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# The Analysis of Capital Structure
## Based on Two Hypothetical Firms

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**ABSTRACT**
A corporate's financing of its real investments has recently attracted considerable attention from academics who have proposed various theories. This paper firstly studies the Gordon growth model and the weighted average cost of capital formula with the empirical tests and the sensitivity analysis. Then, we exam changes in a company's market value concerning different combinations of capital structures and identify an arbitrage opportunity based on the Modigliani-Miller propositions. Finally, we investigate the relationship between a firm's funding sources and its profit using panel data regression to figure out the optimal way to financing. These studies enable us to realize the fundamental relationship between a corporate's price and its capital structure. In light of those studies, we could have some insightful ideas regarding funding decision-making in reality.

***Keywords:*** *Gordon growth model; WACC; Modigliani-Miller propositions; arbitrage opportunity; capital structure*

## 1. INTRODUCTION

Recent years have seen several theories that have been proposed to exam the variation in debt as a way to finance a corporate's real investments [1]. For example, the trade-off theory states that there is an optimal capital structure. A company needs to trade-off between the tax deduction and the costs of financial distress resulting from debt financing [2]. In contrast, the pecking order theory claims that due to adverse selection, a firm chooses to exert internal finance such as retained earnings [3]. When outside funding is inevitable, it will prefer debt and treat equity as a last resort [4]. Though there is no universal view leading to the most appropriate and beneficial financing choice [2], the modern theory of sources of funding indeed begins with the celebrated paper of Modigliani and Miller [5]. They represent a logically consistent proof that given the particular conditions, a corporate's market value and its overall cost of capital are independent of the amount of debt it issues [6].

The Gordon growth model, initially articulated by Gordon and Shapiro [7] in their paper, is an approach to estimate the market value of a mature, slowly growing corporate whose operating income is expected to grow perpetually and constantly. It uses a company's future dividends or income per share divided by the spread between its cost of equity and the rate at which its dividends or income per share are expected to grow to determine its current price [8]. As stated by Miller [9], a firm's weighted average cost of capital involves the computation that its cost of equity and cost of debt are averaged using as weights the proportions of those separate financing sources [10]. The celebrated paper was first written by Modigliani and Miller [11] stated that when transaction costs and corporate taxes are non-existent in the market, any company's market value, as well as its overall cost of capital, is utterly unaffected by its combinations of capital structures due to the opportunity for individual investors to make their homemade leverages in an efficient and complete market [12]. As claimed by Hall and Weiss, since long-term variable cost per unit is U-shaped, with an increase in a firm's scale, its profit increases as well until the scale reaches a threshold value [13].

This paper first studies how to obtain a corporate's market value by using the Gordon growth model and determining a company's overall cost of capital by

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means of the weighted-average method. In addition, this paper exams changes in a firm's price concerning various ways to finance its real investments and finds that two corporates have identical investment projects. However, different combinations of capital structures should have the same value based on the Modigliani-Miller propositions; otherwise, there exists an arbitrage opportunity that we investigate in this paper later. Furthermore, we exert the sensitivity analysis to figure out that a company's cost of equity is the most sensitive factor in both the Gordon growth model and the determination of the weighted average cost of capital. Finally, we conduct the fixed effect model of panel data to find a particular relationship between a firm's sources of funding and its profit. So, an eclectic solution exists to the optimal way to finance by trading off a corporate's profit and its weighted average cost of capital.

The paper's structure is studying the Gordon growth model first and then the weighted average cost of capital formula. After that, we exam the relationship between a corporation's market value and its ways to finance its real investments and identify an arbitrage opportunity. Finally, this paper investigates the relationship between a company's profit and its combinations of capital structures.

## 2. BACKGROUND

We first need to make some assumptions. In a frictionless market where transaction costs and corporate taxes are absent, we assume that two companies are both publicly listed with equally risky investment projects that generate perpetual operating income of CF million p.a. One is entirely financed by issuing common shares with a cost of equity of $r_E(U)$ p.a., while it is involved in the other one's capital structure that debt consists of perpetual bonds with a market value of debt, D(L) million, a market value of equity, E(L) million, and a cost of debt, $r_D(L)$ p.a.

