

# Standard Option and Power Option of Amazon.Inc and Aensitivity Analysis

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## ABSTRACT

Option, used by speculators in directional trades, volatility trades and combination trades is one of the most important tools in the development of finance. It can be divided into various types such as the standard option, the power option, the double option. In this article, based on the model of BSM and the data of Amazon from Yahoo finance, we use three parameters including the stock price, volatility and the strike price to do the sensitivity analysis of the standard call option and the power call option. The analysis is meaningful for the reason that people can use it to decide which one they should to buy in an effort to gain more profits. We find that the power option has a higher value and can bring more returns to consumers than the standard option because of the higher volatility and the greater price gap between the spot price and strike price compared to the standard option. This paper provides detailed comparisons and convinced reasons to support an idea that people can profit more from the power option than the standard option.

**Keywords:** *The standard option, The power option, BSM model, Sensitivity analysis, High profits.*

## 1. INTRODUCTION

An investment portfolio is any combination of financial assets which held by the investors and financial institutions, which can involve stocks, bonds, real estates and other assets. The main purpose of investment portfolios is separating and hedging risks. As an effective way to diversify the assets, portfolios can help the investors beware of the possible large fluctuations in the future and optimize the expected return. Options, as a kind of financial derivatives, are common and popular in the contemporary investment portfolios. An option refers to a contract, which provides the buyer the right to buy or sell an underlying asset or financial instrument at an previous assigned strike price on or before the maturity date. In the trading process of options, the rights and obligations of two sides are not equal. After the long side paid the premium, the short side need to take unlimited liability and deliver the asset if the buyer wants it, which is different from the traditional monetary instruments. In the current global financial market, there are various types of existed options, which have obvious distinctions between their calculations of payoffs.

Therefore, it is of great significance for the issuers to develop the specific pricing method for each type of option, as this will ensure their profits by taking the premiums. This paper has carried out related research on the assumed pricing model of the power option of Amazon.

The pricing models of options are crucial and have attracted the attention of many researchers. In the existing literature, the pricing methods are mainly based on the Binomial Option Pricing Model, Monte-Carlo Simulation and Black-Scholes Model. Binomial Option Pricing Model is developed in 1979. After that, Simon and Zvi [1] price the options by assuming that the stock price follows a simple, stationary binomial process. At each moment in time, the price can either increase or decrease by a specific percentage. Odegbile et al. [2] found that this model is based on an assumption about the evolution of the price of the underlying asset and the so called "no-arbitrage principle". The model determines an option price that does not permit arbitrage opportunities. Emmanuel et al. [3] proposed that the activities of arbitrageurs or speculators ensure that financial assets

such as options are priced within a narrow tolerance of their theoretical values in the Binomial Option Pricing Model.

In contrast, the Monte-Carlo simulation is a much more sophisticated method to value options. Boyle et al. [4] claimed that the method simulates the process generating the returns on the underlying asset and invokes the risk neutrality assumption to derive the value of the option. Grant et al. [5] found that through incorporating optimal early exercise in the Monte Carlo method of valuing options by linking forward-moving simulation and the backward-moving recursion of dynamic programming through an iterative search process, the model can be available to price the American options. Charnes et al. [6] proposed that the availability of powerful workstations and recent advances in applying the Monte-Carlo simulation makes it accessible to a huge amount of users who would otherwise avoid programming the algorithms necessary to value derivative securities.

