 1 \le r_4 \le l_4(y))$, $y(r_1, r_2, r_3, r_4) = x(r_1 + i_1 \le 1, r_2 + i_2 \le 1, r_3 + i_3 \le 1, r_4 + i_4 - 1)$. (We call $x[(i_1, i_2, i_3, i_4), (i'_1, i'_2, i'_3, i'_4)]$ the $[(i_1, i_2, i_3, i_4), (i'_1, i'_2, i'_3, i'_4)]$-*segment* of x.)

We let each sidelength of each input tape of these automata be equivalent in order to increase the theoretical interest.

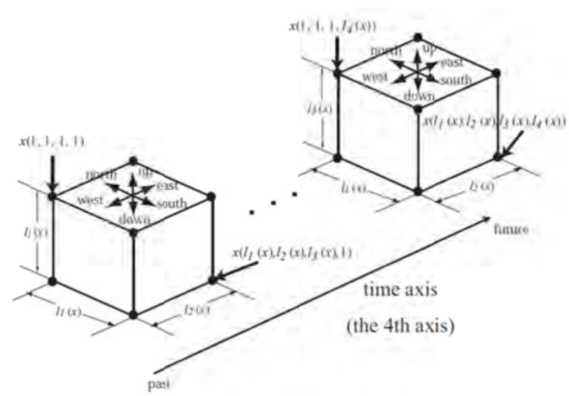

Fig. 1:Four-dimensional Input Tape.

2. Main Result

This section investigates how the difference of configuration-reader affects the accepting powers of $\mathcal{A}$-3*BCA*(k)'s. First, we start to investigate the case when the converter is deterministic.

Lemma 1. *Let* $T_1 = \{x \in \{0,1,2\}^{(4)} | \exists n \ge 1\ [l_1(x) = l_2(x) = l_3(x) = l_4(x) = n+1\ \&\ \exists i(1 \le i \le n)[x(i,n+1,n+1,n+1) = 2\ \&$ (*each symbol on the remaining parts is* "0"or"1")$\&x[(i,1,n+1,n+1),(i,n,n+1,n+1)] \neq x[(n+1,1,n+1,n+1),(n+1,n,n+1,n+1)]]]\}$. *Then*, (1) $T_1 \in \mathcal{L}$ [*NA*-3*DBCA*(1)], *and* (2) $T_1 \notin \mathcal{L}$ [*DA*-3*DBCA*(27)].

Proof: It is easily seen that there exists a nondeterministic three-dimensional finite automaton accepting the set of three-dimensional tapes which are obtained by extracting the bottom plane from the tape contained in T_1. Therefore, (1) holds. On the other hand, the proof of (2) is similar to that of Lemma 2(2) in [7].

□

Lemma 2. *Let* $T_2 = \{x \in \{0,1\}^{(4)} \mid \exists n \geq 1\ [l_1(x) = l_2(x) = l_3(x) = l_4(x) = 2n\ \&\ x[(1,1,2n,2n), (2n,n,2n,2n)] = x[(1,n+1,2n,2n), (2n,2n,2n,2n)]]\}$. *Then*, (1) $T_2 \in \mathcal{L}[DOP\text{-}3DBCA(1)]$, *and* (2) $T_2 \notin \mathcal{L}[NO\text{-}3DBCA(27)]$.

Proof: (1) Note that there exists a deterministic one-way
parallel sequential array acceptor accepting the set of two-dimensional tapes obtained by extracting the bottom plane of the first cube from the tape contained in T_2. It is easily seen from this fact that (1) holds. (2) The proof is similar to that of Lemma 2(2). Suppose that there exists an NO-$3DBCA(27)$ $M = (R, B)$ accepting T_2, where R is a converter and B is a configuration-reader. Let K be the set of each cell of $B \in NO$, and $|K| = s$. For each $n \geq 1$, let $V(n)=\{ x\in\{0,1\}^{(4)} \mid l_1(x) = l_2(x) = l_3(x) = l_4(x) = 2n\ \&\ x[(1,1,1,1), (2n,2n,2n,2n\text{-}1)] \in \{0\}^{(4)}$, $V'(n) = V(n) \cap T_2$, $W(n) = \{w \in K^{(2)} \mid l_1(w) = 2n\ \&\ l_2(w) = 1\}$ ($K^{(2)}$ means the set of all two-dimensional tapes over Σ .). For each $x \in V(n)$, let $\rho(x) \equiv$ the configuration of R just after reading x, $\rho_W(x)$ = the west half of $\rho(x)$, and $\rho_E(x) \equiv$ the east half of $\rho(x)$. Further, for each $x \in V'(n)$, let $\mathrm{Run}(x) = \{z \in K^{(2)} \mid z$ is a run of B on $\rho(x)$ whose lower right corner symbol is an accepting state of B.$\}$ and $r(x) = \{z[(1,n),(2n,n)] \mid z \in \mathrm{Run}(x)\} \subseteq W(n)$. Then, the following proposition must hold.

