Exploring the Critical Thinking Process of Prospective Teachers with High Mathematics Ability in Solving Ill-Structured Problems

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ABSTRACT
This study explored the features of critical thinking process skills of prospective teachers with high mathematics abilities in the process of solving ill-structured problems in mathematics classes of private universities in Makassar, Indonesia. Participants solved ill-structured problems by following the analyze, browse, create, decision-making, and evaluate stages. Thirty teacher candidates were selected to complete a math proficiency test. Prospective teachers were grouped based on high mathematical ability. Then they were interviewed based on the results of the Ill-Structured problem-solving. Based on the analysis and discussion, the results showed that the subjects could carry out critical thinking processes in solving ill-structured problems through five (5) stages/phases as follows. Analyze; Subjects could understand the problem by identifying the information needed and defining the problem given. Browse Subjects had the ability to mention or write by grouping relevant information to solve the given problem. *Create*; Subjects were able to formulate and create alternative problem-solving in written form accompanied by reasoning. *Decision-making*; The subjects were able to solve the problem by connecting the information obtained to find a common thread of a problem so that the end can be described from a tangled condition. *Evaluate;* The subjects were able to evaluate the steps to solving the problem and obtain the results or answers in accordance with the target they wanted to achieve. This study's findings suggest that this process can be used to help detect the thought process features of teacher candidates for solving Ill-Structured problems in mathematics education and for future research.

Keywords: *Ill-structured problems, Mathematics education, Prospective teachers, Critical thinking.*

1. INTRODUCTION

Critical thinking competence in problem-solving is one of the basic skills that prospective teachers must possess to learn mathematics. Five process skills that prospective teachers must possess through learning mathematics are included in the standard process, namely: (1) problem solving; (2) reasoning and proof; (3) communication; (4) connection; (5) representation [1]–[3]. These skills include high-level mathematical thinking that must be developed in learning mathematics.

Prospective teachers in mathematics education are specifically prepared to become professional teachers and they need to have critical thinking skills in solving mathematical problems [4]. Because after graduation, they must have professional pedagogical competence as teachers. Professional and pedagogic competencies that must be achieved are reflected in the formulation of learning outcomes, both in terms of knowledge and special skills [5]–[7]. Therefore, prospective teachers must possess the ability to think critically in solving mathematical problems. Watson and Bills say that experience in solving mathematical problems can support the formation of professional competence and pedagogic competence, including learning outcomes [8], [9]. Problem-solving is not only the goal but also the core of learning mathematics [1], [10]-[12].

The findings of Mairing's investigation show that the ability of prospective mathematics teachers in problem-solving is at a score of 64.5 on a scale of 0 – 100 [13]. The ability to think in order to solve problems is beneficial not only for prospective teachers, but also for students communicating their thought processes in overcoming problems afterward. Critical thinking is also a component of this research.

1.1. Critical Thinking and Ill-Structured Problems in Problem Solving

Critical thinking based on 21st-century abilities requires someone to know how to think. When the person is faced with a relatively new problem, his critical thinking skills will develop over time. Prospective teachers, scholars, and everyone involved in achieving the main goal must master critical thinking skills [14]–[16]. This is useful for generating ideas for various fields of study and disciplines. In addition, the combination of temperament and mental capacity constitutes critical thinking [17]. Therefore, one of the alternative thinking processes in this study is very important to evaluate the critical thinking process of prospective teachers in overcoming difficulties in solving ill-structured problems. The critical thinking process consists of four stages: clarification, assessment, inference and strategy [18], [19]. The measurement of prospective teachers' capacity to think critically is conducted through the four phases.

Furthermore, research on future mathematics teacher candidates needs to have ill-structured problem-solving skills, namely the problem of incomplete form. The findings of the prospective teachers work and interviews will be used for building strategies, namely through a critical thinking process. It will show that prospective mathematics teachers can master the geometry courses studied in the last semester because prospective mathematics teachers take advantage of their learning experiences. This data is stored in long-term memory to recall the experience when faced with a challenge.

In addition to practical interest, it is possible to clarify and refine preliminary studies and past research in the context of critical thinking processes and steps in solving ill-structured problems in this study. Il-Structured problems require teacher candidates to identify the information and skills needed to solve problems [20], [21].

1.2. Contribution

Based on the description above, it appears that the ability to solve ill-structured problems through a critical thinking process is crucial for prospective mathematics teachers and can be applied as a mathematics learning strategy in the future. Prospective mathematics teachers have various backgrounds and different in problem-solving abilities [22]. Therefore, prospective mathematics teachers need to have critical thinking process skills in solving ill-structured problems.