In addition, the Black-Scholes model was developed mainly for pricing European options on stocks by the economists Fischer Black and Myron Scholes in 1972. Golbabai [7] use price variation of the correlated fractal transmission system is used to deduce the fractional Black-Scholes model that has an  $\alpha$ -order time fractional derivative. The fractional Black-Scholes model is employed to price American or European call and put options on a stock paying on a non-dividend basis. Roul [8] is concerned with the design of a high order numerical approach based on a uniform mesh for efficient numerical solution of time-fractional Black-Scholes equation, governing European options. This scheme is used to price three different European options governed by a time-fractional Black-Scholes model. This article focuses mainly on capturing the discrete behavior of linear and nonlinear Black-Scholes European option pricing models. Halperin [9] presents a discrete-time option pricing model that is rooted in reinforcement learning (RL), and more specifically in the famous Q-Learning method of RL. Using Q-Learning and related methods, once created in a parametric setting, the model can go model-free and learn to price and hedge an option directly from data, without an explicit model of the world. This suggests that RL may provide efficient data-driven and model-free methods for the optimal pricing and hedging of options. Gulen [10] presents a combined method; a sixth order finite difference (FD6) scheme in space and a third-order strong stability preserving Runge-Kutta (SSPRK3) over time.

In this paper, we use Black-Scholes Model as the foundation for power option pricing. We consider the risk-free rate, volatility, spot price, time to expiration, dividend yield and strike price to calculate the payoffs of standard option and power option related to Amazon.Inc, in order to find out the difference in corresponding

pricing methods, and then power index and downsizing scale are then included to increase the pricing rationality of Amazon's power option. Result analysis suggests that, compared to traditional standard option base on Amazon's stocks, the new power option provides higher expected returns with the same investment scale. In sensitivity analysis, the results prove that when  $S_t$  and strike price become larger, the positive changes in profits of Amazon's power option are more obvious than standard option's. However, for power option, with all other parameters remain the same, the volatility will be expanded for "power index" times, which enlarges the substantial risks of investors. In other words, they are more likely to lose their premiums because of inaccurate predictions of the Amazon's future.

The remainder of the paper is organized as follows: Section 2 describes the background information about Amazon; Section 3 introduces the pricing methods of standard option and power option by using the Black-Scholes model as the reference; Section 4 conducts the result analysis of the potential returns of two types of investments; Section 5 shows the sensitivity analysis related to the  $S_t$ , strike price and volatility. The last section presents our conclusions.

## 2. BACKGROUND INFORMATION

Amazon is the largest online e-commerce company in the United States, located in Seattle, Washington. It is one of the first companies on the Internet to start operating e-commerce. Under the name of the company, it also includes three subsidiaries: Alexa Internet, a9.com and IMDb.

Amazon is a company founded by Jeff Bezos in July 1995. In the early days, Amazon was also called Cadabra.com. In the beginning, Amazon only operated the online book sales business, but now it has expanded to a wide range of other products, including DVDs, music CDs, and computer software. The nature of Amazon is a basic online bookstore, but the far-sighted Bezos sees the potential and characteristics of the Internet. When a large physical bookstore offers 200,000 books, the online bookstore can provide readers with more choices than 200,000 books. Therefore, Bezos renamed Cadabra.com after the Amazon River, which is home to the most diverse organisms on the planet, and reopened it in 1995.

Jeff Bezos, the founder and CEO of Amazon, is considered a visionary entrepreneur, but for a long time, he was not recognized or favored by the outside world. Amazon also adheres to its global strategy in China, does not assess short-term income and profit, continues to invest on a large scale, and do what they think is right-in their view, this is a marathon. Amazon has always been tepid in China. It does not fight price wars like other e-commerce companies and does not rush into advertising. Therefore, it has not been able to achieve the rapid growth

of JD.com and Suning. Amazon China adheres to its global strategy, not as a business company but as a technology company's development philosophy. The supply chain management capabilities represented by warehousing and logistics are the long-term competitiveness of e-commerce. Supply chain management is actually to move goods from manufacturers to consumers, and complete this process in the shortest, most economical, and most effective way. The process is optimized through technological means.

Amazon is the most customer experience-centric company in the world. The support of Amazon Flywheel is based on the three pillars of customer experience, rich choices, convenience and low price, strong technical capabilities, and data-based operations are the engine that drives the acceleration of the flywheel. Everyone at Amazon knows that the starting point of "flywheel" is customer experience, which refers to a virtuous circle that takes customer experience as the starting point. Bezos believes that in Amazon's flywheel, when the customer experience is good, more traffic will naturally increase and more traffic will attract sellers to sell things online, so that consumers have more and more abundant choices and more convenient services. It will also further enhance the customer experience. As the flywheel continues to grow, Amazon's operating costs will be shared. The cost structure will be more reasonable. The money saved can be returned to consumers to form a low price. This is also an important factor in improving customer experience.

