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Unpacking Primary Teachers' Initial Knowledge of Realistic Mathematics Education: A Case of Iceberg Model of Fraction Division

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ABSTRACT
Iceberg model of realistic mathematics education (RME) is known as a metaphor to illustrate how informal, pre-formal, and formal mathematical models and strategies are used by students to develop a "floating capacity" for the understanding of formal representations of mathematics. This is a survey study that explores Indonesian primary teachers' initial knowledge of RME through iceberg model created for learning fraction division in primary classrooms. A total of forty-five elementary school teachers in Sidoarjo, Indonesia worked on a questionnaire about creating an RME iceberg in the initial agenda of teacher professional training. The created iceberg models were analysed based on the extent to which the learning stages meet the admitted learning trajectory of fraction division in RME. Results indicate the iceberg models created by the primary teachers tend to illustrate the skeleton of learning stages in formal mathematics, instead of pre-formal mathematics. Most of the teacher participants concern with providing a learning experience on the formal operation of fraction division at the beginning of learning stages followed by contextual learning experience, which is inversely related to the standard model of iceberg in RME.

Keywords: *Realistic mathematics education, Fraction division, Inservice teacher, Iceberg.*

1. INTRODUCTION

Being in the 21st century, teachers are faced with new challenges in teaching concepts and knowledge to their students. The focus of education that used to be only about transferring knowledge has now shifted to teaching skills such as critical thinking, creativity, collaboration, communication [1] and even now it is added to teach citizenship and character [2]. Learning these skills cannot be taught in one day in one lesson. However, these skills need to be taught as early as possible and taught continuously. So that since elementary school these skills should also have been trained through the given mathematics lessons.

But in fact, there are still many teachers who have not applied this learning pattern in the teaching and learning process. Several sources commented that many teachers in Indonesia teaching mathematics still use a mechanistic approach with an emphasis on drill and practice as well as procedural using abstract formulas and algorithms. In general, the evidence also reveals that many teachers, including primary school level (SD) teachers, do not meet the requirements to apply appropriate approaches to problem solving in their classrooms [3].

This view shows a learning pattern that is in accordance with the instrumentalist view, where mathematics needs to be taught as a science that is ready and ready to be used by the user [4]. Teaching mathematics with Mechanistic Approach, or 'traditional approach', which is based on questions and exercises, which treat people like computers or machines Ref). This is not in line with the demands of the 2013 Curriculum Process Standards contained in the Regulation of the Minister of Education and Culture Number 65 which is about the implementation of an interactive, inspiring, fun, challenging, and motivating learning process for students to participate actively [5]

In addition, learning always with an explanation-example-drill pattern certainly has its own impact on students' views on mathematics and student achievement. Several studies have shown that learning mathematics with this pattern causes students to be reluctant to learn mathematics and does not produce optimal learning outcomes [6-7].

The teacher's approach to teaching mathematics can be categorized into four categories, namely, mechanistic, structuralistic, empirically and realistic [8]. Mechanistic, or the 'traditional approach', is based on drills and patterns, which treat people like computers or machines (mechanics). Empirically, the world is reality, for students are provided with materials from the world of their lives. Students are faced with situations that force them to perform horizontal mathematization. Structuralistic, or the 'New Math' approach which is based on set theory, flowcharts and games which are a type of vertical but expressed mathematization of the created world, which has nothing to do with the student's living world. Realistic, real-world situations or contextual problems serve as a starting point for learning mathematics. Then this situation is explored by performing horizontal mathematization.

The Realistic Mathematics approach is an alternative learning approach that provides opportunities for students and fully supports students to find their own essential concepts through exploration activities. This approach is in accordance with educational goals and meeting the demands of the 2013 curriculum process standards. In addition, realistic mathematics is also considered to be able to teach the skills needed in the 21st century in addition to the mathematical concept itself. This is because with realistic mathematics students are taught to reason and develop strategies, think critically and creatively, solve problems, collaborate, communicate, and express ideas [9]. Some research results also reveal that realistic mathematics can make students happier with mathematics and motivated to solve mathematical problems [10-11].