By knowing how the critical thinking process in solving ill-structured problems is based on high mathematical abilities, a prospective teacher will contribute to developing the theory of problem-solving abilities in particular and cognitive psychology in general. Therefore, the main objective of this study was to describe the critical thinking process of prospective mathematics teachers in solving ill-structured problems based on high mathematical abilities.

2. METHOD

Qualitative research methods were used to describe the critical thinking process of prospective mathematics teachers in solving ill-structured problems. In this qualitative research, the subject was the main instrument, and the data obtained were presented in descriptive form [23].

2.1. Participants

This study involved thirty teacher candidates from a mathematics education study program at a private university in Makassar, Indonesia. They were assumed to have completed basic geometry courses and had adequate knowledge and expertise in solving general geometry problems. To control the level of mathematical ability of the research subjects, teacher candidates with a temporary achievement index of more than 3.50 were selected as prospective subjects based on the results of the mathematical ability test, and then one teacher candidate was selected as the subject (with high mathematical ability). Subjects with high mathematical ability criteria were predicted and expected to indicate critical thinking when given ill-structured problems.

2.2. Instrument

The instrument used to collect the data was a geometric problem developed with its steps; the subjects wrote their instrument and alternative solution, then submitted the instrument to the validator for the assessment and revision. The "ill-structured" problem is shown in Figure 1 below.

Right triangle ΔABCat$C$. $D \in \overline{AB}$. so that $\overline{AD}$: $\overline{BD} = 1 : 2$. Prove that$9\overline{CD}^2 = a^2 + 4b^2$!

Figure 1 *Ill-structured* Problem

2.3. Data Analysis

The interactive model of Miles, Huberman, and Saldana was used to analyze the data in this study [23]. Data analysis used an interactive model, it started with the data collection process and continued at each stage of the research until it was completed. Data reduction, data presentation, and conclusion drawing/verification were three qualitative data analysis processes. Finally,

based on high mathematical ability, math teacher candidates obtained an overview of the critical thinking process in solving ill-structured problems.

3. FINDINGS AND DISCUSSION

This study aimed to describe the critical thinking process of prospective mathematics teachers in solving ill-structured problems. As a result, the following is data analysis of prospective mathematics teachers with high mathematics abilities based on the review of answers and interviews. Furthermore, the findings of the subject's answers in solving ill-structured problems are shown below for each phase of the critical thinking process, namely analyze, browse, create, decision-making, and evaluate.

AnalyzePhase

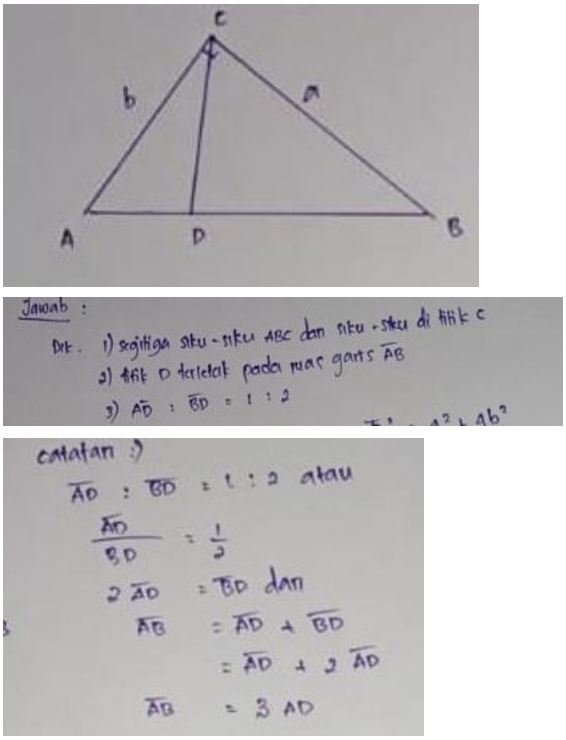
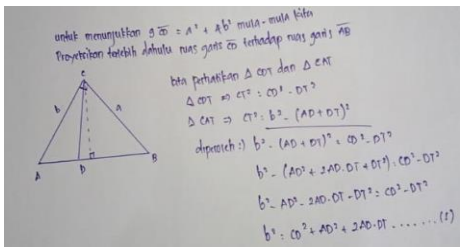