With the outbreak of the 2020 epidemic, Amazon's financial situation has also undergone some changes. Although workers at one of Amazon's factories protested the company's failure to provide adequate safety equipment, Amazon's published earnings detail measures taken by online retailers to protect workers. Amazon executives said that the decline in customer demand in the hotel and travel industry limited growth, but revenue still increased by 32.8% from US\$7.696 billion to US\$10.219 billion, and operating income increased by 38.3% to US \$3.075 billion, accounting for the company's total business of US\$3.989 billion. More than 77% of income.

As many stores close and shoppers turn to the Internet to buy magazines and other products, Amazon's sales will rise. Delivery costs in the first three months of 2020 soared 34.1% to 11.531 billion U.S. dollars, higher than 8.601 billion U.S. dollars a year ago. This reduced net income from US\$3.561 billion in the first quarter of 2019 to 28.8% to US\$2.535 billion. Lipsman said: "The bottom line performance is relatively loose, but given the rising labor costs and delivery logistics costs of commercial operations, and the shift to lower-margin categories such as grocery stores, this is not entirely unexpected." "High profit margins The fact that China's cloud and advertising businesses are well maintained should help offset the rise in business costs in the coming quarters. "This pandemic

may accelerate multiple trends that benefit Amazon in the long run, including e-commerce, cloud Computing and digital advertising.



**Fig.1.** stock price of amazon

According to the stock price of Amazon, we choose 1 year as a period. As we can see that the stock price has fluctuated a lot. From Aug 26, 20 to nowadays, the stock prices weren't stable. What's more, the overall trend of its prices shows an upward tendency. The changing of price was regular which means that the lower price won't last for a long period of time, as well as higher price. The graph demonstrates the prices which are approximately between 3,000,00 to 3,800,00. The change of price is always sharply. In the figure 1, it is difficult to find that price change is stable which means decrease year by year or increase gradually.

### 3. STANDARD OPTION

#### 3.1. Definition

Standard option is a kind of contract which provide equity holders with the right to buy or sell an asset at a fixed price on a specified date.

Standard option is different from other derivative financial assets. Generally, there are three obvious characteristics in the standard option. Firstly, the asymmetry of rights and obligations. In option trading, the rights and the obligations of buyers and sellers are not equal. By paying the premium, the buyers acquire the right to buy or sell, but they do not have the obligation to trade. However, after receiving the premium, sellers must buy or sell certain items at the request of buyers. Secondly, the asymmetry between risk and return. The risk of the option buyers is known, which is limited to the payment of the premium and without additional obligations. But buyers' potential benefits are theoretically unlimited. Nevertheless, the sellers' profits (premium received) are limited, but the potential losses is theoretically infinite. Therefore, in an effort to keep balance, the option is commonly designed to increase the possibility of the seller's profit that can be much greater than that of the buyer. At last, the standard option has a leverage effect. In option trading, the risks and losses faced by the buyers are limited and predictable, and the biggest loss of buyers is only the premium. Therefore, the

buyers do not need to pay the margin. The seller is in a short position of a commodity or financial asset after the option is sold as the contract, the risk is unknown. But sellers only need to deposit a certain amount of margin to the exchange, generally a certain percentage of the amount in the contract. So the standard option is speculative.

### 3.2. Category

The standard option can be divided into two categories including call option and put option. The call option can help buyers purchase shares in fixed prices though the stock prices have increased. And the put option can help holders sell their stocks at the same prices as they bought them when the stock prices decreased fiercely, which is beneficial to avoid great losses. Take Apple's share as an example, we suppose Apple's share is trading at \$350, when the price rises to \$400, someone who own the call option can buy the stock at the initial price. When the price fall to \$300, the person who has bought can sell it at \$350.

