Erratum
Errata on “Kansei Impression and Automated Color Image Arrangement Methods”

(i) Page 62, Line 24 (left)
Error
From Eq.(3), we obtain Eq.(4) and (5) because $y_0=f(x_0)$ and $y_0 + dx = \phi(x_0 + dx)$.

Corrected
From Eq.(3), we obtain Eq.(4) and Eq.(5) because $y_0=\phi(x_0)$ and $y_0 + dx = \phi(x_0 + dx)$.

(ii) Page 62, Line 2 (right)
Error
Since $\phi'(x) = L\phi(x)$, according to Eq. (5), we derive following Eq. (7).

Corrected
Since $\phi'(x) = Lf(x)$, according to Eq.(5), we derive the following Eq. (7).

(iii) Page 63, Fig. 2
Error
Fig. 2. Conceptual image of the transform from uniform distribution (PDF) to the desired one.

Corrected
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Kansei Impression and Automated Color Image Arrangement Methods

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Abstract
This paper proposes a new color image arrangement method using an elastic transform on some kinds of axes. In this paper, we present the principle of our method using HMGD (Histogram Matching based on Gaussian Distribution). And we describe that the automated method applies the HMGD to input color image only when the image has single-peakedness in its histogram on the focused axis. And we describe about HPA-HMGD (Histogram Peakedness Adaptive HMGD) as improvement HMGD. We also show that the method gives a good Kansei effect in the case of applying the HMGD onto Brightness axis. Moreover, we compare processing results of the HPA-HMGD and HMGD, we show that HPA-HMGD is better.

Keywords: Automated color arrangement, elastic transform, histogram matching, HMGD, Curvature, HPA-HMGD

1. Introduction
Automated image processing for enhancement and/or arrangement of color images has been more familiar to us according to the spreading of Digital Camera, Smart Phone, DVD, etc.1-3 However, we consider that the research on the automated arrangement method that brings about good sensibility effect (or Kansei effect) is still on the way to practical use.
In this paper, we propose a novel color image arrangement method using an elastic transform based on histogram on some kinds of axes.4-6 As for the axes, there are Brightness axis and principal component one that can be obtained by Principal Component Analysis (PCA) in the RGB three-dimensional vector space that is an attribute space of color image. Especially, we explain the main principle of the proposed method in the case of using HMGD (Histogram Matching based on Gaussian Distribution).7 And we show that a curvature computation for the cumulative histogram of original image will be effective, in order to automatically detect whether the original image has single peak histogram or not. Then, we illustrate that the method using the HMGD works very well for input image that has single peak histogram on the Brightness axis.
Also, we illustrate that the results of improved HMGD which applies peak detection works better than present HMGD, which is HPA-HMGD (Histogram Peakedness Adaptive HMGD).

2. Elastic Transform based on Histogram Matching

2.1. Principle

We describe the principle of histogram based elastic transform in the following. Let \( f(x) \) and \( f(y) \) be two probabilistic density functions on real variables \( x \) and \( y \), respectively. The probabilistic density function (PDF) is corresponding to histogram of gray level image. However, the histogram is defined on discrete variable. In addition, let \( y = \phi(x) \) be a continuous and monotonous increase function between variables \( x \) and \( y \) as shown in Fig. 1.

In addition, let value of \( x \) be the range from 0 to \( L \). Accordingly, variable \( y \) ranges from 0 to \( \phi(L) \). Let \( P \) mean the probability. From the above definitions and Fig.1, we have Eq. (1) \~ (3).

\[
P(0 \leq x \leq L) = \int_{x=0}^{x=L} f(x)dx = 1 \quad (1)
\]

\[
P(0 \leq y \leq \phi(L)) = \int_{y=0}^{y=\phi(L)} g(y)dy = 1 \quad (2)
\]

\[
f(x)dx = P(x_0 \leq x \leq x_0 + dx) = P(\phi(x_0) \leq y \leq \phi(x_0 + dx)) = P(y_0 \leq y \leq y_0 + dy) = g(y)dy \quad (3)
\]

From Eq. (3), we obtain Eq. (4) and (5) because \( y_0 = \phi(x_0) \) and \( y_0 + dx = \phi(x_0 + dx) \).

\[
f(x)dx = g(y)dy = g(y)\phi'(x)dx \quad (4)
\]

\[
f(x) = g(y)\phi'(x) \quad (5)
\]

Thus, if we know the \( y = \phi(x) \) and \( g(y) \), then we have the \( f(x) \). Using the above equations, we derive the principle of HE. Let \( \phi(x) \) be defined by Eq. (6).

\[
\phi(x) = L \int_{0}^{x} f(x)dx \quad (6)
\]

Since \( \phi'(x) = Lf(x) \), according to Eq. (5), we derive following Eq. (7).

\[
f(x) = g(y)Lf(x) \quad (7)
\]

Therefore, we understand that if we take the transform function as Eq. (6), \( g(y) \) becomes uniform distribution. It corresponds to the HE processing, which means that function defined by cumulative histogram transforms the original histogram into the uniform one.