Figure 2 Subjects' answers in solving ill-structured problems phase analyze

BrowsePhase



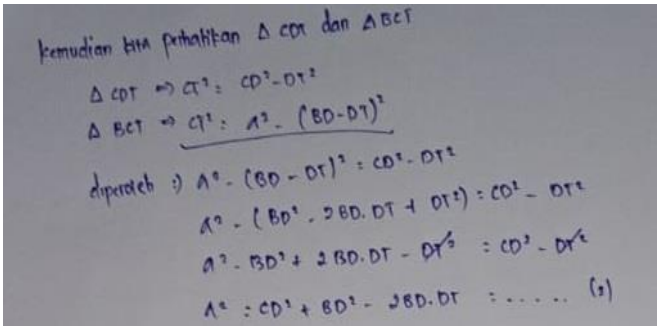


Figure 3 Subjects' answers in solving ill-structured problems phase browse

Create Phase

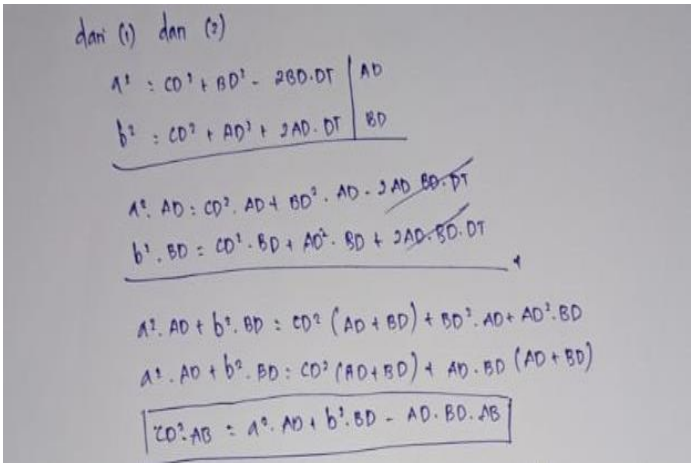


Figure 4 Subjects' answers in solving ill-structured problems phase create

Decision-Making Phase

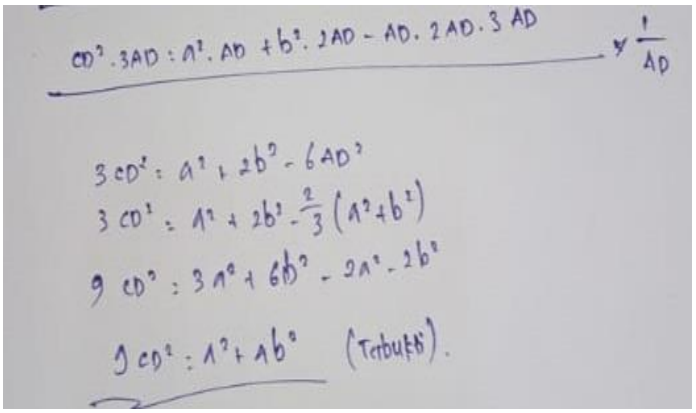


Figure 5 Subjects' answers in solving ill-structured problems phase decision-makin.

Evaluate Phase

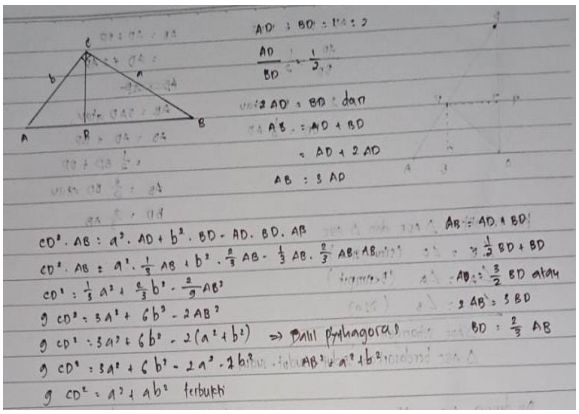


Figure 6 Subjects' answers in solving ill-structured problems phase evaluate

As illustrated in Figure 7, the critical thinking process of prospective teachers with high mathematics abilities can be represented by the structure of critical thinking in solving ill-structured problems.

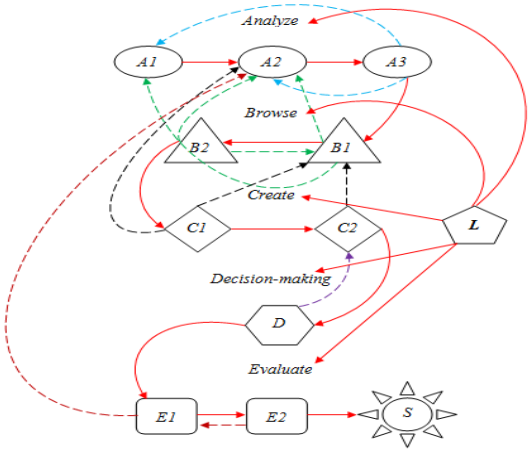


Figure 7 Structure of the Critical Thinking Process of Prospective Teachers in Solving ill-structured Problems

Table 1. Information Structureof Critical Thinking

Symbol	Description
→	The common thread of the critical thinking structure of prospective teachers who are highly capable in solving ill-structured problems.
- - - - →	The relationship between aspects of each critical thinking indicator.