### 3.3. Pricing

In 1973, Fischer Black and Myron Scholes published the BSM model which assumed future stock prices had a lognormal trend meaning the future spot price could be represented as a function of "z" [9].

$$S_T = S_0 e^{(\alpha - \frac{1}{2}\sigma^2)T + Z\sigma\sqrt{T}} \quad (1)$$

$S_0$  = stock price

$\alpha = \gamma - \delta$  for stocks

$\alpha = 0$  for futures

### 3.4. Calculation

We did a large number of calculations on the excel in order to elaborate the standard option more clearly and compare the standard option and the power option. First of all, we performed a thousand simulation, then we added a random value to keep the stock price being random. After that, we used the data that we have collected from websites and a formula to calculate the final spot price. Subsequently, we calculated the payoff of the standard call option. In addition, we also figured out the average value and present value of the standard option. Afterwards, we made data table to do sensitivity analysis which showed the relationship between the value of stock price and the standard call option. Then, we created a function.

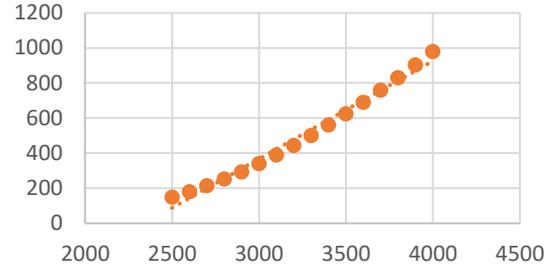


Fig. 2. Sensitivity analysis of stock price and PV of the standard call option

From Fig. 2., we drew a conclusion that when the stock price goes up by \$1, the option's price increases by \$0.56 according to our data.

Specifically, we also analyzed two other parameters: the strike price (Fig. 3.) and the volatility (Fig. 4.).

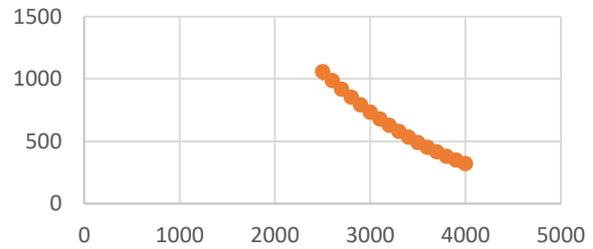


Fig. 3. Sensitivity analysis of strike price and PV of the standard call option

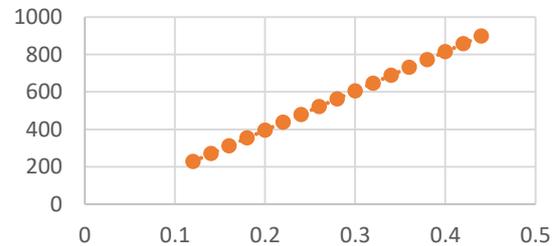


Fig. 4. Sensitivity analysis of volatility and PV of the standard call option

### 3.5. Usage

The standard option has various usages. For instance, people can use the standard option to speculate. They can buy standard call option or standard put option in an effort to benefit from rising or falling prices respectively using leverage and having downside protection. Besides, people can buy shares at low prices and sell them at high prices to gain more profits if they believe that options are overvalued or undervalued. Additionally, the standard option can be considered as a hedge against meaning that it can reduce the losses.

### 3.6. Choice

There are two main reasons for people choosing to

buy the standard option [10]. For one thing, the stimulating function of the standard option. Taking managers of modern companies as examples, It can be seen that after the introduction of the standard option system, the level of stock price directly affects the manager's return. Managers can enjoy the growth of benefit but bear the corresponding risks brought by the increase of companies' stock price. In this way, managers' personal income and business performance can establish a positive correlation with the future development of the enterprises, so as to encourage managers to attach more attention to the long-term sustainable development of the establishments, rather than just focusing on short-term financial indicators.