According to Freudenthal, in the Realistic Mathematics Education (RME) approach, there are three principles that can be used as research references on learning design, namely: (1) Guided rediscovery and progressive mathematization, (2) Didactic Phenomenology, and (3) Developed models. alone. The combination of Van Hiele's three levels, Freudenthal's didactic phenomenology and Treffer's progressive mathematization [12] resulted in five characteristics (principles) of RME [13-15], two of which are the use of context in phenomenological exploration. and the use of models to facilitate progressive mathematization which means the development of intuitive, informal, context-bound ideas towards more formal mathematical concepts [16].

The essence of RME is the didactic construction of progressive formalization [17]. This means that RME not only offers support for progress from concrete to abstract situations, but this progress is described as an instructional sequence that places the context of the problem as a starting point for obtaining students' informal reasoning. From this context, the teacher invites students to explore pre-formal strategies and visual models that represent situations in that context, which are then progressively directed to more formal ones to support their mathematical understanding. This strategy is often more concise and efficient, although sometimes it is not as efficient as the formal model. For example, when working on a multiplication problem with natural numbers, the "grouping" strategy with the rectangular model will be easier to do than directly counting the members of a group of larger values when solving the problem. In addition, pre-formal models can often be used to solve problems in various contexts, such as the use of ratio tables to solve problems involving proportional reasoning. Treffers [8] describes this as "horizontal" and "vertical" mathematization, where horizontal mathematization is the process of developing mathematical tools to solve problems in realistic contexts and advanced vertical mathematization in the mathematical domain. These principles and characteristics are contained in the Iceberg Model for realistic mathematics learning.

The iceberg model [18] is a metaphor that shows the role of context, models, representations, and strategies in the process of developing students' understanding of certain mathematical concepts. This Iceberg model is based on the principles of Realistic Mathematics or RME (Realistic Mathematics Education). Where mathematics learning activities begin by providing meaningful real contexts for students, involving models, mathematical representations and strategies made by students which will then be brought to formal representations. The iceberg model can be said to be divided into three parts: the peak which contains the formal purpose or concept; the middle section is full of student representations, models, and pre-formal strategies related to goals; and at the bottom contains real contexts that support students' informal reasoning [14].

The choices we make to teach mathematics are influenced by various factors, one of which is our previous experience in learning mathematics [19]. Another thing that influences the way we teach mathematics is the availability of facilities and infrastructure, including technology. In addition, what is no less important is that the knowledge of mathematics and pedagogy that we (teachers or prospective teachers) have also affects the way we teach mathematics [20]. If we only know the formal concepts of mathematics without wanting to know how these concepts are constructed or discovered, then what we teach students is also only the tip of the iceberg (formal).

Treffers [21] presents that the mechanistic (or arithmetic) trend, there are no real phenomena used as sources of mathematical activity in learning, little attention is paid to applications in life, and the emphasis is on rote learning so there is no horizontal and vertical mathematization. Meanwhile, the empirical trend places a strong emphasis on only horizontal mathematization where the goals of formal mathematics are not prioritized so that it will result in less activity that directs students to vertical mathematization.

Contrary to the empiricist trend, learning with the structuralist trend emphasizes the very dominant vertical component, by not focusing on activities that support students' understanding through horizontal mathematization [21]. In this trend, any mathematical activities are carried out within the mathematical system, and even if there are real phenomena, this does not function as a model to support a shift in activities that lead to formalization in the mathematical system [21].

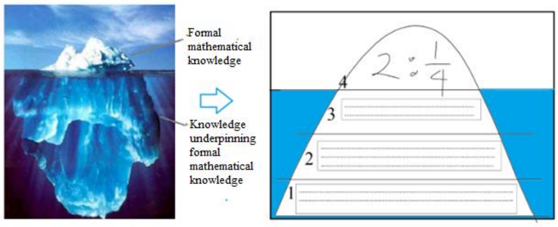
RME, in this case adheres to the last trend, which is realistic, where horizontal and vertical mathematization is carried out both continuously [21]. This means that the phenomena from which mathematical concepts and structures emerge are implicitly used as both source and application domains. This, according to theoretical principles, creates for the learner the possibility of conceptual achievement by orienting himself to various phenomena, which benefits the construction of concepts and structures of formal mathematics and their application [21].