Proposition 1. *For any two different tapes x and y in* $V'(n)$, $r(x) \cap r(y) = \phi$.

[**Proof:** The proof is similar to that of Proposition 4 in [8].

□]

Proof of Lemma 2 (*continued*): As is easily seen,

$$|V'(n)| = 2^{2n^2} \text{ and } |W(n)| \leq s^{2n}$$

Therefore, it follows for large n that $|V'(n)| > |W(n)|$. Consequently, it follows for such large n that there must be two different tapes x and y in $V'(n)$ such that $r(x) \cap r(y) \neq \phi$. This contradicts Proposition 1. □

Lemma 3. *Let* $T_3 = \{x\in\{0,1\}^{(4)} \mid \exists n \geq 1\ [l_1(x) = l_2(x) = l_3(x) = l_4(x) = 2n\ \&\ x[(1,1,2n,2n), (n,2n,2n,2n)] = x[(n+1,1,2n,2n), (2n,2n,2n,2n)]]\}$. *Then,* (1) $T_3 \in \mathcal{L}[DP\text{-}3DBCA(1)]$, *and* (2) $T_3 \notin \mathcal{L}[NOP\text{-}3DBCA(27)]$.

Proof: (1) Note that there exists a deterministic four-way parallel sequential array acceptor accepting the set of three-dimensional tapes which are obtained by extracting the bottom plane of the last cube from the tape contained in T_3. It is easily seen, from this fact, that (1) holds.

(2) Suppose that there exists an NOP-$3DBCA(27)$ $M = (R,B)$ accepting T_3. Let s be the number of states of each cell of $B \in NOP$. For each $n \geq 1$, let $V(n) = \{x\in\{0,1\}^{(4)} \mid \exists n \geq 1\ [l_1(x) = l_2(x) = l_3(x) = l_4(x) = 2n\ \&\ x[(1,1,1,1), (2n,2n,2n,2n\text{-}1)] \in \{0\}^{(4)}]\}$, $V'(n) = V(n) \cap T_3$. For each $x \in V(n)$, let $\rho(x) \equiv$ the configuration of R just after reading x, $\rho_N(x) \equiv$ the north half of $\rho(x)$, and $\rho_N(x) \equiv$ the south half of $\rho(x)$. Furthermore, for each $x \in V'(n)$, let conf(x) $\equiv$ the set of possible configuration of B just after $\rho_N(x)$ is read, when $\rho(x)$ is accepted by B. (Note that $\rho(x)$ is accepted by B since each tape in $V'(n)$ is accepted by M.) Then, the following two propositions must hold. (The proofs are omitted here. If necessary, see proofs of Lemmas 7 and 8 in [6].)

Proposition 2. (i) *For any two tapes x and y in* $V'(n)$ *such that their* [(1, 1, 2n, 2n), (n, 2n, 2n,2n)]-*segments are identical*, $\rho_N(x) = \rho_N(y)$, *and* (ii) *For any two tapes x and y in* $V'(n)$ *such that their* [(n + 1, 1, 2n, 2n), (2n, 2n, 2n, 2n)]-*segments are identical*, $\rho_S(x) = \rho_S(y)$.

Proposition 3. *For any two different tapes x and y in* $V'(n)$, conf(x) $\cap$ conf(y) $= \phi$.

