

## Research Article

# Regret Theory-Based Case-Retrieval Method with Multiple Heterogeneous Attributes and Incomplete Weight Information

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## ABSTRACT

Case retrieval is a crucial step in case-based reasoning (CBR), which is related to decision-making effectiveness. To improve decision support, CBR usually calculates case similarity and evaluates utility. However, the psychological behavior of decision makers is seldom considered in case retrieval. This paper proposes a novel case-retrieval method that deals with multiple heterogeneous attributes and incomplete weight information based on regret theory (RT). First, we define the function of the perceived utility based on attribute similarity and RT. Next, a mathematical programming model is constructed to determine the attribute weights based on linear programming technique for multidimensional analysis of preference (LINMAP). Based on this, we can calculate the perceived utility and determine a set of similar historical cases. Furthermore, the utilities of the evaluated attributes are calculated based on RT and LINMAP. Subsequently, we compute the comprehensive utilities of similar historical cases and obtain the ranking order of similar historical cases. Thus, the most suitable historical case is obtained. Finally, a case study of a gas explosion is conducted to illustrate the use of the proposed method.

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## 1. INTRODUCTION

Case-based reasoning (CBR) is a type of comparative reasoning, wherein new problems are solved by referring to the solutions of old problems. Because CBR can find solutions quickly, it is widely used in many fields such as emergency decision-making [1,2], fault detection [3,4], and medicine [4,5], and the historical case with the highest similarity is used as a reference. However, some studies have proved that the most similar historical case may not be the most suitable for a target case [6–8]. To this end, the most suitable historical case is typically retrieved by first evaluating case similarity and then retrieving the most suitable historical case based on the evaluation criteria. Therefore, a good case-similarity evaluation and a good retrieval method are vital to identify the most suitable historical case.

The case information in the decision-making process is often qualitative and quantitative, which are usually described by heterogeneous information [9–11]. There are two main research directions related to case-similarity evaluation with multiple heterogeneous attributes. The first is to propose a new case-similarity evaluation method for a heterogeneous multi-attribute problem.

For example, Fan *et al.* [12] proposed a hybrid similarity evaluation method with five formats of attribute values: crisp symbols, crisp numbers, interval numbers, fuzzy linguistic variables, and random variables; Zheng *et al.* [13] presented a hybrid multi-attribute case-similarity evaluation method with four formats of attribute values: crisp numbers, interval numbers, multi-granularity linguistic variables, and intuitionistic fuzzy numbers; and Yu *et al.* [14] considered crisp numbers, interval numbers, crisp symbols, linguistic terms, and probabilistic linguistic term sets for case-similarity evaluation, and in another study, Yu *et al.* [15] considered crisp numbers, crisp symbols, interval numbers, and fuzzy linguistic variables for case-similarity evaluation. The second is to determine attribute weights. For example, Zheng *et al.* [16] proposed a new case-retrieval method based on double-frontier data-envelopment analysis (DEA): DEA models determined the attribute weights automatically without the need to be specified; Yan *et al.* [17] presented a method of optimizing weights during case retrieval to improve problem-solving; and Wu *et al.* [18] proposed a weight-determination method using particle swarm optimization. However, existing studies have rarely considered the psychological behavior of decision makers. Various emotions, such as rejoicing, regret, or dislike, may affect the decision-making process [19]. Therefore, it is necessary to consider the decision maker's psychological behavior when evaluating case similarities.

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To improve problem-solving, some studies have begun to investigate how the most appropriate historical case can be retrieved from similar historical cases according to evaluation criteria. For example, Qi *et al.* [6] used the technique for order of preference by similarity to ideal solution (TOPSIS) to calculate the utility of similar historical cases. Zheng *et al.* [8] proposed a DEA model to obtain the priority vector of the evaluation matrix to determine the most suitable historical cases. Wang *et al.* [1] also considered the utilities of similar historical cases. However, these selections unrealistically assumed that the decision makers were completely rational.

Recently, several theories, such as prospect theory (PT) [20,21], regret theory (RT) [21,22], and cumulative prospect theory (CPT) [23], have been proposed to model psychological behavior in the decision-making process. PT and CPT are commonly used but have some limitations; for example, reference points must be given in advance but are difficult to determine, and many parameters must be set in the calculation formulas. In contrast, RT does not require determining the reference points in advance and has fewer parameters. RT is a model of psychological behavior that focuses on regret and rejoicing from the selection of one alternative over another. RT has been used in many studies to solve decision-making problems, considering the decision makers' psychological behavior. Zhang *et al.* [22] integrated RT into group decision-making to consider the regret aversion of decision makers. Zhou *et al.* [24] proposed a new method based on TOPSIS and RT to solve the gray random multi-attribute decision-making problem. Peng and Yang [21] presented a new method based on RT and PT to solve random multi-attribute decision-making problems. Therefore, RT is an effective tool for solving decision-making problems considering psychological behavior.

The following three challenges exist in applying an RT-based case-retrieval method to select the most suitable historical case: (1) evaluating case similarities based on RT, (2) determining utility according to the evaluation based on RT, and (3) evaluating attribute weights considering the decision makers' psychological behavior. Furthermore, the attributes are often only partially known because of the associated complexity, lack of data, or lack of knowledge [25].

This paper proposes a RT-based case-retrieval method based on case retrieval and RT for scenarios with multiple heterogeneous attributes and incomplete weight information. First, we calculate case similarity considering the psychological behavior of decision makers using RT. Second, we construct a set of similar historical cases according to the perceived utility based on case similarity. Third, we select the most suitable historical case, considering the psychological behavior of decision makers, and RT is incorporated to evaluate utility. Finally, we calculate the comprehensive case utilities according to the perceived utility based on case similarity and the comprehensive perceived utility based on the evaluated information. Moreover, the weights of historical case attributes and evaluated attributes are considered incomplete, and the decision makers assign their preferences for similar historical cases. With the linear programming for multidimensional analysis of preference (LINMAP), we construct a mathematical programming model based on consistency and inconsistency measurements to determine the attribute weights.