For another thing, The investment decision function of the standard option. To be more specific, under the option method, the value of managers' decisions will be examined and evaluated, and it can provide investors with alternative state for future investment. Transforming investment decision into effective option execution can make the decision more scientific.

**4. POWER OPTION**

**4.1. Black-scholes model (BSM model)**

In 1973, Fischer Black and Myron Scholes proposed the Black-Scholes Model. The model was designed to price an option (usually European Options, which investors can only execute at the date expiration) as a mathematical function based on certain variables, generally the current risk-free rate, time to expiration, strike price, dividend yield (added in 1976), volatility and stock price. The formula assumes the future stock prices are lognormally distributed.

To price an non-dividend-paying European Call Option, firstly, multiplying the stock price by the cumulative standard normal probability distribution function. Secondly, implementing the same arithmetic into the the net present value of the strike price and subtracting the resulting value from that of the previous calculation, can be written as:

$$C = N(d_1)S_0 - N(d_2)Ke^{-rt} \tag{2}$$

where:  $d_1 = \frac{\ln \frac{S_t}{K} + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$

and  $d_2 = \frac{\ln \frac{S_t}{K} + (r - \frac{\sigma^2}{2})t}{\sigma\sqrt{t}} = d_1 - \sigma\sqrt{t}$

where:

$S_0$  = stock price

$K$  = strike price

$t$  = time to expiration

$r$  = current risk-free rate

$\sigma$  = volatility measured by annual standard

deviation

$N(x)$  = cumulative standard normal probability distribution

**4.2. power option**

Power options are a class of exotic options in which the payoff at maturity is related to the certain positive power of the underlying asset price, which allows investors to a provide high leverage strategy and to hedge nonlinear price risks.

In order to price Amazon's power option (call), it was decided construct a spreadsheet model in Microsoft Excel. The model is hoped to simulate the cumulative standard normal probability distribution (1,000 times), calculate the  $S_t$  through other six parameters, predict the potential payoffs and estimate the present value of Amazon's power call option.

As a technology giant, Amazon never pays dividends to shareholders, but chooses to use existing capital to achieve internal growth through acquisitions and investment in business. Therefore, dividend yield is 0 in this type of option. Then the  $S_t$  can be expressed as:

$$S_t = \text{spot price} \times e^{((r-0.5\sigma^2)\times t + Z \times \sigma \times \sqrt{t})} \tag{3}$$

For the long side of the options trade, they can only be profitable if  $S_t$  is higher than strike price. Additionally, the payoff of such an option will be magnified in relation with a specific benchmark rate, which causes a standard power option has a non-linear payoff. Thus, the payoff of a call option is:

$$\text{Payoff} = (\text{Max}(S_t - K, 0))^i \tag{4}$$

where:  $\text{power} > 0$

In addition, if the market view turns out to be true, the potential payoff received by the buyer of Amazon's power call option would be much higher than that on an equivalent due to the relatively higher leverage ratio. In that way, a particular scale is then needed to enhance the rationality of this type of option's return. Otherwise, the power call option's payoffs related to the company's stock will tend to be unreasonable, which are usually around  $i$  times of standard option's.

**Table 1.** Payoffs of the Amazon's power call option and standard call option with 7 same parameters

| Payoff(call)(powered option) | Payoff(call)(regular option) |
|------------------------------|------------------------------|
| 0.00                         | 0                            |
| 0.00                         | 0                            |
| 0.00                         | 0                            |
| 0.00                         | 0                            |
| 45.55                        | 213.4221828                  |
| 0.00                         | 0                            |
| 5.14                         | 71.69507504                  |
| 0.00                         | 0                            |
| 0.00                         | 0                            |

5832.25                      2415.005794  
 · 15 simulations  
 · power i = 2, scale = 1000

Therefore, the payoff should be written as:

$$Payoff = \frac{(MAX (s_t - k, 0))^i}{scale} \quad (5)$$

where:  $power > 0$

After completing the simulation, the average result of 1,000 payoffs should be taken out and adjusted according to the current risk-free rate to obtain the present value of the option, which can be represented as:

$$Present Value = Average Payoff \times e^{-rt} \quad (6)$$

### 4.3. Result analysis

Table 2 shows that when all parameters are the same,  $k=2$ , scale = 1000, Amazon's power call option's present value is about \$1266.42. In contrast, the present value of standard option is about \$533.75. Therefore, it is concluded that power call option on Amazon's stock has a higher value and can bring investors more substantial returns than standard option.

