Optimal Trading Strategy for Gold Based on Optimal Stopping Time

Chiahao Chuen[1, a †], Yuning Dong[2, b†], Chenghao Qiu[3, c†] and Ai Wang[4, d*, †]

[1] Shenzhen Senior High school, high school, Shenzhen, China
[2] Anhui University, School of Applied Statistics, Hefei, China
[3] Sun Yat-sen University, School of Mathematics, Guangzhou, China
[4] Sun Yat-sen University, School of Mathematics, Zhuhai, China
†These authors contributed equally.

[a]sid@cn-school.com
[b]r22014004@stu.ahu.edu.cn
[c]qiuchh6@mail2.sysu.edu.cn
[*d]wangai5@mail2.sysu.edu.cn

Abstract. The optimal stopping time is of profound significance in statistics, mathematics and finance, and can be used to derive optimal choices from uncertain problems such as volatile golden market. The paper mainly focuses on studying the problem of optimal stopping time by using the basic theory of Brownian motion and resolving the optimal time node for trading gold at a specified time in which tendency of golden price is known. Brownian motion, one of the basic theories in the stock market, solves probabilistic random problems and helps calculate the time node closet for best-selling time in formula. This paper uses Brownian motion as a research method to calculate optimal stopping time. Then the data of golden price is selected from 2021 to 2022 as a model and used for analysing boundary and retracement state of specific golden price to determine the optimal stopping time that maximal trading revenue generates. Therefore, this paper provides a common method to study best stopping time in bull market to derive the optimal profit. The empirical results verify the feasibility and operability of the optimal stopping time model in the investment market. Investors can use boundary value and retracement value as reference data to sell gold in time in order to avoid huge losses. Investors can also adjust the parameters of the model according to their own investment strategies, so that the calculated results of the model can meet individual needs.

Keywords: Optimal Stopping Time; Brownian Motion; Golden Price.

1 Introduction

The optimal stopping time, a mathematical problem combined with probability and the control theory, aims to find the best solution to some applications with actual problems. The optimal stopping time, studied extensively in recent years, has a wide range of

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applications such as finance and science. Because of the existence of gambling, opportunity cost and information asymmetry, optimal stopping time are widely used for many particular problems, such as hiring choices, finding suitable rents, the stock market and gold trading. In real life, there always exists problems that considered having the characters of randomness, uncertainty and unpredictability. It is undeniably necessary to consider when to stop optimally among a large amount of data and complex processes. Optimal stopping time can induce the best solution finally and be utilized to provide an optimal strategy to some specific and intricate problems. What's more, the price of gold trade is highly concerned in the economically international trade. This paper mainly focuses on analysing price tendency of gold, using Brownian motion to determine the optimal stopping time of gold price. Gold prices are volatile but relatively stable compared with stock price in competitive markets. It is necessary to do sufficient research on the best stopping time over a period of time to obtain maximum benefits. Finding the best time to stop when the choice of gold price is optimal becomes a significant problem to people who have to make international transaction in gold. Therefore, this paper studies the gold price movements from 2021 to the present and gives the optimal stopping time to maximize profits by the method of Brownian motion.

Using equivalent functions of semi-Markov processes, the optimal stopping time is solved by Chen, Guo and Liao, which is of great reference significance to the study of this paper [1]. Optimal stopping time can be applied in many practical problems, best time to choose stock price and gold trading, for example. In the stock market, Consumer can use optimal stopping problem to achieve the goal of maximizing the profits [3]. They reject random walk model. This article studies time series dependence in the direction of stock prices for bull and bear market, by modelling the probability in a mathematical way, such as interest rates. Yun and Li studied semi-Markov decision process by using data-transformation method, which can also be used to an optimization model for semi-Markov decision processes (SMDPs) [9]. By the process of approximating the continuous random process from the general discrete process, a strict proof of the change of stock price according to the geometric Brownian motion with drift rate is given by Xie, Chen and Bi. In this paper, the general model of stock price process is pointed out [8]. Pemy and Zhang used geometric Brownian motions and a finite-state Markov chain to study the problem of optimal selling rule, which illustrated the optimal selling time in the field of finance explicitly [6]. Zhan studied the best-selling time of Chinese stock market by using the optimal stop-time method by verifying the validity of the optimal stop-time model in the two biggest bull markets, providing a new analytical perspective which is of great importance to our study of optimal stopping time in gold price [11]. Zhao, Xu, and Gu improved the traditional time series analysis method and proposed a time series analysis method based on the graph model [12]. Using this method, the short-term volatility of the gold price is predicted, pointing out that the volatility of the gold price tends to increase. However, under the influence of other factors in the market, it will not grow indefinitely, and eventually show a rapid decline after a high point every period of time. Wang, Chen and Li used Markov chain to make a prediction model for the fluctuating gold price for scientific prediction [7]. Zeng 's article pointed out that the price of gold has maintained an upward trend in the long run, giving buyers more confidence in holding gold [13]. Livieris, Pintelas and