Inversely, if we define the transform function \( \phi(x) \) as an integral of desired PDF \( f(x) \) for example, Gaussian distribution, we can obtain the desired PDF using the \( \phi(x) \) and the uniform distribution such as Fig.2.

The abovementioned theory means that, if we combine the both transform, we can obtain the transformation from an original distribution (PDF) to a desired one. This means that an image with original histogram can be transformed into another image with desired histogram. We consider that it is the principle of the Histogram Matching (HM).
2.2. Elastic Transform on Axis

We can choose the abovementioned transform such as HE (Histogram Equalization) and HM (Histogram Matching) on arbitrary axis (for example, principal component axis) in the color attribute (RGB) space as shown in Fig. 3. For example, in the case where the HE processing on the Brightness axis is applied, the HE brings about image enhancement by contrast stretching. In addition, in the case of HE on a principal component axis in RGB space, we guess that the contrast stretching will be done along to a certain tone of color.

Fig. 4 shows examples of the HE on Brightness axis and PC (Principal Component) axis. Fig. 5 shows resultant examples of the HE and HMGD (Histogram Matching based on Gaussian Distribution) on Brightness axis. The HMGD processing shows a more moderate effect than HE. In this paper, we show that the HMGD is useful for the automated color image arrangement.

2.3. Detection of Histogram Peakedness by using Curvature Computation

In this section, we describe how we detect the histogram peakedness by using curvature. Let \( y \) be a function with respect to \( x \), the definition of the curvature \( R \) is given by Eq. (8).

\[
R = \frac{\frac{d^2 y}{dx^2}}{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}} \tag{8}
\]

Let \( g(x) \) be a Gauss density function with variance \( \sigma^2 \) and average \( a \). And also let \( g(x) \) be representing a histogram of input image whose pixel values from 0 to \( L \). That is,

\[
g(x) = \frac{K}{\sigma\sqrt{2\pi}} e^{-\frac{(x-a)^2}{2\sigma^2}} \tag{9}
\]

In Eq. (9), \( K \) means a coefficient that satisfies following Eq. (10).

\[
\frac{K}{\sigma\sqrt{2\pi}} \int_0^L e^{-\frac{(u-a)^2}{2\sigma^2}} du = 1 \tag{10}
\]

Let \( y = f(x) \) be a function representing the cumulative histogram. Then \( f(x) \) can be represented Eq. (11).

\[
f(x) = \int_0^x g(u)du = \frac{K}{\sigma\sqrt{2\pi}} \int_0^x e^{-\frac{(u-a)^2}{2\sigma^2}} du \tag{11}
\]
Since \( y = f(x) = \int_0^x g(u) \, du \), \( g(x) \) can be described as Eq. (12).

\[
\frac{dy}{dx} = g(x) = \frac{K}{\sigma \sqrt{2\pi}} \cdot e^{\frac{-(x-a)^2}{2\sigma^2}} \quad (12)
\]

By the same way,

\[
\frac{d^2y}{dx^2} = \frac{dg(x)}{dx} = \frac{K}{\sigma \sqrt{2\pi}} \cdot e^{\frac{-(x-a)^2}{2\sigma^2}} \cdot \left(-\frac{1}{2\sigma^2}\right) \cdot \left[2(x-a)\right]
\]

\[
= \left(\frac{K}{\sigma \sqrt{2\pi}} \cdot e^{\frac{-(x-a)^2}{2\sigma^2}}\right) \cdot \left(-\frac{(x-a)}{\sigma} \cdot (a-x)\right) = g(x) \cdot \left(\frac{a-x}{\sigma}\right) \quad (13)
\]

Hence, we obtain the following Eq. (14) and we can approximate to Eq. (8).

\[
R = \frac{(a-x)^2}{\sigma^2} g(x) \approx \frac{(a-x)^2}{\sigma^2} \cdot \frac{g(x)}{\left[1 + g(x)^2 \right]^{\frac{1}{2}}}
\quad (14)
\]

The curvature \( R \) varies the sign according to the value of \( x; (x < a) \ R > 0, (x = a) \ R = 0, (x > a) \ R < 0 \). So we consider that a histogram with single peak shows the same characteristic regarding the curvature \( R \) of the cumulative histogram.

Fig. 6 shows an example of the curvature computation.\(^7\)

The real curvature computation is performed on the discrete values. We can find that there are largely changing points in the curvature from plus value to minus value at more than one point. So, we easily understand that the histogram of the original image is not a single peak type.