The novelties of the developed approach include the following aspects: (1) The proposed method considers the psychological

behavior of decision makers by using RT. The proposed method is more consistent with the actual decision and can provide better decision results. (2) The weights of the problem attribute and evaluate attribute are determined by LINMAP. The results of case retrieval are more objective and accurate. (3) The proposed method not only considers the perceived utility based on case similarity between the historical cases and the target case, but also the evaluation utility of the decision makers. Therefore, the results are more suitable for the target case.

The remainder of this paper is organized as follows. Section 2 reviews basic concepts and notations. Section 3 describes the proposed RT-based case-retrieval method based on case retrieval and RT with multidimensional preference information and incomplete weight information. Section 4 presents a case study of a gas explosion to illustrate the use of the proposed method and compares the proposed method with other methods. Section 5 concludes this paper.

## 2. PRELIMINARIES

This section describes the concepts of fuzzy numbers including intuitionistic fuzzy numbers and single-valued neutrosophic numbers. Next, incomplete weight information is introduced. Finally, RT is described.

### 2.1. Fuzzy Numbers

**Definition 1.** [26] Let  $U = \{u_1, u_2, \dots, u_m\}$  be a finite universe of discourse. An intuitionistic fuzzy set  $A$  in  $U$  can be defined as  $A = \{ \langle u_i, u_A(u_i), v_A(u_i) \mid u_i \in U \rangle$ , where functions  $u_A : U \rightarrow [0, 1]$  and  $v_A : U \rightarrow [0, 1]$  represent the degree of membership and degree of nonmembership, respectively, and they satisfy the condition  $0 \leq u_A(u_i) + v_A(u_i) \leq 1$ . Let  $\pi_A(u_i) = 1 - u_A(u_i) - v_A(u_i)$  be the hesitant degree of the intuitionistic fuzzy set, satisfying the condition  $0 \leq \pi_A(u_i) \leq 1$ .

For convenience, we define the intuitionistic fuzzy number as  $\alpha = (u_\alpha, v_\alpha)$ , which satisfies the conditions  $u_\alpha \in [0, 1]$ ,  $v_\alpha \in [0, 1]$ ,  $0 \leq u_\alpha + v_\alpha \leq 1$ . The score function can be defined as  $s(\alpha) = u_\alpha - v_\alpha$ .

**Definition 2.** [27] Let  $X$  be a set of objects. A single-valued neutrosophic set can be defined as

$$A = \{x (T_A(x), I_A(x), F_A(x)) \mid x \in X\},$$

where  $T_A(x)$ ,  $I_A(x)$ , and  $F_A(x)$  represent the truth-membership function, indeterminacy-membership function, and falsity-membership function, respectively, satisfying the following condition:  $\forall x \in X, T_A(x), I_A(x), F_A(x) \in [0, 1]$  and  $0 \leq T_A(x), I_A(x), F_A(x) \leq 3$ .

For convenience, we define the single-valued neutrosophic number as  $(T_A(x), I_A(x), F_A(x))$ , recorded briefly as  $A = (T_x, I_x, F_x)$ .

### 2.2. Incomplete Weight Information

In the real world, case information is often uncertain; thus, the attribute weights may also be incomplete. The incomplete weight information can be described in the following five forms, which

are denoted by a subset,  $\Lambda_i (i = 1, 2, 3, 4, 5)$ , of attribute weights in  $\Lambda_0$  [28,29]. Let  $W = \{w_1, w_2, \dots, w_m\}$  be a vector of the attribute weights such that  $\sum_{i=1}^m w_i = 1$  and  $0 \leq w_i \leq 1, i \in M, M = \{1, 2, \dots, m\}$ .

**Form 1 Weak ranking:**  $\Lambda_1 = \{w \in \Lambda_0 | w_i \geq w_j, \text{ for all } i \in I_1 \text{ and } j \in J_1\}$ , where  $I_1$  and  $J_1$  are two disjoint subsets of the subscript index set  $M$  of all attributes.

**Form 2 Strict ranking:**  $\Lambda_2 = \{w \in \Lambda_0 | u_{ij} \geq w_i - w_j \geq \lambda_{ij}, \text{ for all } i \in I_2 \text{ and } j \in J_2\}$ , where  $u_{ij} > 0$  and  $\lambda_{ij} > 0$  are constants, satisfying  $u_{ij} > \lambda_{ij}$ ;  $I_2$  and  $J_2$  are two disjoint subsets of the subscript index set  $M$ .

**Form 3 Difference ranking:**  $\Lambda_3 = \{w \in \Lambda_0 | w_i - w_j \geq w_k - w_l, \text{ for all } i \in I_3, j \in J_3, k \in K_3 \text{ and } l \in L_3\}$ , where  $I_3, J_3, K_3,$  and  $L_3$  are four disjoint subsets of the subscript index set  $M$ .

**Form 4 Multiplication ranking:**  $\Lambda_4 = \{w \in \Lambda_0 | w_i \geq \eta_{ij} w_j, \text{ for all } i \in I_4 \text{ and } j \in J_4\}$ , where  $\eta_{ij} > 0$  is a constant;  $I_4$  and  $J_4$  are two disjoint subsets of the subscript index set  $M$ .

**Form 5 Interval ranking:**  $\Lambda_5 = \{w \in \Lambda_0 | \lambda_i \leq w_{ij} \leq \lambda_i + \varepsilon_i, \text{ for all } i \in I_5\}$ , where  $\lambda_i > 0$  and  $\varepsilon_i > 0$  are constants;  $I_5$  is a subset of  $M$ .

### 2.3. Regret Theory

In the case-retrieval process, decision makers are often not completely rational because of the uncertainty and limitations of their knowledge. They generally make rational choices; however, they have some emotions (such as regret and rejoicing, rewards, and costs) when making decisions. Emotions should be considered in the decision-making process because they tend to influence decisions. RT considers not only the utility of the selected alternative but also the effects of the other alternatives on the decision-making process [30,31]. Therefore, RT consists of two parts: the utility function of the current result and the regret/rejoice function in comparison with other results.