Pintelas proposed a new deep learning model and uses convolution formula to predict gold price with volatility and movement. The fluctuation, dynamics and volatility are analysed by the utilization of LSTM layers. In their research, a new method, deep learning, is proposed to offer a more significant forecasting and predicting model that deserves to be further studied [4]. The gold price fluctuation has been explored from the perspective of complex network theory, utilizing corresponding sequence of price fluctuations. The time series and gold price series are explored to demonstrate that this multifractal series originates from persistent and short-term factors. The mechanism of gold price fluctuation has been studied, which is helpful to better understand gold price fluctuation over time in this paper [10]. Hussin, Rahman and Bahar utilized geometric Brownian model, which is based on historical and discrete model to do reference estimation and statistical analysis of gold price samples because of the importance of predicting gold price fluctuations. The utilization of geometric Brownian motion provides an important reference for the optimal stoppage problem in this paper [2].

Brownian motion, a common physical way to solve the problem of randomness, can make the optimal choice solved in different situation and it can be used in different fields like IT, medical treatment and commerce. What's more, this kind of method is very prevailed in finance. However, it is found that the study in the most recent gold price analysis of the optimal stopping time problem for 2021 and 2022 is incomplete. Therefore, this research aims to solve the optimal stopping time by the method of Brownian motion under the model of recent international basic gold price.

2 Methodology

2.1 Preparations and Theoretical Foundation

The price of gold is volatile. Hence, it is very important for investors to choose the best time for selling. Considering that the fluctuation of gold price is random, the investor's goal is to find the best-selling point of the overall situation according to the real-time value information. In this paper, the optimal stopping time method is adopted to evaluate the price of gold through the existing data, and calculate the time node closest to the best time for selling. It is known that the stock price obeys geometric Brownian motion. As an important financial product, the price fluctuation of gold is similar to other financial products such as stocks and options. This paper will use the Brownian motion model to study the optimal stopping time of gold trading.

2.2 Theoretical Basis of Method and Processing

Suppose $B = (B_t)_{t\geq 0}$ is Brownian motion defined in probability space, $B^{\mu} = (B_t^{\mu})_{t\geq 0}$ represents Brownian motion with drift term, and it satisfies:

$$B_t^{\mu} = \mu t + B_t \tag{1}$$

When the value of gold reaches the maximum, investors can get the maximum profit by selling it. Define the maximum value process S_t^μ to indicate the maximum value of B_t^μ on $[0,t]$:

$$S_t^\mu = \sup_{0\le s\le t} B_t^\mu \tag{2}$$

Assuming that s is at any time on $[0,t]$, it is uncertain whether S_t^μ is S_s^μ. This *shows* that the local maximum on $[0,s]$ cannot be used to directly determine the maximum on a larger interval $[0,t]$. In other words, the global maximum of gold price is not measurable in real time. This is an *optimal* stopping time problem. It is necessary to build a related mathematical model to find the selling point that is closest to the maximum price of gold and can be measured in real time, so as to maximize the profit from selling gold.

Without losing generality, only the case on the interval $[0,1]$ is considered here. Define the time to get the global maximum of B_t^μ as follows:

$$\theta = arg\max_{0\le t\le 1} B_t^\mu \tag{3}$$

In order to get the maximum profit, it is ideal to sell gold at the moment closest to θ. The optimal stopping time problem described here is:

$$V_\mu = inf\, E(|\tau - \theta|) \tag{4}$$

Following gives the solution of the optimal stopping time problem when the drift term $\mu = 0$.