To conduct empirical tests, we exert two hypothetical exchanged-listed corporates, U and L, as our empirical cases. They both have utterly identical investment projects that yield perpetual operating income of 20 million p.a. The all-equity financed company U has a cost of capital of 10% p.a. Taking on debt that consists of perpetual bonds, the firm L has a 50 million market value of debt, a 170 million market value of equity and a 6% costs of debt p.a.

## 3. GORDON GROWTH MODEL

In this part, we exert the Gordon growth model to obtain a corporate's market and its cost of equity.

### 3.1. Model

The Gordon growth model is used to value the stock of a mature, slowly growing corporate, assuming that its operating income is expected to grow perpetually at a constant rate [7]. In accordance with Gordon and Shapiro [8], a company's market value is equal to its future dividends or income per share divided by the spread between its cost of equity and the rate at which its dividends or income per share are expected to grow. It could be written as follows:

$$P = \frac{CF}{r_E - g} \tag{1}$$

The condition is that $r_E > g$; otherwise, P will be infinite or negative, which is unreasonable. If a corporate's growth rate is equal to zero, the Gordon growth model will become the perpetuity model as follows:

$$P = \frac{CF}{r_E} \tag{2}$$

### 3.2. Empirical Test

In accordance with equation (2), we obtain the corporate U's equity market value, which is 200,000,000 million.

To determine the equity cost of capital for the corporate L, we need to rearrange the perpetuity model, and it could be written as follows:

$$r_E = \frac{CF}{P} \tag{3}$$

Since the corporate L is financed by some proportions of debt that consists of perpetual bonds, the borrowing costs need to be excluded from its operating income to calculate its cost of equity. Thus, the income for its shareholders is 17,000,000 million p.a. Therefore, according to equation (3), we could acquire the equity cost of capital for company L, which is 10%.

### 3.3. Sensitivity Analysis

We choose corporate L as our analytical object to conduct the sensitivity analysis to determine how the operating income p.a., the cost of equity, and the growth rate impact a company's market value. We assume that the firm's initial growth rate is 5% constantly and perpetually. The sensitivity coefficient could be mathematically expressed as follows:

$$Sensitivity\ Coefficient = \frac{Relative\ Difference}{Relative\ Change\ of\ Object} \tag{4}$$

**Table 1.** Sensitivity Coefficient of Operating Income p.a., the Cost of Equity and Growth Rate

Million

| | Benchmark scheme | Change of CF | | Change of $r_E$ | | Change of g | |
|---|---|---|---|---|---|---|---|
| | | -10% | 10% | -10% | 10% | -10% | 10% |
| Market Value | 340,000 | 306,000 | 374,000 | 425,000 | 283,333 | 309,090 | 377,777 |
| Difference with the Benchmark Scheme | | -34,000 | 34,0000 | 85,000 | -56,666 | -30,909 | 37,777 |
| Relative Difference | | -10% | 10% | 25.00% | -16.67% | -0.09 | 0.11 |
| Sensitivity Coefficient | | 1 | 1 | -2.50 | -1.67 | 0.91 | 1.11 |

In accordance with equation (4), when the value of the cost of equity and that of growth rate have been fixed at 10% and 5% separately, the sensitivity coefficients are 1 in both conditions that the relative changes in operating income p.a. are -10% and 10%. This means that a corporation's market value changes as much as its operating income p.a. does, and it could be shown in Figure 1:

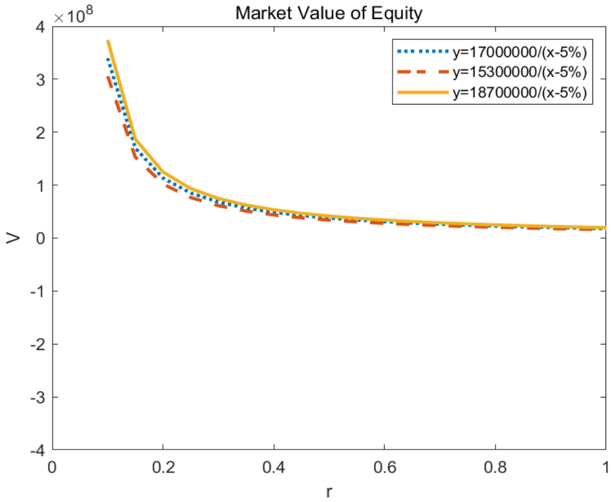


**Figure 1.** Changes in Market Value for Changes in Operating Income

According to equation (4), if the value of operating income p.a. and that of growth rate have been fixed at 17,000,000 and 5%, respectively, the sensitivity coefficients are -2.50 and -1.67 separately in the situations that the relative changes in the cost of equity are -10% and 10%. This indicates that the changes in a corporate's market value and its cost of equity are in different directions. It could be depicted in Figure 2:

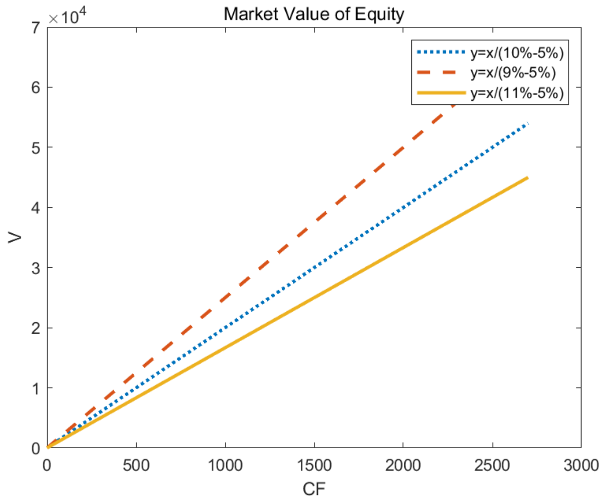


**Figure 2.** Changes in Market Value for Changes in the Cost of Equity

In accordance with equation (4), after the value of operating income p.a. and that of the cost of equity has been fixed at 17,000,000 and 10%, respectively, the sensitivity coefficients are 0.91 and 1.11 separately in the conditions that the relative changes in growth rate are -10% and 10%. This suggests that the changes in a corporate's market value and its growth rate are in the same directions, and it could be shown in Figure 3:

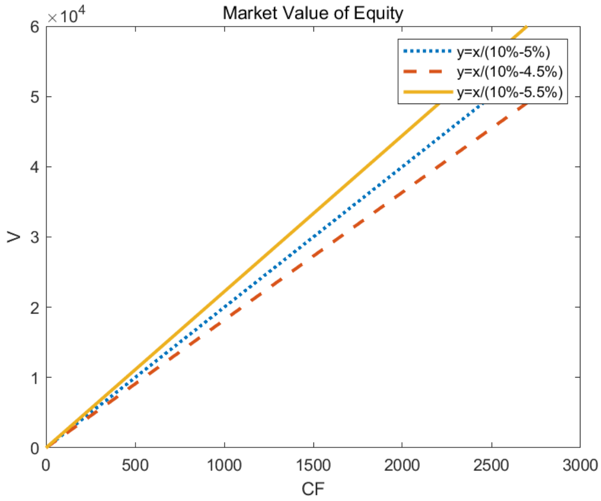


**Figure 3.** Changes in Market Value for Changes in Growth Rate

Overall, a corporate's cost of equity is the most sensitive factor in the Gordon growth model, so the result comes that if a company is eager for a higher market value, its priority is to control its cost of equity.

# 4. WEIGHTED AVERAGE COST OF CAPITAL

In this part, we utilize the weighted average method to determine a corporation's overall cost of capital.