Let  $A = \{A_1, A_2, \dots, A_n\}$  be a set of alternative, where  $A_i$  represents the  $i$ th alternative. Let  $x_i$  be the possible result obtained after selecting alternative  $A_i$ . Then, the decision maker’s perceived utility for alternative  $A_i$  is defined as

$$u_i = v(x_i) + R(v(x_i) - v(x^*)), \tag{1}$$

where  $x^* = \max \{x_i | i = 1, 2, \dots, n\}$ . This implies that the decision maker will regret selecting alternative  $x_i$  rather than the best alternative. Here,  $v(x_i)$  is the utility function of the current result, which satisfies the conditions  $v'(x_i) \geq 0$  and  $v''(x_i) \leq 0$ . In this paper, we use a power function to define the utility function, and it can be defined as

$$v(x_i) = (x_i)^\alpha, \tag{2}$$

where  $\alpha$  is the risk aversion coefficient of decision makers,  $0 \leq \alpha \leq 1$ ; a smaller  $\alpha$  indicates a greater degree of risk aversion of decision makers.  $R(u(x_i) - u(x^*))$  represents the regret value, which satisfies the condition  $R(u(x_i) - u(x^*)) < 0$ . In this paper, we define

the regret/rejoice function as follows [22]:

$$R(u(x_i) - u(x^*)) = 1 - \exp(-\delta(u(x_i) - u(x^*))), \tag{3}$$

where  $\delta$  is the decision maker’s risk aversion coefficient. A smaller  $\delta$  indicates a greater degree of risk aversion of decision makers. Figure 1 shows the effect of  $\delta$  on the regret/rejoice function  $R(u(x_i) - u(x^*))$ .

## 3. PROPOSED METHOD

This section presents the proposed case-retrieval method based on RT with heterogeneous attributes (such as crisp numbers, interval numbers, intuitionistic fuzzy numbers, and single-valued neutrosophic numbers). Moreover, the weights of case attributes and evaluated attributes are incomplete. Figure 2 shows the basic procedure of the proposed method.

### 3.1. Calculation of the Perceived Utility Based on Attribute Similarity

The following notations are used.  $C = \{C_1, C_2, \dots, C_m\}$  is a set of historical cases, where  $C_i$  represents the  $i$ th historical case, and  $C_0$  is the target case. The problem attribute vector with regard to the historical cases and the target case is  $X = \{X_1, X_2, \dots, X_n\}$ , where  $X_j$  represents the  $j$ th problem attribute. The problem attribute value of the historical case is  $x_{ij}$ , where  $i \in \{1, 2, \dots, m\}$  and  $j \in \{1, 2, \dots, n\}$ , and the problem attribute value of the target case is  $x_{0j}$ .  $W^p = \{w_1^p, w_2^p, \dots, w_n^p\}$  is the vector of attribute weights, where  $w_j^p$  represents the  $j$ th attribute weight, such that  $\sum_{j=1}^n w_j^p = 1$  and  $0 \leq w_j^p \leq 1, w_j^p \in \Lambda_0$ . Moreover,  $d_j(C_0, C_i)$  denotes the attribute distance between the target case  $C_0$  and historical case  $C_i$  with regard to the problem attribute  $X_j$ . The problem attribute distance  $d_j(C_0, C_i)$  is defined in four scenarios, depending on the attribute type, as follows:

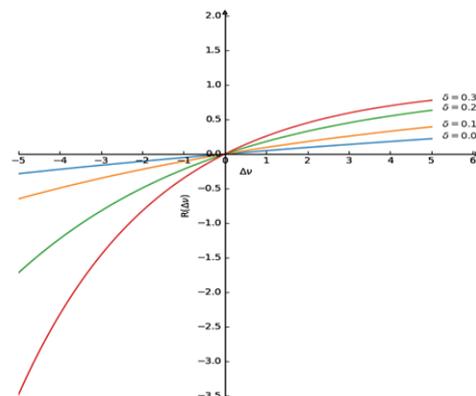


Figure 1 | Regret/rejoice function of regret theory (RT).

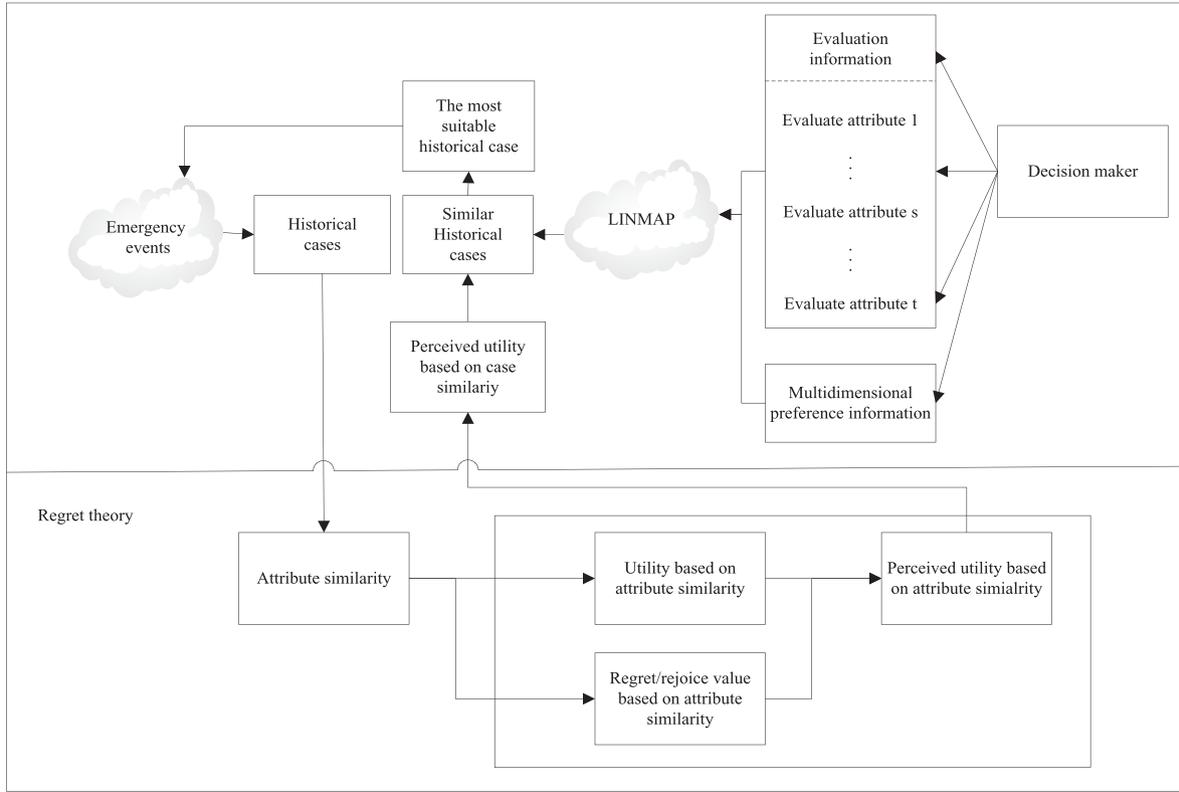


Figure 2 | Flowchart of the proposed RT-based case-retrieval method.