This research aims to explore Indonesian primary teachers' initial knowledge of RME through iceberg model created for learning fraction division in primary classrooms.

2. METHODS

This is a survey study that explores Indonesian primary teachers' initial knowledge of RME through iceberg model created for learning fraction division in primary classrooms. A total of forty-five elementary school teachers in Sidoarjo worked on a questionnaire about creating an RME iceberg in the initial agenda of teacher professional training. The created iceberg models were analysed based on the extent to which the learning stages meet the admitted learning trajectory of fraction division in RME. Such a learning trajectory, at least, is examined whether it represents the realistic approach, mechanistic approach, empiricist approach, or structuralist approach as mentioned in Table 1.

Task



Iceberg in mathematics learning describes a model of student knowledge in which the formal form of a concept in mathematics is described as the tip of the iceberg, while the knowledge that underlies formal mathematical knowledge is described by the part of the ice that is below the sea surface. The figure above shows the formal mathematical concept that is aimed at students' knowledge of 2:1/4 learning. There are three initial learning steps needed to get to this knowledge, namely step 1, step 2, and step 3. Write down the three learning steps.

Figure 1 Iceberg task.

All the teacher responses on the iceberg task were then analyzed by categorizing each of them into one of such four possible approaches as indicated in Table 1.

Table 1. Mathematics learning approach

Approach	Characteristics
Mechanistic	Based on drills and patterns, which treat people like computers or machines (mechanics).
Empiricist	Students are provided with materials from the world of their lives. Students are faced with situations that force them to perform horizontal mathematization.
Structuralistic	Based on set theory, flowcharts and games which are a type of vertical but expressed mathematization of the created world, which has nothing to do with the student's living world.
Realistic	Real-world situations or contextual problems serve as a starting point for learning mathematics. Then this situation is explored by performing horizontal mathematization.

3. RESULTS AND DISCUSSION

3.1. Teacher Preferences of learning approach

Of the 45 responses there are 38 iceberg models created by the primary teachers illustrate the skeleton of learning stages in formal mathematics. The most common way in teaching division of fraction by the respondents is indicated by iceberg suggesting mechanistic approach (n=25) like shown in Figure 2.

1. 2/1 : 1/4
2. 2/1 x 4/1
3. 8/1 = 8

1. menanamkan konsep. 2. memberikan latihan soal. 3. memberikan contoh soal yang terjadi pada kehidupan sehari-hari

Figure 2 Learning stages suggesting mechanistic approach.

In summary, Table 2 presents the distribution of responses which regards to each of learning approaches.

Table 2. Teacher preferences of learning approach

Learning approach	Number of responses
Mechanistic	25
Empiricist	5
Structuralist	2
Realistic	6

It is obvious that most of the responses regards to mechanistic, while only few of the responses regards to the other three approaches. This means that teaching fraction division by explaining the concepts, followed by giving examples and further exercises, which does not promote students' mathematization, is commonly found within teacher responses. However, realistic approach, which encourages students' mathematization, both horizontal and vertical is not commonly found within teacher response. This phenomenon suggest challenging questions which analyze initial knowledge and beliefs of teacher respondent as the basis of providing any teacher development within realistic mathematics education.

Table 3. Example of teacher responses on learning approach preference

Learning approach	Teacher Response
Mechanistic	1) Explain the concept. 2) provide practice questions. 3) give examples of questions that occur in everyday life
Empiricist	1) Present contextual issues associating with everyday life, 2) Students solve problems by themselves and the teacher as motivation, 3) Comparing the results of students and teachers
Structuralist	1) Divide 2 into two separate number, namely 1 and 1, 2) divide every "1" by ¼ , 3) = 2/1 x 4/1 step = multiplying 2 x 4 = 8
Realistic	2) Connect division topic with real life division-based problem, 2) use models to present the concept of division, 3) Represent the concept into mathematical symbols

Table 3 points out some examples of respondents' responses taken from their iceberg models. No matter they understand the principles or characteristics of realistic mathematics education in their iceberg models, their responses can be indicated to follow one of the four approaches: mechanistic, empiricist, structuralist, or realistic. For example, teacher responses of starting lesson with the activity of dividing 2 into two separate numbers and ending with multiplying the first number by second fraction whose numerator and denominator is inverted indicates a shifting sign which brings an already formal mathematics into a more formal mathematics. This progression indicates a structuralist approach.