## 4.1. Model

In accordance with Miller [9], a corporate's weighted average cost of capital is a standard treatment, when the company has issued both common stock and bonds as its sources of financial capital, which involves the computation that its cost of equity and cost of debt are averaged using as weights the proportions of those separate funding sources. Thus, the weight average cost of capital is mathematically expressed as follows [10]:

$$r_A = \frac{D}{D+E} \times r_D + \frac{E}{D+E} \times r_E \quad (5)$$

## 4.2. Empirical Test

In accordance with equation (5), we could determine the overall cost of capital for the corporate L, which is 9.1%.

## 4.3. Sensitivity Analysis

We continue to choose the corporate L as our analytical object to conduct the sensitivity analysis to determine how the cost of debt and the cost of equity impact a company's weighted average cost of capital, respectively. We assume that the firm's proportions of debt and equity in its capital structure is unmodified, which is 5/22.

**Table 2.** Sensitivity Coefficient of Cost of Debt and Cost of Equity

| | Benchmark Scheme | Change of $r_D$ | | Change of $r_E$ | |
|---|---|---|---|---|---|
| | | -10% | 10% | -10% | 10% |
| WACC | 9.09% | 8.95% | 9.23% | 8.32% | 9.86% |
| The difference with the Benchmark Scheme | | -0.14% | 0.14% | -0.77% | 0.77% |
| Relative Difference | | -2% | 1% | -8.50% | 8.50% |
| Sensitivity Coefficient | | 0.15 | 0.15 | 0.85 | 0.85 |

In accordance with equation (4), the sensitivity coefficients are 0.15 in both conditions that the relative changes in the cost of debt are -10% and 10%, after we have fixed the value of the cost of equity at 10%. This means that the changes in a corporate's cost of debt have a slight impact on the value of the weighted average cost of capital, and it could be shown in Figure 4:

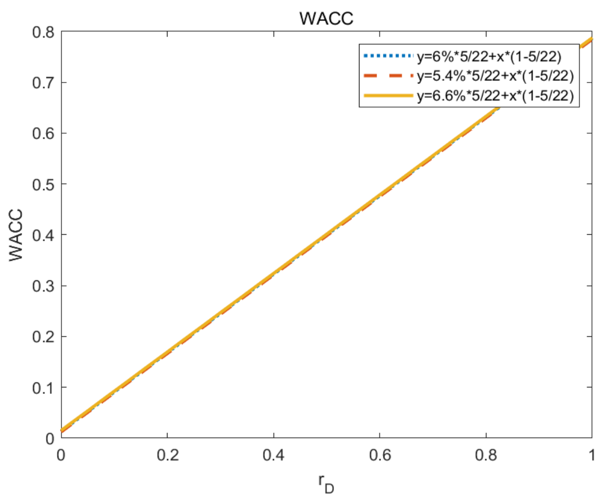


**Figure 4.** Changes in WACC with for Changes in the Cost of Debt

According to equation (4), the sensitivity coefficients are 0.85 in both situations that the relative changes in the cost of equity are -10% and 10%, after we have fixed the value of the cost of debt at 6%. This indicates that the changes in a corporate's cost of equity have a significant effect on the value of the weighted average cost of capital, and it could be depicted in Figure 5:

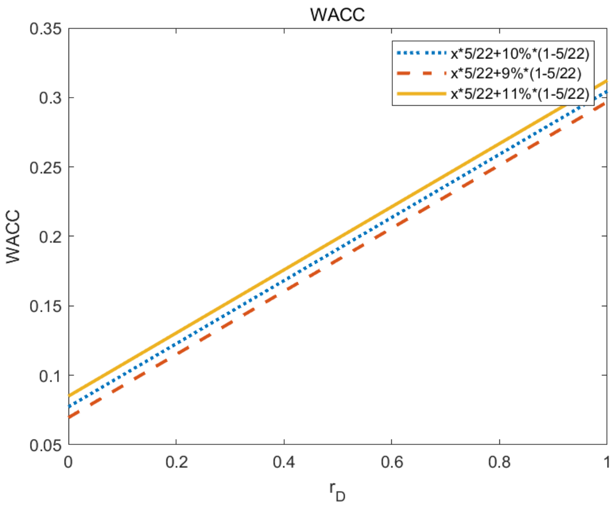


**Figure 5.** Changes in WACC for Changes in the Cost of Equity

In conclusion, a corporate's cost of equity is a more sensitive factor than its cost of debt in determining the weighted average cost of capital, so it is crucial to control the cost of equity to minimize the company's weighted average cost of capital.