- (1) When attribute  $X_j$  is a crisp number, then

$$d_j(C_0, C_i) = \frac{|x_{ij} - x_{0j}|}{d_j^{\max'}}$$

where  $d_j^{\max'} = \max \{|x_{ij} - x_{0j}| | i \in \{1, 2, \dots, m\}\}$ .

- (2) When attribute  $X_j$  is an interval number, i.e.,  $x_{ij} = [x_{ij}^-, x_{ij}^+]$ ,  $x_{0j} = [x_{0j}^-, x_{0j}^+]$ , then

$$d_j(C_0, C_i) = \frac{\sqrt{(x_{ij}^- - x_{0j}^-)^2 + (x_{ij}^+ - x_{0j}^+)^2}}{d_j^{\max''}}$$

where

$$d_j^{\max''} = \max \left\{ \sqrt{(x_{ij}^- - x_{0j}^-)^2 + (x_{ij}^+ - x_{0j}^+)^2} \mid i \in \{1, 2, \dots, m\} \right\}$$

- (3) When attribute  $X_j$  is an intuitionistic fuzzy number, i.e.,  $x_{ij} = \langle u_{ij}, v_{ij} \rangle$ ,  $x_{0j} = \langle u_{0j}, v_{0j} \rangle$ , then

$$d_j(C_0, C_i) = \frac{\sqrt{\frac{1}{2} \left( (u_{ij} - u_{0j})^2 + (v_{ij} - v_{0j})^2 + (\pi_{ij} - \pi_{0j})^2 \right)}}{d_j^{\max'''}}$$

where  $\pi_{ij} = 1 - u_{ij} - v_{ij}$ ,  $\pi_{0j} = 1 - u_{0j} - v_{0j}$ ,

$$d_j^{\max'''} = \max \left\{ \sqrt{\frac{1}{2} \left( (u_{ij} - u_{0j})^2 + (v_{ij} - v_{0j})^2 + (\pi_{ij} - \pi_{0j})^2 \right)} \mid i \in \{1, 2, \dots, m\} \right\}$$

- (4) When attribute  $X_j$  is a single-valued neutrosophic number, i.e.,  $x_{ij} = \langle T_{ij}, I_{ij}, F_{ij} \rangle$ ,  $x_{0j} = \langle T_{0j}, I_{0j}, F_{0j} \rangle$  then

$$d_j(C_0, C_i) = \frac{\sqrt{\frac{1}{3} \left( (T_{ij} - T_{0j})^2 + (I_{ij} - I_{0j})^2 + (F_{ij} - F_{0j})^2 \right)}}{d_j^{\max''''}}$$

where

$$d_j^{\max''''} = \max \left\{ \sqrt{\frac{1}{3} \left( (T_{ij} - T_{0j})^2 + (I_{ij} - I_{0j})^2 + (F_{ij} - F_{0j})^2 \right)} \mid i \in \{1, 2, \dots, m\} \right\}$$

Furthermore, according to [12], the inverse exponential function is used to calculate the attribute similarity. Specifically, attribute similarity  $Sim_j(C_0, C_i)$  is defined as

$$Sim_j(C_0, C_i) = \exp(-d_j(C_0, C_i)).$$

When calculating the case similarity, RT considers emotions. If the value of the attribute similarity is not the largest, the decision maker

will regret; otherwise, they will rejoice. Therefore, RT is introduced to calculate the decision maker's psychological behavior toward case similarity. The perceived utility, based on case similarity, is calculated as follows:

**Step 1:** Suppose that the ideal attribute similarity with regard to problem attribute  $X_j$  is  $Sim_j^+$ , given by

$$Sim_j^+ = \max \{ Sim_j(C_0, C_i) \mid i \in \{1, 2, \dots, m\} \}. \quad (9)$$

**Step 2:** Calculate the perceived utility based on the attribute similarity. According to Section 2.3, the perceived utility consists of two parts: the utility value and the regret/rejoice value. Let  $u_{ij}$  be the utility based on attribute similarity, given by

$$u_{ij} = (Sim_j(C_0, C_i))^\alpha. \quad (10)$$

**Step 3:** Suppose that the regret/rejoice value based on attribute similarity is  $r_{ij}$ , given by

$$r_{ij} = \begin{cases} 1 - \exp \left[ -\delta \left( Sim_j(C_0, C_i) - Sim_j^+ \right) \right], & Sim_j(C_0, C_i) < Sim_j^+ \\ 0, & Sim_j(C_0, C_i) \geq Sim_j^+ \end{cases} \quad (11)$$

Based on this, the perceived utility based on attribute similarity,  $v_{ij}$ , can be defined as

$$v_{ij} = r_{ij} + u_{ij}. \quad (12)$$

**Step 4:** The perceived utility based on case similarity,  $\Phi_i$ , can be calculated as

$$\Phi_i = \sum_{j=1}^n w_j^p \Phi_{ij}. \quad (13)$$

Obviously,  $\Phi_i \in [0, 1]$ . A higher value of  $\Phi_i$  corresponds to a more suitable historical case.