Let the probability density function of standard normal distribution be expressed as:

$$\varphi(x) = \frac{1}{\sqrt{2}} e^{-\frac{x^2}{2}} \tag{5}$$

Let the distribution function be expressed as:

$$\Phi(x) = \int_{-\infty}^{x} \varphi(y)\, dy \tag{6}$$

Optimization function V satisfies:

$$V = 2\Phi(z^*) - 1 \tag{7}$$

And z^* is the only solution to the following equation:

$$4\Phi(z) - 2z\varphi(z) - 3 = 0 \tag{8}$$

Because of $S_t^0 = \sup_{0\le s\le t} B_s^0$, the optimal stopping time τ^* satisfies:

$$\tau^* = inf\{0 \le t \le 1 : S_t^0 - B_t^0 \ge z^*(1-t)^\beta\} \tag{9}$$

The formula (9) is the criterion for judging the optimal stopping time.

If the fluctuation of gold price meets the Brownian motion pattern, $S_t^\mu - B_t^\mu$ indicates the retracement size of the gold price, let $\Delta = S_t^\mu - B_t^\mu$. If $S_t^\mu = \sup_{0 \le s \le t} B_t^\mu$, $\Delta = 0$. Else, $\Delta > 0$. The optimal stopping time τ^* is the moment when the retracement of gold price exceeds the boundary $z^*(1-t)^\beta$ for the first time. This solution is similar to the method of Lunde and Timmermann. However, there differences. They are shown as follows. First, the boundary value of this paper is a monotonically decreasing function of time, which indicates that the probability of gold price falling increases with the increase of time. Second, coefficient z^* is determined by normal distribution function. Third, this paper uses the optimal stopping time problem and get the global inference according to the real-time data. Peskir and Shiriaev gave a detailed proof of the final solution when $\mu = 0$ [5].

3 Results and discussion

3.1 Data Selection and Analysis

The gold price data used in this paper comes from the National Bureau of Statistics. The two selected sample intervals are from January 2, 2006 to August 29, 2006, and from July 24, 2013 to March 12, 2014. The price changes in these two periods are obvious and have practical significance. Both of these two sets of data will demonstrate the optimal stopping time for gold price in the bull market. Therefore, this paper selects them as the sample interval of the experiment. Gold prices, which is the fundamental data source of the research, are firstly shown to have a brief look at the upward and downward trend. Boundary and retracement curves are the main research results that are supported by rigorous analysis and theory of Brownian motion, so that the moment of maximum profit in the bull market is found. Tests of the both time intervals are shown as follows.

3.2 First Test from January 2006 to August 2006

In the sample interval that is selected from January 2, 2006 to December 31, 2008, according to the interval length, set time range T=172 days. Figure 1 is the trend chart of gold price in this interval. According to the calculation method of drift term μ=0 mentioned in the method, the curves of retracement value and boundary value are drawn respectively. When the retracement value is above the boundary value for the first time, it is the optimal stopping time. Therefore, the intersection of the two curves is the key point of the whole chart. It means both ultimately the optimal stopping moment and the market price of gold at which the maximum benefit is achieved. Figure 2 is a graph of retracement value and boundary value in this sample interval. According to the calculation of this model, the optimal stopping time in this section is on May 18th, 2006. It can be seen from the image that near the beginning of the interval, the boundary function value is larger, and the corresponding tolerable retracement degree is higher. Since May 12th, 2006, the price of gold in the back has shown a downward trend. On May

18th, 2006, the retracement value exceeded the boundary value, and the optimal stopping time was obtained. If investors sell gold in time on this day, they can avoid further losses.