## 5. MODIGLIANI-MILLER PROPOSITIONS

This part investigates a corporate's market value and its overall cost of capital for various ways to finance its real investments based on the Modigliani-Miller Propositions.

### *5.1. Proposition 1*

In accordance with Modigliani and Miller [11], in the absence of transaction costs and corporate taxes, any company's market value and its overall cost of capital are entirely unaffected by its ways to finance its real investments because, in an efficient and complete market, individual investors could create their homemade leverages to offset any particular debt-equity ratio chosen by the firm [12] thus, if two corporates that have identical investment projects but different combinations of capital structures must have the same price as well as the identical overall cost of capital.

### *5.2. Proposition 2*

It is stated by Modigliani and Miller [11] that, in a market where transaction costs and corporate taxes are non-existent, a levered company's expected rate of return on equity is equal to the appropriate capitalization rate related to its business risks, plus a premium associated with its financial risks, which is equal to the debt-to-equity ratio times the difference between the capitalization rate and the interest rate to compensate the extra risk. It could be written as follows:

$$r_E = r_A + (r_A - r_D)\frac{D}{E} \tag{6}$$

### *5.3. Empirical Test*

We use the two corporates, U and L, as our empirical cases to present the Modigliani-Miller propositions. In accordance with proposition 1, since the levered company's market value is 220 million, which lies above that for equivalent unlevered firm, 200 million, we could conclude that the corporate L is overpriced relative to the company U. Also, according to proposition 2, the unlevered firm's cost of equity, 10%, is equal to that for the equivalent levered corporate, 10%, whose shareholders require a higher expected return to compensate the financial risks, so the share prices of the two companies are not in equilibrium. There exists an arbitrage opportunity that we analyze in the next part.

## 6. ARBITRAGE

In this part, we identify an arbitrage opportunity and analyze two arbitrage strategies in violation of the Modigliani-Miller propositions.

### *6.1. Arbitrage Opportunity*

As claimed by Modigliani and Miller [11], if proposition 1 did not hold, in an efficient and complete market, individual investors could purchase and sell shares and bonds to exchange one income stream for another that has the same relevant respects as the earlier one. This enables the investors who are quite independent of their attitude towards risk to earn risk-free profits. As the investors continuously exploit those arbitrage opportunities, the market value of the overpriced shares will fall. That of under-priced shares will rise, consequently inclining to eliminate the discrepancy between the prices of the corporates.

### *6.2. Arbitrage Strategy*

The arbitrage strategy is as follows:

**Table 3.** Arbitrage Strategy

Million

| Strategy | $t = 0$ | $t = 1, 2, \ldots$ |
|---|---|---|
| Step 1: Short a of L's equity | $+170a$ | $-(20 - 50 \times 6\%)a = -17a$ |
| Step 2: Short a of L's bonds | $+50a$ | $-(50 \times 6\%)a = -3a$ |
| Step 3: Long $a\frac{V(L)}{V(U)}$ of U's equity | $-\frac{220}{200}a \times 200 = -220a$ | $+\frac{220}{200}a \times 20 = +22a$ |
| Net Cash Flow: | 0 | $+2a$ |

The first step is to short sell $a$ of the corporate L's equity. The second step is to short sell $a$ of the company L's bonds, which is equivalent to borrowing privately an amount of $50a$ million costing 6% p.a. The third step is to exert the proceeds in total to buy the maximum fraction of the firm U's equity, so today's net cash flow is zero. As shown in the table, the above strategy generates an arbitrage profit of $2a$ million each year in the future.

An alternative arbitrage strategy is to replace the third step above by longing a fraction of the corporate U's equity. This costs $200a$ today but entails perpetual cash inflows of $20a$ each year, which could exactly offset the total cash outflows of $20a$ generated by the first two steps. This strategy creates a costless arbitrage profit of $20a$ million today.