### 3.2. Perceived Utility Value of Case-Similarity Consistency and Inconsistency Measurements

Let  $\Omega = \{(k, l) \mid C_k \geq C_l, k, l = 1, 2, \dots, m\}$  be the multidimensional preference information given by the decision maker. If  $\Phi_k \geq \Phi_l$  for the pair of historical cases  $(k, l) \in \Omega$ , then historical case  $C_k$  has more perceived utility than historical case  $C_l$ . Thus, the ranking order of historical cases  $C_k$  and  $C_l$ , determined by  $\Phi_k$  and  $\Phi_l$  based on  $(w_j^p, Sim_j^+)$ , is consistent with the preferences given by the decision maker. Conversely, if  $\Phi_k < \Phi_l$ , then  $(w_j^p, Sim_j^+)$  is not chosen because the ranking order determined by  $\Phi_k$  and  $\Phi_l$  is inconsistent with the preferences given by the decision maker. Subsequently, we define an index to measure the degree of consistency between the ranking order of historical cases  $C_k$  and  $C_l$  determined by  $\Phi_k$  and  $\Phi_l$ . The consistency index is defined as

$$(\Phi_l - \Phi_k)^+ = \begin{cases} \Phi_l - \Phi_k, & \Phi_l \geq \Phi_k \\ 0, & \Phi_l < \Phi_k \end{cases}. \quad (14)$$

Next, the consistency index can be rewritten as

$$(\Phi_l - \Phi_k)^+ = \max \{0, \Phi_l - \Phi_k\}. \quad (15)$$

Furthermore, the total consistency index is defined as

$$G = \sum_{k,l \in \Omega} (\Phi_l - \Phi_k)^+ = \sum_{k,l \in \Omega} \max \{0, \Phi_l - \Phi_k\}. \quad (16)$$

Clearly, a bigger G corresponds to a higher degree of total consistency.

Similarly, an index to measure the degree of inconsistency between the ranking order of historical cases  $C_k$  and  $C_l$  determined by  $\Phi_k$  and  $\Phi_l$  is defined as

$$(\Phi_l - \Phi_k)^- = \begin{cases} \Phi_k - \Phi_l, & \Phi_l < \Phi_k \\ 0, & \Phi_l \geq \Phi_k \end{cases}. \quad (17)$$

Subsequently, the inconsistency index can be rewritten as

$$(\Phi_l - \Phi_k)^- = \max \{0, \Phi_k - \Phi_l\}. \quad (18)$$

Furthermore, the total inconsistency index is defined as

$$B = \sum_{k,l \in \Omega} (\Phi_l - \Phi_k)^- = \sum_{k,l \in \Omega} \max \{0, \Phi_k - \Phi_l\}. \quad (19)$$

Similarly, a bigger B corresponds to a higher degree of total inconsistency.

### 3.3. Mathematical Programming Model Based on LINMAP

For case retrieval, it is crucial to determine attribute weights because they are linked to the perceived utility values. Recent research has mainly used two approaches: development of an optimization model [13,32] and machine learning [17,33]. However, these two approaches are not suitable for solving our problem because we consider fuzzy reference information. Moreover, our weight information is incomplete and contains subjective multidimensional attributes. The LINMAP method is based on a pairwise comparison of the alternatives given by the decision maker. It generates the best alternative when the solution is close to the ideal solution. Moreover, the LINMAP method determines the attribute weights by constructing a mathematical programming model. Therefore, we use LINMAP to determine the attribute weights. According to [22,34], the mathematical programming model should ensure that the consistency degree of the perceived utility is as large as possible, and the inconsistency degree of the perceived utility is as small as possible. Based on this, we construct the following model to determine the attribute weights:

$$\begin{aligned} \text{Min } B &= \sum_{(k,l) \in \Omega} \max \{0, \Phi_k - \Phi_l\} \\ \text{s.t. } G - B &\geq \eta, \\ w_j^p &\geq \epsilon, \end{aligned} \quad (20)$$

$$W^p \in \Lambda,$$

where  $\eta > 0$  is a priori threshold, which is given by the decision makers. Moreover,  $\varepsilon > 0$  is a sufficiently small positive number to ensure  $w_j^p > 0$ .

For each pair of  $(k, l) \in \Omega$ , let  $\lambda_{kl} = \max\{0, \Phi_k - \Phi_l\}$ . Hence,  $\lambda_{kl} \geq 0$  and  $\lambda_{kl} \geq \Phi_k - \Phi_l$ ; i.e.,  $\lambda_{kl} \geq 0$  and  $\Phi_l - \Phi_k + \lambda_{kl} \geq 0$ . We can obtain

$$\begin{aligned} G - B &= \sum_{(k,l) \in \Omega} \left[ (\Phi_k - \Phi_l)^+ - (\Phi_k - \Phi_l)^- \right] \\ &= \sum_{(k,l) \in \Omega} (\Phi_k - \Phi_l). \end{aligned} \tag{21}$$

Thus, (20) can be rewritten as the following mathematical programming model:

$$\begin{aligned} \min \quad & \sum_{(k,l) \in \Omega} \lambda_{kl} \\ \text{s.t.} \quad & \sum_{(k,l) \in \Omega} (\Phi_k - \Phi_l) \geq \rho, \\ & \lambda_{kl} - \Phi_k + \Phi_l \geq 0, \end{aligned} \tag{22}$$

$$\lambda_{kl} \geq 0,$$

$$w_j^p \geq \varepsilon,$$

$$W^p \in \Lambda.$$

Optimal attribute weights  $w_j^{p*} = (w_1^{p*}, w_2^{p*}, \dots, w_m^{p*})$  can be obtained by solving this problem.

### 3.4. Set of Similar Historical Cases

After determining the attribute weights  $w_j^{p*} = (w_1^{p*}, w_2^{p*}, \dots, w_m^{p*})$ , we can obtain the perceived utility based on case similarity  $\Phi_i$ . Subsequently, we can retrieve similar historical cases. According to [35,36], a set of similar historical cases can be constructed using  $\Phi_i$ . We select historical cases with a high perceived utility based on case similarities. Therefore, we set a perceived utility based on the case-similarity threshold. Let  $\xi$  denote the perceived utility,  $\xi \in [\min\{\Phi_i\}, \max\{\Phi_i\}]$ , which is given by the decision maker based on their knowledge and experience. A large value of  $\xi$  indicates that the decision maker has high expectations of perceived utility.

A historical case  $C_i$  is selected if it satisfies the condition  $\Phi_i > \xi$ . Furthermore, the selected historical cases form a similar case set  $S_v, v \in \{1, 2, \dots, h\}$ , such that  $\{S_v = \{C_i\} | i \in N_v\}$ , which represents the index set of all similar historical cases.