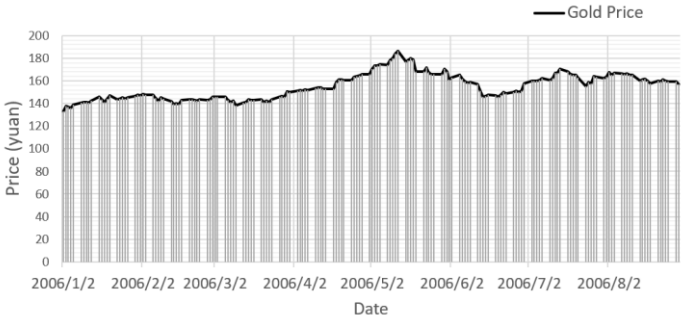

Fig. 1. Gold Price from January to August 2006

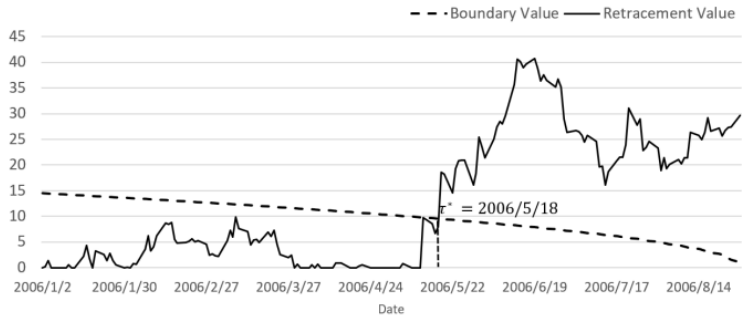

Fig. 2. Boundary and Retracement during 2006

3.3 The Second Test from July 2013 to March 2014

In the sample interval from July 24, 2013 to March 12, 2014, according to the interval length, set time range T=172 days. Figure 3 is the trend chart of gold price in this interval. Figure 4 is a graph of retracement value and boundary value in this sample interval in the same way. As is shown in Figure 2, the optimal stopping time in this section is on September 11th, 2013 according to calculation method. According to the comparison between images and data, even greater losses can be avoided in this interval.

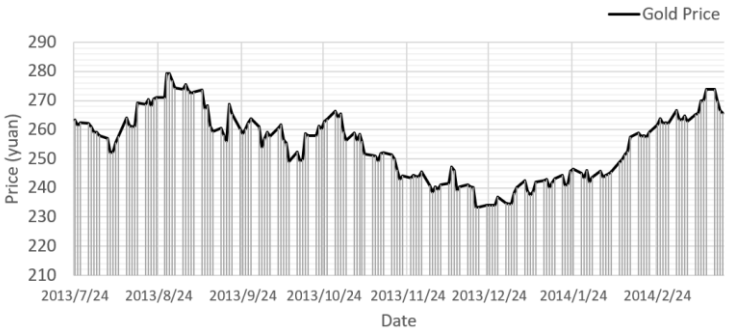

Fig. 3. Gold Price from July 2013 to March 2014

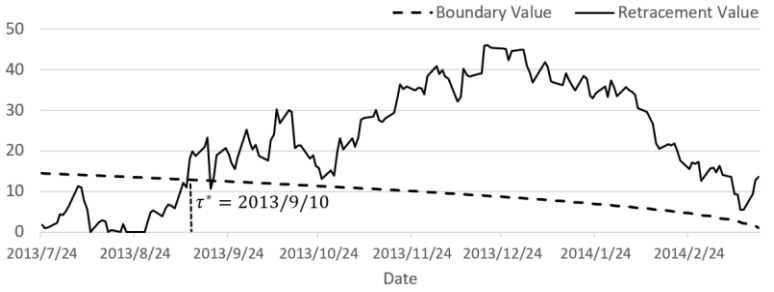

Fig. 4. Boundary and Retracement from 2013 to 2014

3.4 Discussion about the Results

From the empirical results, the optimal stopping time model not only considers the fluctuation characteristics of the gold price itself, but also the duration of the bull market. Obviously, for gold, which is less volatile, the optimal stop time is more traditional, classic and intuitive. For more volatile gold prices, forecasting models are harder to control. At the initial stage, the boundary function value is large, so the disturbance and retracement caused by market noise can be filtered out. However, with the prolongation of the duration, the inertia and momentum of the gold price gradually weaken. When the market retreats to a certain extent, it will easily lead to investors' panic, and the possibility of a sharp decline is constantly increasing. Therefore, the boundary function decreases monotonously correspondingly, so that a smaller retracement will also trigger the selling signal, so as to avoid investors from taking the risk of possible drastic stock price fluctuations.

3.5 Expanded Research

The above two experiments were conducted under the condition that $\beta=0.5$ in formula (9). Investors can further adjust the value of β according to the risks they can bear. Here, this paper will conduct a simulation experiment to simply demonstrate the function of

parameter β so that it offers a deeper understanding and insight about the selection of the value of β and the method of obtaining optimal stopping moment. Now assume that the stock price and time show a linear decreasing relationship. In this simulation experience, suppose investors buy gold on the first day when the price of gold is per gram of 300 yuan, and then the price of gold per gram drops by 0.2 yuan every day for 170 days. Figure 5 is a curve drawn according to the simulated gold price data. Figure 6-8 is a graph of retracement value and boundary value when β is 0.25, 0.5 and 0.75 respectively. It is shown that for different values of β, the optimal stopping time tends to move backward as the value of β goes up.