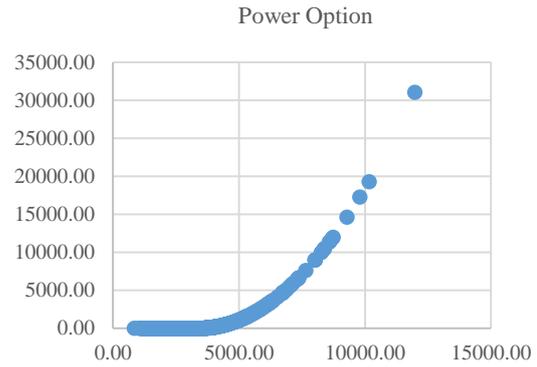
**Table 2.** The present values of Power Call Option and Standard Call Option

|         | Payoff(call)(powered option) | Payoff(call)(regular option) |
|---------|------------------------------|------------------------------|
| Average | 535.8914227                  | 1271.50                      |
| PV      | 533.7521384                  | 1266.42                      |

· risk-free rate = 0.20%, volatility = 26.58%, spot price = 3555.34, time to expiration = 2, dividend yield = 0, strike price = 3405.00

In this case, the premium of the option will also be higher than that of standard option. The writer of a call option has an obligation and unlimited liability because he must deliver the underlying if the option buyer wants it. In the pricing process, sellers need to set a higher price to ensure that their own interests will not be negatively effected.

To the long side, buyers must have a correct assessment of Amazon's stock prospects before buying the option. According to Figure 5, affected by the high leverage ratio, high potential profits are also accompanied by high risks, which also means that the possibility of losing premium is large for purchasing the company's power call option.

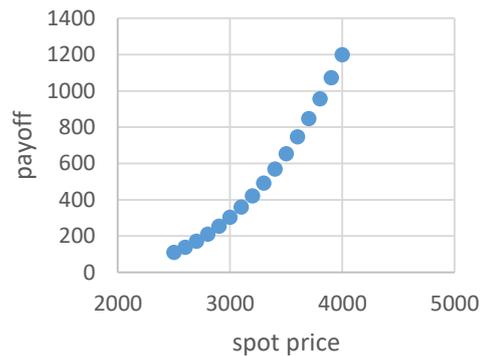


**Fig.5.** Amazon's Power Call Option (Gross: premium is not included)

### 5. SENSITIVITY ANALYSIS

The power option will be affected by 3 factors. They are spot price, strike price and volatility. A change in each one of them will lead to a change on your payoff.

According to the figure and data below, we can see that as the spot price becomes higher, the power option payoff will turn higher in parallel. In the table, the spot prices rise from 2500 to 4000, the payoff you can get rises from 111.03 to 1199.8. What is the reason behind? The spot price refers to the contract price agreed upon by both parties in the transaction of actual goods according to the principle of fairness. When the spot price goes up, the price gap between the spot price and strike price becomes higher. This price difference, in the table which is payoff, is what you can earn. On the contrary, as the spot price decreases the price gap between spot and strike price will gradually narrow down. That means the payoff decrease along with the spot price goes down.



**Fig. 6.** Effect of spot price on payoff

According to the Fig.7. , we can see that as the strike price increases, the payoff buyers get will become lower . In the table, when the strike price rise from 2500 to 4000, the payoff falls with the change of strike price from 1125.6 to 323.4. The strike price is the price specified in the option contract, no matter how high or how deep the futures price rises or falls in the future, the buyer has the right to buy or sell at the execution price. The payoff is

the price difference between strike price and spot price. Consumers expect the spot price higher than the strike price that they pay so that they can get profit. The greater the price gap between the spot price and strike price, the more of the profit or payoff you can get. Otherwise the higher the strike price you pay for the future, the less payoff you will get.