### 3.5. Evaluation of Utility of Similar Historical Cases

According to the similar case set, the  $p$ th decision maker  $D^p, p = \{1, 2, \dots, h\}$  evaluates the effects of the alternative solutions applied to the target case. Suppose that the evaluated attributes are denoted by  $R = \{R_1, R_2, \dots, R_t\}$ , where  $R_s$  is an evaluated attribute, and  $s = \{1, 2, \dots, t\}$ . Let  $r_{vs}^p$  denote the evaluated attribute value of  $D^p$  for a similar historical case  $S_v$ . Let  $W_v^R = \{w_{v1}^R, w_{v2}^R, \dots, w_{vt}^R\}$  be

a vector of the evaluated attribute weights  $w_{vs}^R$ , such that  $\sum_{s=1}^t w_{vs}^R = 1$

and  $0 \leq w_{vs}^R \leq 1, w_{vs}^R \in \Lambda_0$ . In the case retrieval, decision makers mainly consider linguistic variables and crisp numbers to express the evaluated attribute values. Then, the steps of evaluating the utilities of similar historical cases are shown as follows.

**Step 1:** Convert linguistic variables into triangular fuzzy numbers and defuzzify them. Let  $Y$  be a linguistic term set with odd cardinalities; i.e.,  $Y = \{y_q | q \in \{1, 2, \dots, T\}\}$ . According to [37], the linguistic variable  $y_q, y_q \in Y$  can be converted into a triangular fuzzy number as follows:

$$\begin{aligned} \tilde{y}_q &= (y_q^a, y_q^b, y_q^c) \\ &= (\max((y-1)/T, 0), y/T, \min((y+1)/T, 1)), \end{aligned} \tag{23}$$

where  $y_q^a, y_q^b$ , and  $y_q^c$  are real numbers, and  $y_q^a \leq y_q^b \leq y_q^c$ .

Subsequently, we defuzzify the triangular numbers as follows:

$$\eta_q = (y_q^a + 4 \cdot y_q^b + y_q^c) / 6. \tag{24}$$

**Step 2:** Normalize the evaluated attributes. When the linguistic variables are transformed into crisp numbers, all the evaluated attributes are crisp. Next, we normalize the evaluated attributes as follows:

$$\tilde{r}_{vs}^p = \begin{cases} (r_{vs}^p - g_s^p) / (k_s^p - g_s^p), & r_{vs}^p \in N^b \\ (k_s^p - r_{vs}^p) / (k_s^p - g_s^p), & r_{vs}^p \in N^c \end{cases}, \tag{25}$$

where  $k_s^p = \max\{r_{vs}^p | v \in \{1, 2, \dots, h\}\}$ ,  $g_s^p = \min\{r_{vs}^p | v \in \{1, 2, \dots, h\}\}$ ,  $N^b$  is the benefit attribute value, and  $N^c$  is the cost attribute value.

**Step 3:** Calculate the utility of the evaluated attribute  $\pi_{vs}^p$ . According to Eq. (2),  $\pi_{vs}^p$  can be obtained as follows:

$$\pi_{vs}^p = (\tilde{r}_{vs}^p)^\alpha. \tag{26}$$

**Step 4:** Calculate the regret/rejoice value based on the evaluation attribute utility  $R_{vs}^p$ . According to Eq. (3),  $R_{vs}^p$  can be obtained as follows:

$$R_{vs}^p = 1 - \exp(-\delta(\pi_{vs}^p - \pi_s^{p*})), \tag{27}$$

where  $\pi_s^{p*} = \max\{\pi_{vs}^p | v \in \{1, 2, \dots, h\}\}$ .

**Step 5:** Calculate the perceived utility based on the evaluation criteria  $V_v^p$ . According to Eq. (1),  $V_v^p$  can be obtained as follows:

$$V_v^p = \sum_{s=1}^t w_{vs}^R (\pi_{vs}^p + R_{vs}^p). \quad (28)$$

**Step 6:** Calculate the consistency index  $G'$  and the inconsistency index  $B'$  based on  $V_{vs}^p$  (according to Section 3.2), as follows:

$$G' = \sum_{p=1}^h \sum_{(\beta,\gamma) \in \Omega^p} \max \left\{ 0, V_{\beta}^p - V_{\gamma}^p \right\}, \quad (29)$$

$$B' = \sum_{p=1}^h \sum_{(\beta,\gamma) \in \Omega^p} \max \left( 0, V_{\gamma}^p - V_{\beta}^p \right), \quad (30)$$

where  $(\beta, \gamma) \in \Omega^p$  represents the multidimensional preference information given by decision maker  $D^p$ .

**Step 7:** Construct the mathematical programming model to determine the attribute weights (according to Section 3.3), as follows:

$$\begin{aligned} & \min \sum_{(\beta,\gamma) \in \Omega^p} \lambda_{\beta\gamma}^p \\ \text{s.t. } & \sum_{p=1}^h \sum_{(\beta,\gamma) \in \Omega^p} (V_{\beta}^p - V_{\gamma}^p) \geq \rho, \\ & \lambda_{\beta\gamma}^p - V_{\beta}^p + V_{\gamma}^p \geq 0, \\ & \lambda_{\beta\gamma}^p \geq 0, \\ & w_{vs}^R \geq \varepsilon, \\ & W^R \in \Lambda, \end{aligned} \quad (31)$$

where  $\lambda_{\beta\gamma} = \max \left\{ 0, V_{\gamma}^p - V_{\beta}^p \right\}$ . Attribute weights  $w_{vs}^{R*}$  are evaluated by evaluating (31).

**Step 8:** Calculate the comprehensive perceived utility  $U_v$  based on the evaluation criteria, as follows:

$$U_v = \sum_{p=1}^h V_v^p. \quad (32)$$

### 3.6. Comprehensive Case Utility and Ranking Historical Cases

To retrieve the most suitable historical case, we need to measure the comprehensive case utility and rank similar historical cases. Thus, we use a simple additive method to aggregate the perceived utility based on case similarity  $\Phi_i$  ( $i \in N_v$ ), i.e.,  $\Phi_v$ , and the comprehensive perceived utility based on evaluation criteria  $U_v$ . Therefore, the comprehensive case utility  $\Gamma_v$  can be defined as

$$\Gamma_v = \Phi_v \cdot U_v. \quad (33)$$

Clearly, a larger value of  $\Gamma_v$  indicates that the historical case  $C_v$  is a better alternative for the target case.