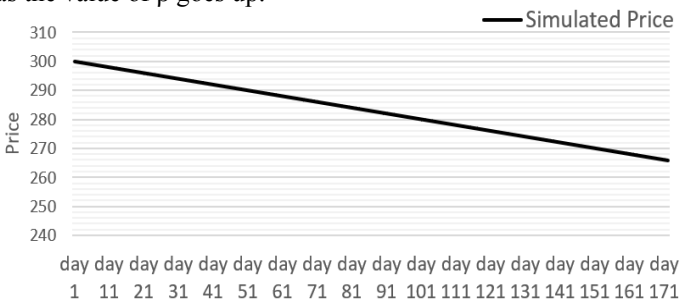

Fig. 5. Simulation of Price Trend

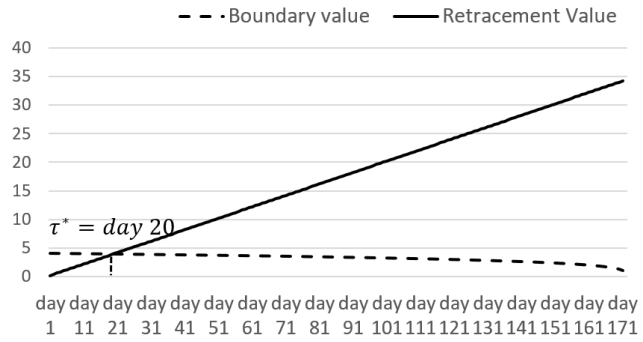

Fig. 6. Boundary and Retracement with $\beta = 0.25$

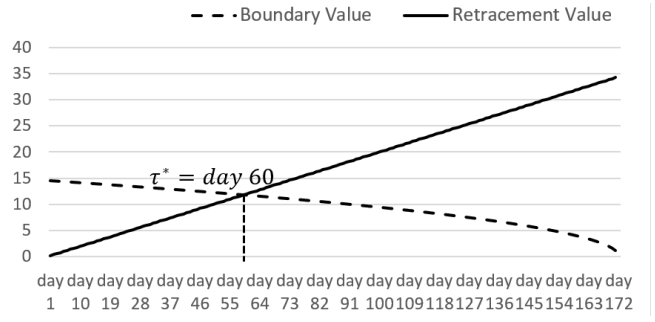

Fig. 7. Boundary and Retracement with $\beta = 0.50$

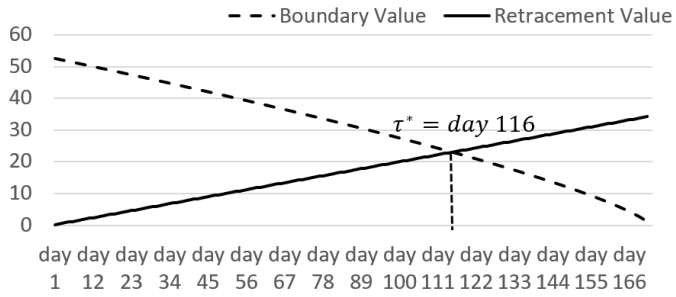

Fig. 8. Boundary and Retracement with β=0.75

The optimal stopping time and loss when β takes different values as shown in TABLE 1. The larger β is, the greater the tolerable retracement value is. If investors choose short-term investment and don't want to take great risks, they can choose the best stopping time when β is small. If investors have a long-term vision, are committed to long-term investment, and can bear a certain amount of losses in a certain period of time, they can refer to the optimal stopping time when β is large. Investors can adjust the size of parameter β according to their own strategies and financial resources. The optimal stopping time model in this paper can be adjusted according to investors' investment orientation, and has good practicability and operational space.

Table 1. Optimal Stopping Time and Loss Rate at Different Values of β

β	τ^*	Loss Rate
0.25	20	-1.27%
0.50	60	-3.93%
0.75	116	-7.67%

Due to the purpose and timeliness of investment, investment can be divided into long-term investment and short-term investment. The difference between two kinds of investment and two kinds of financial management is not only the length of time. For the futures investment of gold, stocks and so on, three elements should be considered: income, risk and time.

Long-term investment is wise for many investors, avoiding many unnecessary risks. However, the risk of investment cannot be completely avoided, so what investors can do is to minimize the risk of loss. The most easily encountered risks in long-term investment are stock market crash, economic recession, company change and industrial change, etc. Short-term investment returns quickly and has high liquidity. However, the corresponding risk is high and the income is not high, so it is difficult to make a profit as long as it is.