### 2.4. Improvement of HMGD

In the previous section, we described how we detect histogram peakedness by using curvature computation. Therefore, we consider that the HMGD will work better if the peakedness of the original image histogram is fit the peakedness of HMGD-reference histogram.

The definition of the approximation of curvature \( R \) is given by Eq. (14) as shown in the previous section. From Eq. (14), we can find the peakedness brightness values of the original image.

Let \( P_a \) be a peakedness brightness value of the image which have single peak histogram. From Eq. (9) and (10), we can derive the reference Gaussian distribution function \( g_r(x) \) that the average \( a \) correspond to \( P_a \), as shown in Eq. (15).

\[
g_r(x) = \frac{K}{\sigma \sqrt{2\pi}} \cdot e^{\frac{(x-P_a)^2}{2\sigma^2}} \quad (15)
\]

In above equation, \( K \) means a coefficient that satisfies Eq. (16).

\[
\frac{K}{\sigma \sqrt{2\pi}} \int_0^L e^{\frac{(u-P_a)^2}{2\sigma^2}} \, du = 1 \quad (16)
\]

Fig. 7 shows an example of the HPA-HMGD (Histogram Peakedness Adaptive Histogram Matching based on Gaussian Distribution). The HPA-HMGD processing shows a better result than HMGD.
3. Experimentation

3.1. HMGD

Fig. 8 (a), (b) shows an example of results by HMGD processing on Brightness axis, where original image has almost single peak histogram. The set of histogram and cumulative histogram (original image and HMGD image) are shown in Fig. 9(a), (b). And the curvature of cumulative histogram of input image is shown as well in Fig. 9(c). From Fig. 9 (b), we understand that the cumulative histogram of HMGD image clearly approaches to Gaussian distribution.

Similarly, Fig. 10 (a), (b) shows another example of results by HMGD processing, where original image has almost single peak histogram. For these cases where the histogram of input image has almost single peak, the HMGD works very well for resulting a good feeling impression (or Kansei effect).

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Fig. 8. Example of the result by HMGD.

(a) Original image

(b) HMGD image

Fig. 9. Histogram, cumulative histogram, and curvature of Fig. 8.

(a) Histogram (left) and cumulative histogram (right) of original image (Fig. 8(a))

(b) Histogram (left) and cumulative histogram (right) of HMGD image (Fig. 8(b))

(c) Curvature of cumulative histogram of original image (Fig. 8(a))
3.2. Comparison of HMGD and HPA-HMGD

Fig. 11 (a) ~ (c) shows example of results of HMGD and HPA-HMGD processing which the original image have multiple peakedness histogram. The set of histogram and cumulative histogram (original image, HMGD image and HPA-HMGD image) are shown in Fig.12 (a) ~ (c). And the curvature of cumulative histogram of input image is shown as well in Fig.12 (d).

From Fig.11, we consider that the result of HPA-HMGD processing image (Fig.11 (c)) is better than the result of HMGD processing image (Fig.11 (b)). Especially, tone of color of Fig.11 (c) is the most natural.
(a) Histogram (left) and cumulative histogram (right) of original image (Fig. 11(a))

(b) Histogram (left) and cumulative histogram (right) of HMGD image (Fig. 11(b))

(c) Histogram (left) and cumulative histogram (right) of HPA-HMGD image (Fig. 11(c))

(d) Curvature of cumulative histogram of original image (Fig. 11 (a))

Fig. 12. Histogram, cumulative histogram, and curvature of Fig. 11.

Fig. 13. Example of the results by HMGD and HPA-HMGD.
Fig. 13 (a) ~ (c) shows example of results of HMGD and HPA-HMGD processing which the original image has single peakedness histogram.

From Fig. 13, we understand that the result of HMGD processing image (Fig. 13 (b)) is brighter than HPA-HMGD processing image (Fig. 13 (c)). And, we have to take into account that the tone of HPA-HMGD image (Fig. 13 (c)) is close to original image.

In these cases HPA-HMGD have a good feeling impression (or Kansei effect) than HMGD. And we found that HPA-HMGD is corrected image into natural tones than HMGD.

4. Conclusion

Aiming at automated affective color image arrangement, we have proposed a concept of Elastic Transform (ET) method based on histogram and/or histogram matching, which can be applied to some kinds of axis such as Brightness axis. And we have suggested that HMGD (Histogram Matching based on Gaussian Distribution) is regarded as one of the ET method.

As for the concrete method for automated color image arrangement, we have proposed a method that applies HMGD processing to input image after detecting whether the input image has single peak histogram or not. In this paper, we have also provided the curvature computation for the peak detection in histogram. And also we have shown some experimental results.

Furthermore, we suggested the HPA-HMGD as the improvement of HMGD. Through some experimental, we understand that the tone of HPA-HMGD image is more natural than the tone of HMGD image. We consider that HPA-HMGD processing method will be useful and promising than HMGD.

References