## 4. ILLUSTRATIVE EXAMPLE

This section presents a case retrieval for a gas explosion as a case study to demonstrate the applicability and practicability of the proposed method.

### 4.1. Case Study

In recent years, various gas-explosion emergencies have occurred in China, which have led to huge losses for society. Such emergencies involve similar problems that may be solved by similar solutions. Thus, CBR can be used to quickly generate alternative solutions for the target case.

Company A is a coal company in Fujian Province, China. When a new gas-explosion emergency occurs, the company uses CBR to retrieve the most similar historical case to generate an alternative. To this end, this company collects 10 historical cases ( $C_1, C_2, \dots, C_{10}$ ), containing 8 attributes ( $X_1, X_2, \dots, X_8$ ), namely the number of underground personnel ( $X_1$ ), area of impact of the explosion ( $X_2$ ), degree of damage to the ventilation system ( $X_3$ ), degree of landslide ( $X_4$ ), scope of the fire ( $X_5$ ),  $O_2$  concentration ( $X_6$ ), CO concentration ( $X_7$ ), and  $CH_4$  concentration ( $X_8$ ). Among them,  $X_1, X_6, X_7$ , and  $X_8$  are crisp numbers;  $X_2$  is an interval number;  $X_3$  is an intuitionistic number; and  $X_4$  and  $X_5$  are single-valued neutrosophic numbers. Table 1 describes the historical cases and the target case ( $C_0$ ). The objective of this study is to retrieve the most suitable historical case and help decision makers to generate alternative solutions for the target case. Subsequently, we describe the computation process and the results obtained using the proposed method. The computation processes and results are presented as follows.

**Step 1:** We calculate the attribute similarities using Eqs. (4–8), and the computation results are listed in Table 2. We set the multidimensional preference information to  $\Omega = \{(2, 1), (3, 4), (5, 4), (6, 3), (6, 10), (7, 4), (10, 9)\}$ . Moreover, we assume that the attribute weights are incomplete.  $\Lambda = \{w \in \Lambda_0 | w_1 > 2w_2, 0.01 \leq w_2 - w_3 \leq 0.2, 0.1 \leq w_4 \leq 0.2, w_4 - w_5 \geq w_1 - w_2, w_5 \geq w_6, 0.1 \leq w_7 \leq 0.1, 0.1 \leq w_7 \leq 0.1, w_8 \geq 2w_7\}$ .

**Step 2:** We calculate the perceived utility based on attribute similarity  $v_{ij}$  using Eqs. (9–12). The computation results are listed in Table 3. Next, we set  $\rho = 0.01$ ,  $\varepsilon = 0.05$ ,  $\alpha = 0.88$ , and  $\delta = 0.3$ . Then, we construct the mathematical programming model according to Eqs. (14–19), and the attribute weights can be obtained as  $W^p = \{0.1200, 0.0600, 0.0500, 0.0200, 0.1400, 0.1300, 0.1000, 0.2000\}$ .

**Step 3:** We obtain the perceived utility based on case similarity according to Eq. (13), as follows:

$$\Phi_1 = 0.4535, \Phi_2 = 0.5802, \Phi_3 = 0.6380, \Phi_4 = 0.5382, \Phi_5 = 0.6036, \Phi_6 = 0.5165, \Phi_7 = 0.5141, \Phi_8 = 0.5216, \Phi_9 = 0.6117, \Phi_{10} = 0.6750.$$

**Step 4:** The decision makers set the perceived utility threshold  $\xi = 0.6$ , and a set of similar historical cases can be obtained as  $S = \{S_1, S_2, S_3, S_4\} = \{C_3, C_5, C_9, C_{10}\}$ . Subsequently, three decision makers evaluate three attributes: the effect of emergency

**Table 1** Information of historical cases and target cases.

	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	$X_6$	$X_7$	$X_8$
$C_1$	45	[26, 29]	<0.8, 0.16>	(0.7, 0.3, 0.1)	(0.7, 0.3, 0.1)	12	31	7
$C_2$	68	[25, 30]	<0.8, 0.14>	(0.9, 0.1, 0.1)	(0.8, 0.2, 0.1)	13	29	9
$C_3$	41	[22, 27]	<0.6, 0.3>	(0.9, 0.1, 0.2)	(0.85, 0.15, 0.05)	28	32	12
$C_4$	75	[20, 28]	<0.7, 0.2>	(0.8, 0.2, 0.2)	(0.85, 0.15, 0.05)	27	42	13
$C_5$	70	[29, 38]	<0.75, 0.2>	(0.9, 0.1, 0.2)	(0.7, 0.3, 0.1)	22	23	10
$C_6$	42	[32, 40]	<0.8, 0.15>	(0.75, 0.25, 0.1)	(0.75, 0.25, 0.2)	28	28	8
$C_7$	43	[26, 32]	<0.9, 0.05>	(0.8, 0.2, 0.2)	(0.65, 0.35, 0.3)	31	27	15
$C_8$	37	[15, 23]	<0.75, 0.2>	(0.95, 0.05, 0.1)	(0.8, 0.2, 0.2)	30	32	13
$C_9$	60	[17, 23]	<0.7, 0.25>	(0.65, 0.35, 0.1)	(0.75, 0.25, 0.2)	25	29	10
$C_{10}$	65	[25, 34]	<0.6, 0.35>	(0.95, 0.05, 0.15)	(0.75, 0.25, 0.2)	24	31	11
$C_0$	61	[23, 32]	<0.75, 0.2>	(0.8, 0.2, 0.15)	(0.8, 0.2, 0.15)	21	30	12