This paper does not explain the investment strategies in detail, but only demonstrates the diversity of investment strategies and the importance of timely adjustment of strategies, thus illustrating the practicability of this model. If an investor decides to make a long-term investment at a certain stage, the parameter β can be appropriately raised.

The higher β is, the greater the tolerable retracement value is, which is suitable for investors who are not entangled in the current fluctuations and are not eager to sell. If investors can't bear high risks, they can lower the parameter β moderately and stop the loss in time before the price drops sharply.

3.6 Common Mistakes and Error Analysis

There is a slight error in the choice of best stopping moment because the time at which the two lines intersect is between two exact dates. Therefore, approximate estimation is used to select the date moment closest to the intersection when retracement value curve surpasses the boundary value curve for the first time. However, this type of error is small enough to be ignored because the date obtained can still be defined as the optimal stopping time.

4 Conclusions

Because the global maximum value of gold price can't be judged by the existing data, in order to analyse the best-selling time of gold in real time, this paper constructs the optimal stopping time model, and obtains the optimal stopping time by the correlation between the boundary value and the retracement value. Among them, the boundary value is a monotonically decreasing function with time, which is more in line with reality, indicating that with the increase of the duration of the bull market, the possibility of a bear market is greater. This paper uses this model to empirically analyse the gold price trends from January 2, 2006 to August 29, 2006 and July 24, 2013 to March 12, 2014. The results show that, based on certain preconditions, this model can sell gold before the price of gold drops sharply, which is better than the "buy and hold" strategy. For investors and listed companies, the retracement of stock price is a risk signal. During the bull market, when the retracement of the stock price exceeds the boundary value, it means that there may be a sharp decline. Investors can sell stocks in time to avoid losses, and listed companies can also make preparations for risk prevention and control.

The monetary property of gold is special, and its value is relatively stable. It leads to the result that the probability of violent fluctuation is small. However, for investors and listed companies in the stock market, the retracement of the stock price can be regarded as a risk signal, and a suitable boundary function can be calculated by using this model. In this paper, a simple simulation experiment verifies the operational space of parameter. Investors and listed companies can make different optimal stopping time models according to their own strategies and economic strength. In this way, investors can sell stocks in time to avoid losses, and listed companies can also make preparations for risk prevention and control.

The following are suggestions for future research. First, use more data to test the validity of the model. Although the data used in this paper is typical, more data with different properties can make the model more reliable, and it can also be found that the model has wide application significance, so that the model can play a role in more fields. Second, different data can be used to test the nature of the prediction model itself.

Experiments have found that the predicted optimal stopping time is generally slightly later than the normal peak. Is this phenomenon accidental or persistent? And how long is the delay? Is this time stable with large amounts of data? This is of great significance to model optimization. When selecting data, it is necessary to carry out screening. It is necessary to carry out extensive screening according to the proportion of data appearing, and it is necessary to consider the influence of realistic factors so as to make statistics more realistic and practical. In fact, if it can be shown that all the above-mentioned things exist, it is possible to use some mathematical statistics methods, such as maximum likelihood estimation, to estimate the delay time. Third, the value of βcan be trained by means of reinforcement learning. The specific method is to first select an initial value of β and use different data to find theβwith the best prediction effect from that data and do gradient descent on it. The difficulty of this method lies in the selection of super-parameters and the acquisition of training data. Multiple experiments should be carried out to find the best super-parameters, and great care should be taken in the acquisition of data, otherwise erroneous results may be produced. The knowledge of semi-Markov chains can also be incorporated when the parameter is processed by reinforcement learning. Using the method of semi-Markov chain, the historical data are also added to the research, and different weights can be set for the gold price in different periods.

As the price of gold changes with the changes in the market, it will show different laws under different market conditions in different periods. At present, the only thing we know is that the current market rules will be similar to those of the recent period. Therefore, it is easy to think of setting the weight of the recent gold price higher, so that the recent price rules will have a larger proportion of influence. In addition, in this process, the trend of gold price in different periods needs to be analysed. For example, whether the previous law is similar to the present, whether it presents a periodic law, and so on. This is beneficial to better processing the data. This is beneficial to better processing the data. If macro analysis and micro treatment are combined, better prediction results may be obtained. At the same time, in the process of this forecast, the impact of the actual situation is also very important and cannot be ignored. The research should combine with the changes of realistic policies and international situation, and reasonably use the model in this paper to forecast the gold price. The two complement each other in order to achieve better results.

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