Proof of Lemma 3 (*continued*): As is easily seen,

$$|V'(n)| = 2^{2n^2}$$

Let $t(n)$ be the total number of different configurations of R just after reading north halves of configurations of R just after reading tapes in $V'(n)$. Clearly,

$$t(n) \leq s^{2n}$$

Therefore, it follows for large n that

$$|V'(n)| > t(n)$$

Consequently, it follows for such large n that there must be two different tapes x and y in $V'(n)$ such that conf(x) ∩ conf(y) ≠ ϕ. This contradicts Proposition 3. □

Lemma 4. *Let T_4 be the set of three-dimensional tapes described in Lemma 1 in* [7]. *Then*, (1) $T_4 \in$[*NB*-3*DBCA*(1)], *and* (2) $T_4 \notin$ 𝓛 [*DOP*-3*DBCA*(27)].

Proof: (1) It is easily seen that there exists a nondeterministic one-dimensional bounded cellular automaton accepting the set of three-dimensional tapes which are obtained by attracting the bottom plane from the tape contained in T_4. Therefore, (1) holds. On the other hand, the proof of (2) is shown Lemma 1 in [7]. □

Lemma 5. *Let* $T_5 = \{x \in \{0,1\}^{(4)} \mid \exists n \geq 1\ [l_1(x) = l_2(x) = l_3(x) = l_4(x) = 2n\ \&\ [x(1,1,2n,2n), (n,n,2n,2n)] \neq x[(n+1,n+1,2n,2n),(2n,2n,2n,2n)]]\}$. *Then,* (1) $T_5 \in$ 𝓛 [*NB*-3*DBCA*(1)], *and* (2) $T_5 \notin$ 𝓛 [*NA*-3*DBCA*(27)].

Proof: (1) It is easily seen that there exists a nondeterministic one-dimensional bounded cellular automaton accepting the set of three-dimensional tapes which are obtained by extracting the bottom plane from the tape contained in T_5. Therefore, (1) holds. On the other hand, the proof of (2) is similar to that of Lemma 2 in [7]. □

From the foregoing lemmas, we can obtain the following theorem when the converter is deterministic.

Theorem 1. *For each* $k \in \{1, 7, 27\}$,

(1) 𝓛 [*DA*-3*DBCA*(k)] ⊊ 𝓛 [*NA*-3*DBCA*(k)] ⊊ 𝓛 [*NB*-3*DBCA*(k)] = 𝓛 [*NO*-3*DBCA*(k)] ⊊ 𝓛 [*NOP*-3*DBCA*(k)] ⊊ 𝓛 [*NP*-3*DBCA*(k)]
(2) 𝓛 [*DB*-3*DBCA*(k)] ⊊ 𝓛 [*NB*-3*DBCA*(k)],
(3) 𝓛 [*DO*-3*DBCA*(k)] ⊊ 𝓛 [*NO*-3*DBCA*(k)],
(4) 𝓛 [*DB*-3*DBCA*(k)] ⊊ 𝓛 [*DOP*-3*DBCA*(k)] ⊊ 𝓛 [*DP*-3*DBCA*(k)], *and*
(5) 𝓛 [*DO*-3*DBCA*(k)] ⊊ 𝓛 [*DOP*-3*DBCA*(k)] ⊊ 𝓛 [*NOP*-3*DBCA*(k)].

Proof: It is clear from Proposition 1 in [7] that the inclusion relations hold. Therefore, below, we show that the proper inclusion relations held for each $k \in \{1, 7, 27\}$.

(1): It is obvious from Proposition l in [7] that 𝓛 [*NB*-3*DBCA*(k)] = 𝓛 [*NO*-3*DBCA*(k)]. From Lemma1, 𝓛 [*DA*-3*DBCA*(k)] ⊊ 𝓛 [*NB*-3*DBCA*(k)] holds, and from Lemma 5, 𝓛 [*NB*-3*DBCA*(k)]⊊𝓛 [*NB*-3*DBCA*(k)] holds. In addition, it is obvious from Proposition 1 in [7] that 𝓛 [*DOP*-3*DBCA*(k)] ⊆ 𝓛 [*NP*-3*DBCA*(k)]. It follows from this and Lemma 2 that 𝓛 [*NO*-3*DBCA*(k)] ⊊ 𝓛 [*NOP*-3*DBCA*(k)] holds. Further, it is also obvious from Proposition 1 in [7] that 𝓛 [*DP*-3*DBCA*(k)] ⊆ 𝓛 [*NP*-3*DBCA*(k)]. It follows from this and Lemma 3 that 𝓛 [*NOP*-3*DBCA*(k)] ⊊ 𝓛 [*NP*-3*DBCA*(k)] holds.
(2) and (3) : These are easily proved from Lemma 4 and Proposition 1 in [7].
(4) and (5) : These are also easily proved from Lemmas 4,5,6 and Proposition 1 in [7]. □

Next, we investigate the case when the converter is nondeterministic.