There are 13 iceberg models who have reckon the stage of informal and pre-formal, even though some of them are incomplete (only mention the use of model) or see a model on RME as a manipulative, like indicated in Figure 3. Out of those models, only six were indicated as realistic approach.

1. Orientasi lingkungan secara matematis
2.alat peraga
3. Pembuatan pondasi menuju konsep

Translation:
1. Environmental-oriented in mathematics
2. Manipulative models
3. Constructing stones toward concept understanding

Figure 3 Incomplete ideal iceberg model.

3.2. The existence of informal and formal model within iceberg

From those who responded steps completing the iceberg at Figure 1 realistically, we further analysed the existence of informal and the formal model presented in the steps proposed by the respondents. We are aware of a variety of steps that can be selected to indicate both informal and formal models. Thus, we found that out of 6, there is only

1 response that use the formal and pre-formal stages before moving to the formal one. See Figure 4. The respondent has learnt RME on the postgraduate. However, still the model used is closely related with the use of manipulatives.

Langkah 1. Penanaman konsep melalui pemberian masalah yang kontekstual Langkah 2. Memfasilitasi siswa untuk menggali konsep yang terkait melalui alat peraga benda nyata Langkah 3. Memfasilitasi interaksi antara siswa dan guru, siswa dan siswa untuk menemukan pengetahuan matematika formal dari bentuk informal yang ditemukan sendiri

Translation
Step1. Teach the concept by giving contextual problem 2. Facilitate students to develop the concept by using manipulatives and real thing. Step 3. Facilitate interactions between teacher and students, students-students to get the formal mathematics concept which is derived from the informal one by students themselves.

Figure 4 Learning stages closely associated with realistic approach.

In summary, Table 4 presents the existence of Informal/formal sign in iceberg.

Table 4. Informal and formal models in iceberg

Models in iceberg	Number
Informal/Situational	14
Pre-formal 1 (model of)	10
Pre-formal 2 (model for)	6
Formal	20

It is consistent with the finding of learning approach in Table 3 where most of the models are indicated to formal since the preference of mechanistic approach. Table 4 also indicates that pre-formal models, either model of or model for, are often skipped by teacher respondent. It shows that mathematical progression within mathematization is not explicitly indicated within their knowledge. In the case of fraction division, no explicit model proposed by the teacher respondent which specifically indicates learning stages within realistic approach although all the six teachers provide common stages in realistic approach. Regarding this issue, Peck and Matassa [22] proposed emergent models may be found during learning stages of fraction division with realistic approach. It starts with the use of informal models and strategies to solve problems involving fairsharing situations where multiple items are shared among multiple sharers, activities with progressively more abstract fair-sharing situations, the use of bar model and the partition-distribute-iterate strategy for a more formal stage, then ends up with the use of formal mathematical model of fractions-as-quotients sub-construct.

4. CONCLUSIONS

The iceberg models created by the primary teachers tend to illustrate the skeleton of learning stages in formal mathematics, instead of pre-formal mathematics. Most of the teacher participants concern with providing a learning experience on the formal operation of fraction division (Structuralistic without game) and teachers directly teach the concept and strategy to the students. This shows that teachers tend to teach division on fraction using Mechanistic or traditional approach. There is still urgent need and effort in shifting teachers' way of teaching from traditional to be more facilitate the students to construct their thinking and ideas. As for the relationship with the RME understanding, the result shows that the teachers show lack of understanding on Iceberg model of RME. A Start-Up Workshop could be helpful. In conducting the Start Up Workshop, the pre-formal stage in where the model is build up and emergent, should be paid more attention.

AUTHORS' CONTRIBUTIONS

All the authors have designed this study together. All authors contributed to the revision of the manuscript. The scriptwriter has completed the final version.

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