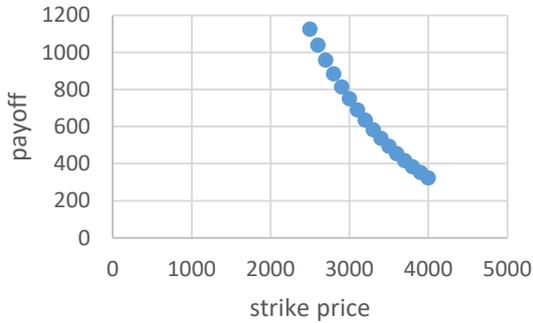


Fig. 7. Effect of strike price on payoff

According to the Fig.8. , we can see that when the volatility becomes bigger, the payoff buyers can get will be higher. In the table the volatility of the power option rises from 0.12 to 0.4 while the payoff rises from 82.7 to 1151.3. The volatility shows the fluctuation of the payoff buyer can get from the power option. The payoff is related to the change of spot and strike price. So the volatility also shows the fluctuation level of two prices. Buyers will use this to estimate the the risk and benefit of this option and make a wise choice. Big volatility means the spot price may rise or fall more rapidly. If the spot price is higher than the strike price you pay, you may get more payoff than that of you buy the option with low volatility. On the other hand, if the spot price is lower than the strike price you pay, you may lose more. The volatility shows people the risk rate of each specific option. High risk option will have high return. More people will choose to buy a option with high volatility for higher payoff.

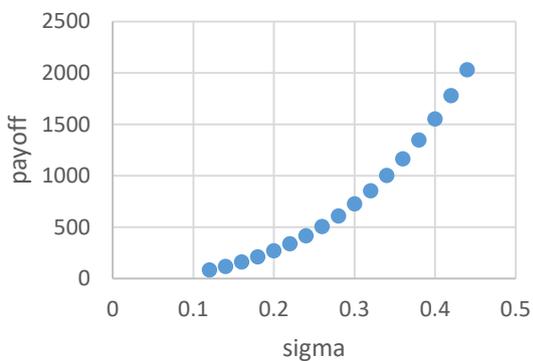


Fig. 8. Effect of sigma on payoff

## 6. CONCLUSION

We have introduced two options, which are standard

option and power option. At the same time,we mention that the model of BSM and the sensitivity of the data. It is obvious that we research the information about Amazon from different aspects. There are two things which called strike price and spot price affect the the power option. The way that we earn money by strike price is use the difference between the money we used to buy the thing and sell the thing. We hope that the spot price become higher after we buy specific products,while the strike price are expected to become lower. Sigma is the representation of volatility which stands for the big fluctuation in stock. However,some people prefer trying to buy stocks with fluctuated volatility because this provide them an opportunity to gain more profit or revenue. The data collected to analysis the power option and call option of Amazon are all form Yahoo finance.Standard option can be used to speculate from its up and down prices. It is beneficial to gain more profits by buying in a lower price but selling at a higher price. Absolutely,standard option is just like a hedge which means that people can reduce the their losses. From another aspect,we built a model to simulate and forecast potential returns and estimate the present value of Amazon's strong call options. The tables in this document showed that call option on Amazon's stock has a higher value and can bring more returns to consumers compared to standard option. The whole article demonstrates the power option and standard option of the company of Amazon. We analyze the influences to both company and customers and evaluate the pros and cons of these two types of options. The knowledge and formula we mentioned can be used to calculate data for other companies since the method is usually universal.

In the future, for Amazon, it is necessary to adjust stock price in time in order to suit customers' preferences. If the company achieve this target, both customers and company will benefit a lot from it, thus leading to a long-run prosperous for Amazon. To ensure the well-operation of a company, evaluation of relevant options is so vital that help some companies check the market positions of themselves. In a word, the survival of Amazon may always depends on the assessment of different options regularly.

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