Lemma 6. *For each* $k \in \{1, 7, 27\}$,
(1) 𝓛 [*NO*-3*NBCA*(k)] ⊆ 𝓛 [*DA*-3*NBCA*(k)],
(2) 𝓛 [*NO*-3*NBCA*(k)] ⊆ 𝓛 [*DB*-3*NBCA*(k)], *and*
(3) 𝓛 [*NO*-3*NBCA*(k)] ⊆ 𝓛 [*DO*-3*NBCA*(k)].

Proof: (1) We prove only 𝓛 [*NO*-3*NBCA*(1)] (The other cases are proved similarly.) Let M= (R, B) be an arbitrary *NO*-3*NBCA*(1), and let K_R and K_B be the set of states of R and B, respectively. Further, let M' = (R', B') be a *DO*-3*NBCA*(1) which acts as follows for a given four-dimensional tape x with each sidelength is n ($n \geq 1$).
(i) Actions of the converter R'

At each time, each (i, j, k, l)-voxel ($1 \leq i, j, k, l, \leq n$) of R' simulates the action of the corresponding voxel of R on x at the same time. In parallel to this action, the voxel selects nondeterministically a state in K_B (we let $q(i, j, k, l)$ be the state) and stores the state in its state, when the voxel reads a symbol on the top plane of the first cube of *x*. Here, *q*(*i,j,k,l*) is a guessed state of *B* which the (*i, j, k, l*)-voxel of *B* will enter by reading the configuration of *R* just after reading *x*; *q*(*i, j, k, l*) will have been stored in the state of the voxel until *x* is completed to read.

(ii) Actions of the configuration reader *B'*

For each *i, j, k, l*($1 \le i, j, k, l \le n$), let *q*(*i, j, k, l*) be a state in K_R which the (*i, j, k, l*)-voxel of *R*' continues to simulate the action of corresponding voxel of R and enters. *B*' accepts a configuration of *R*' just after reading *x* if and only if the following two conditions are satisfied.

① For each *i, j, k, l*($1 \le i, j, k, l \le n$), the (*i, j, k, l*)-voxel can enter *q*(*i, j, k, l*) when it reads (*i, j, k, l*).

② *q*(*n,n,n,n*) is an accepting state of *B*.

It is easily seen that *T*(*M*') = *T*(*M*) for *M* = (*R',B'*).

This completes the proof of the lemma. □

From Lemma 6 and from Proposition 1 in [7], we can obtain directly the following theorem when the converter is nondeterministic.(The proof is omittied here.) It is of great interest to compare the following Theorem 2 with Theorem 1 mentioned for the deterministic case.

Theorem 2. *For each k* ∈ {1, 7, 27}, ℒ [*DA*-3*NBCA*(*k*)] = ℒ [*NA*-3*NBCA*(*k*)] = ℒ [*DB*-3*NBCA*(*k*)] = ℒ [*NB*-3*NBCA*(*k*)] = ℒ [*DO*-3*NBCA*(*k*)] = ℒ [*NO*-3*NBCA*(*k*)].

3. Conclusions

In this paper, we investigated how the difference of the configuration-reader affects the accepting powers of *k*-neighborhood template *A*-type three-dimensional bounded cellular acceptor(abbreviated as *A*-3*BCA*(*k*)). As the results, we showed that when the configuration-reader is deterministic, the *A*-3*BCA*(*k*) which is the converter is nondeterministic is more powerful than the *A*-3*BCA*(*k*) which is the converter is deterministic. However, this tendency is not always true when the configuration-reader is nondeterministic.

We conclude this paper by giving a few open problems.

(1) Accepting powers in the case of alternating version of configuration-reader.

(2) Closure properties of *A*-3*BCA*(*k*).

(3) Recognizability of topological four-dimensional input tapes by *A*-3*BCA*(*k*).

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