### *6.3. Empirical Test*

We continue to use the two corporates, U and L, as our empirical cases to present the process of arbitrage activities. In accordance with the Modigliani-Miller propositions, arbitrage activities trigger the overvalued company's share price to fall. That of the undervalued firm to rise until they reach the market equilibrium with no arbitrage opportunity. In the equilibrium, the corporate U's overall cost pf capital is equal to that for the company L, 10%. The firm U's market value is also equal to that of the corporate L, 200 million. Therefore, company L's cost of equity is 11.33%, which is higher than that for firm U due to the financial risks.

## 7. OPTIMAL CAPITAL STRUCTURE

In this part, we conduct the fixed effect model of panel data to examine the relationship between a corporate's ways to finance its real investments and its profit to work out the optimal combination of capital structure.

### *7.1. Data and Measurement Model*

In addition to minimizing the cost of capital, it is also essential to maximize a corporate's profit in order to obtain the optimal way to finance its real investments. For the sake of studying this problem, after excluding certain companies that have a negative net profit, we take 50 firms in the list of Fortune China 500 in 2020 as our analytical objects. Then we utilize the fixed effect model of short panel data to investigate the relationship between a corporate's combination of capital structure and its profit.

### *7.2. Variable Description*

The main variables involved are corporate's profit, business income, total assets, and owners' equity. The company's profit is net profit after tax, expressed in nop; the business income refers to main business income and other business income, expressed in rev; the total assets are total assets at the end the of the year, expressed in zc; the owners' equity refers to total owners' equity at the end of the year, excluding minority shareholders' equity, expressed in e.

### *7.3. Panel Data Regression Result*

Though the panel data is the focus in econometrics research in recent years, it is often biased and unreliable to simply use the cross-section data or the time series data to find economic rules. Thus, we use the fixed-effect model of short panel data to analyze the relationship between a corporation's capital structure and its profit. The model could be written as follows:

$$nop_{i,t} = c + \alpha_1 rev_{i,t} + \alpha_2 zc_{i,t} + \alpha_3 e_{i,t} + \varepsilon_{i,t} \tag{7}$$

In equation (7), i is the sample of each corporate, and t refers to year time; $nop_{i,t}$ represents the profit of the ith company in year t; $rev_{i,t}$ is the business income of the ith firm in year t; $zc_{i,t}$ refers to the total assets of the ith corporate in year t; $e_{i,t}$ represents the owners' equity of the ith company in year t; $c$ is a constant term; $\alpha_1, \alpha_2, \alpha_3$ are coefficient vectors, and $\varepsilon_{i,t}$ is a random disturbance term. In accordance with equation (7), we could acquire Table 4 as follows:

**Table 4.** Panel Data Regression Result

| Variables | (1) | (2) | (3) |
|---|---|---|---|
| | Fixed Effect | Random Effect | Between Estimator |
| $rev_{i,t}$ | 0.0281373*** | 0.0051836 | -0.013702* |
| | (0.004762) | (0.0047774) | (0.0078536) |
| $zc_{i,t}$ | -0.0077111** | 0.0054403 | 0.012068** |
| | (0.003539) | (0.0035436) | (0.0055522) |
| $e_{i,t}$ | 0.1842186*** | 0.1277429*** | 0.0710421*** |
| | (0.011090) | (0.0108626) | (0.0166257) |
| $c$ | -1513963*** | -574694.4* | 632266.5* |
| | (168370.4) | (297731.4) | (318364) |
| Sample Size | 250 | 250 | 250 |
| $R^2$ | 76.56% | 73.42% | 57.89% |

Note: robust standard errors are shown in bracket; ***, **, * are 1%, 5% and 10% respectively.

In Table 4, we find that all the coefficients are very significant, so the model design is reasonable. Long-term variable cost per unit is U-shaped, and with an increase in the corporate's scale, the company's profit increases as well until the scale reaches a threshold value. When the scale beyond the threshold, the firm will turn into a diseconomy of scale [13]. Also, as a corporation's owners' equity increases, its profit will increase at the same time.