Then, the diagram in Figure 8 can be compared. The researcher designed the problem-solving structure based on critical thinking indicators and aspects of each indicator, as presented below.

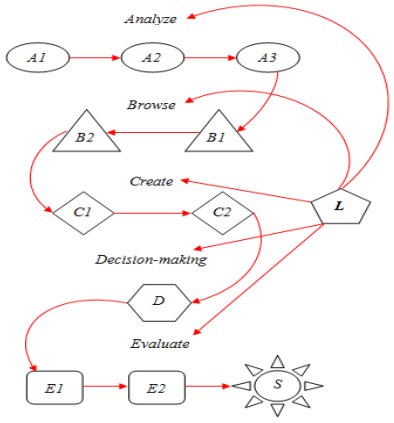


Figure 8 Structure of Problem Solving by Researchers

Table 2. Information Structure Of Problem-Solving

Code	Description
A1	Reviewing the problem
A2	Understanding the problem
A3	Defining the problem
B1	Collecting information
B2	Identifying information relevant and needed
C1	Formulating solution
C2	Creating alternative solution or assumption
D	Fixing problem
E1	Evaluating problem-solving
E2	Identifying idea
S	Finish

The critical thinking process of a prospective teacher with high mathematics abilitywent through five phases or stages, namely A-B-C-D-E. The Analyze stage consisted of three aspects, namely 1) reviewing the problem, 2) understanding the program/problem given, and 3) defining or re-explaining the problem at hand. Then the Browse stage consisted of two aspects, namely 1) identifying the mathematical content needed to solve the given problem, and 2) collecting information needed to solve the problem. The Create stage consisted of two aspects, namely 1) formulating a solution that could meet many conditions, and 2) creating various solutions for the given problem. The decision-making stage consisted of one aspect, namely the justification of the most appropriate solution to solve the given problem. The Evaluate stage consisted of two aspects, namely 1) evaluating, reflecting or thinking about the solution,

and 2) identifying ideas, then modifying and completing the solutions themselves.

This explanation found found that prospective teachers with high mathematical abilities carried out critical thinking processes in solving ill-structured problems. However, prospective teachers were trapped in the form of a given problem or some of the elements that were vaguely defined in the problem. The ill-structured problem given did not inform that Stewart's theorem and the congruence of two triangles were applied. However, prospective teachers could express and write that Stewart's theorem was applied through a line drawn arbitrarily from the vertices of the triangle and intersects with the opposite side. This shows that vaguely defined information caused the subject to be confused about what action to take in solving the problem [18], [24]–[27].

In the critical thinking process, prospective teachers with high mathematics abilities had just realized that they could also use the alternative Stewart's theorem to solve the given problem. It can also be solved by comparing two triangles after looking back at the problem-solving steps and finding the relationship between the two triangles. These findings supported the results of previous research suggesting that the thinking process in ill-structured problem solving requires a different set of skills and attitudes, rather than well-structured problems that have clear goals and known rules to apply in solving the problem [26], [28]–[30].

4. CONCLUSION

The features of ill-structured problems were helpful for prospective mathematics teachers in building assumptions of problem solving or various.Prospective mathematics teachers solved ill-structured problems in various ways and these experiences had a positive effect on the development of critical thinking and creativity. In conclusion, the thinking process of prospective mathematics teachers in solving ill-structured problems was through the Analyze, Browse, Create, Decision Making, and Evaluate stages; 1) *Analyze*, subjects could understand the problem by identifying the information needed and defining the problem given, 2) Browse, subjects had the ability to mention or write by grouping relevant information to solve the given problem, 3) Create, Subjects were able to formulate and create alternative problem-solving in written form accompanied by explanation reasons 4) Decision-making, The subjects were able to justify solving the problem by connecting the information obtained to find a common thread of a problem so that the end could be described from a tangled condition, 5) Evaluate, the subjects were able to evaluate the steps to solving the problem and obtain the results or answers in accordance with what they wanted to achieve. These findings suggest that Ill-Structured problems can be used to help detect the thought process features of teacher candidatesin mathematics education and for future research.

AUTHORS' CONTRIBUTIONS

Conceptualization, A.K., S.A., and A.; data collection, A.K.; data analysis, A.K., S.A., and A.; methodology, A.K., S.A., and A.; writing, review and editing, A.K., S.A., and A.

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