In light of the previous sensitivity analysis, a corporate's cost of equity is the most sensitive factor in the determination of the weighted average cost of capital, so we treat the proportion of equity in a company's capital structure as the independent variable and assume that the firm's market value is fixed in order to explore the relationship between a corporate's ways to finance its real investments and its profit. Thus, any changes in the proportion of equity will trigger changes in the proportion of debt and, in turn, its overall sources of funding, which will affect the company's profit at the same time. Therefore, we can conclude that the greater the proportion of equity in a firm's capital structure, the greater the profit.

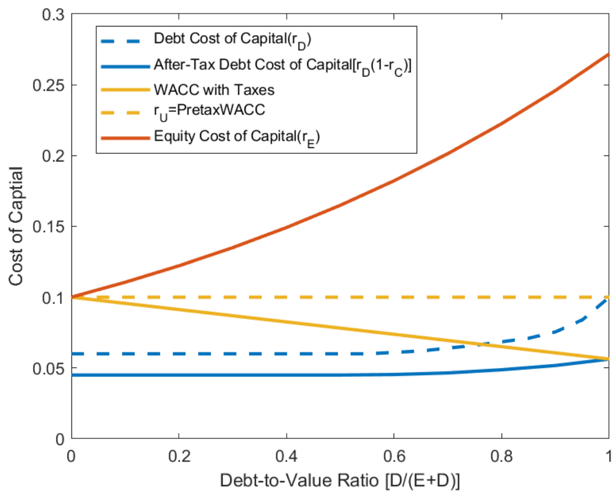


**Figure 6.** Changes in the Cost of Capital for Changes in the Debt-to-Value Ratio

We realize that to figure out the optimal ways to finance a corporate's real investments needs the minimum weighted average cost of capital and the maximum net profit. Suppose a company's proportion of equity is decreasing, equivalent to that its debt-to-value ratio is increasing. In that case, the weighted average cost of capital will be approaching the optimal capital structure. However, as the model that we have already established indicates, such a trend violates the firm's primary goal to maximize its profit. Therefore, we suggest there exists an eclectic solution to the optimal sources of funding.

## 8. CONCLUSION

This paper first examines how to exert the Gordon growth model to obtain a corporate's market value and utilize the average weight approach to acquire a company's overall cost of capital. Then we use the sensitivity analysis to find that a firm's cost of equity is the most sensitive factor in both computations of the Gordon growth model and that of the weighted average cost of capital. In addition, this paper investigates changes in a corporate's price concerning various ways to finance its real investments and figure out that the proportion of debt financing has no impact on its market value as articulated by Modigliani and Miller. Otherwise, there exists an arbitrage opportunity that enables individual investors to earn risk-free profits. Finally, we conduct the fixed effect model of panel data to determine a specific relationship between a company's capital structure and its profit. Thus, when determining the optimal funding sources, a firm should trade-off its profit and its weighted average cost of capital together to reach an eclectic settlement.

**Table 5.** Key Notation

| Notation | Definition |
|---|---|
| $U$ | Company U |
| $L$ | Company L |
| $CF$ | Operating Income p.a. |
| $P$ or $V$ | Firm's Market Value |
| $r_E$ | Cost of Equity |
| $r_D$ | Cost of Debt |
| $r_A$ | Cost of Overall Capital |
| $D$ | Market Value of Debt |
| $E$ | Market value of Equity |
| $nop$ | Firm's Profit |
| $rev$ | Business Income |
| $zc$ | Total Asset |
| $\varepsilon$ | Random Disturbance Term |
| $c$ | Constant Term |
| $\alpha$ | Coefficient Vectors |
| $e$ | Owners' Equity |

However, those studies indeed have some limitations. The most significant one is making some unrealistic assumptions, such as no transaction costs and corporate taxes. In the future, we may exclude some of those unpractical assumptions and take corporate taxes and default risk into consideration, which will enable the model to be more realistic and help us better understand the reality and apply the theory into application.

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