**Table 2** The attribute similarity  $\text{Sim}_j(C_0, C_i)$ .

	$\text{Sim}_1(C_0, C_i)$	$\text{Sim}_2(C_0, C_i)$	$\text{Sim}_3(C_0, C_i)$	$\text{Sim}_4(C_0, C_i)$	$\text{Sim}_5(C_0, C_i)$	$\text{Sim}_6(C_0, C_i)$	$\text{Sim}_7(C_0, C_i)$	$\text{Sim}_8(C_0, C_i)$
$C_1$	0.5134	0.7030	0.6065	0.5025	0.5614	0.4066	0.9200	0.3679
$C_2$	0.7470	0.7907	0.3679	0.5025	0.8249	0.4493	0.9200	0.5488
$C_3$	0.4346	0.6548	1.0000	0.5025	0.6241	0.4966	0.8465	1.0000
$C_4$	0.5580	0.6602	0.6065	0.5025	0.6241	0.5488	0.3679	0.8187
$C_5$	0.6873	0.4943	1.0000	0.5025	0.5614	0.9048	0.5580	0.6703
$C_6$	0.4531	0.3679	0.6065	0.6721	0.7165	0.4966	0.8465	0.4493
$C_7$	0.4724	0.7795	0.6065	0.7950	0.3679	0.3679	0.7788	0.5488
$C_8$	0.3679	0.3679	0.6065	0.3679	0.8249	0.4066	0.8465	0.8187
$C_9$	0.9592	0.4073	0.6065	0.3679	0.7165	0.6703	0.9200	0.6703
$C_{10}$	0.8465	0.7907	0.6065	0.3778	0.7165	0.7408	0.9200	0.8187

**Table 3** The perceived utility based on attribute similarity  $v_{ij}$ .

	$v_{i1}$	$v_{i2}$	$v_{i3}$	$v_{i4}$	$v_{i5}$	$v_{i6}$	$v_{i7}$	$v_{i8}$
$C_1$	0.4131	0.7068	0.5187	0.4540	0.5194	0.2917	0.9293	0.2060
$C_2$	0.7079	0.8133	0.2060	0.4540	0.8442	0.3482	0.9293	0.4448
$C_3$	0.3099	0.6473	1.0000	0.4540	0.5984	0.4098	0.8413	1.0000
$C_4$	0.4706	0.6540	0.5187	0.4540	0.5984	0.4771	0.2346	0.7827
$C_5$	0.6339	0.4449	1.0000	0.4540	0.5194	0.9158	0.4838	0.5993
$C_6$	0.3343	0.2796	0.5187	0.6674	0.7127	0.4098	0.8413	0.3150
$C_7$	0.3596	0.7998	0.5187	0.8172	0.2678	0.2400	0.7592	0.4448
$C_8$	0.2207	0.2796	0.5187	0.2781	0.8442	0.2917	0.8413	0.7827
$C_9$	0.9640	0.3317	0.5187	0.2781	0.7127	0.6304	0.9293	0.5993
$C_{10}$	0.8292	0.8133	0.5187	0.2913	0.7127	0.7175	0.9293	0.7827

rescue ( $R_1$ ), casualty reduction rate ( $R_2$ ), and property loss reduction rate ( $R_3$ ). Table 4 lists the evaluation criteria. The evaluated attribute  $R_1$  is determined from the linguistic variable set,  $s = \{\text{very good} : VG, \text{good} : G, \text{normal} : N, \text{bad} : B, \text{very bad} : VB$ .

**Step 5:** According to Eqs. (23–24), we convert the linguistic variables into crisp numbers. According to Eqs. (25–28), the perceived utility based on evaluation criteria  $V_v^p$  can be obtained, and the results are presented in Table 5.

**Step 6:** For the three decision makers, we set the multidimensional preference information as follows:  $\Omega^1 = \{(1, 2), (1, 3), (2, 3), (3, 4)\}$ ,  $\Omega^2 = \{(3, 1), (2, 4), (3, 4)\}$ ,

and  $\Omega^3 = \{(1, 2), (2, 4), (3, 1)\}$ . According to Eqs. (29–31), the mathematical programming model was constructed to determine the attribute weights, and the results were obtained as follows:  $W^R = \{0.2180, 0.4360, 0.3460\}$ . Subsequently, we calculated the comprehensive perceived utility based on information  $U_v$  evaluated by Eq. (32). The results were  $U_1 = 0.8005, U_2 = 0.7559, U_3 = 0.8844, U_4 = 0.6773$ .

**Step 7:** The comprehensive case utility  $\Gamma_v$  was calculated using Eq. (33). The results were  $\Gamma_1 = 0.5107, \Gamma_2 = 0.4562, \Gamma_3 = 0.5409$ , and  $\Gamma_4 = 0.4572$ . Subsequently, we ranked similar historical cases according to their comprehensive case utility as  $\Gamma_3 \geq \Gamma_1 \geq \Gamma_4 \geq \Gamma_2$ . As a result, the most suitable historical case was  $C_9$ .

**Table 4** | Evaluate information.

	E <sub>1</sub>			E <sub>2</sub>			E <sub>3</sub>		
	R <sub>1</sub>	R <sub>2</sub>	R <sub>3</sub>	R <sub>1</sub>	R <sub>2</sub>	R <sub>3</sub>	R <sub>1</sub>	R <sub>2</sub>	R <sub>3</sub>
s <sub>1</sub>	G	8.5	12	N	9	11	G	9	11
s <sub>2</sub>	N	9	9.6	G	10	10	N	10	9
s <sub>3</sub>	VG	11	7	VG	11	9	VG	12	9
s <sub>4</sub>	B	7	8.7	N	9	12	N	9	10

**Table 5** | Perceived utility based on evaluate information.

	E <sub>1</sub>			E <sub>2</sub>			E <sub>3</sub>		
	R <sub>1</sub>	R <sub>2</sub>	R <sub>3</sub>	R <sub>1</sub>	R <sub>2</sub>	R <sub>3</sub>	R <sub>1</sub>	R <sub>2</sub>	R <sub>3</sub>
s <sub>1</sub>	0.7403	0.7342	1.0000	0.4674	0.7884	0.9039	0.7403	0.7069	1.0000
s <sub>2</sub>	0.4674	0.7884	0.7668	0.7403	0.8951	0.8063	0.4674	0.8063	0.7884
s <sub>3</sub>	1.0000	1.0000	0.5023	1.0000	1.0000	0.7069	1.0000	1.0000	0.7884
s <sub>4</sub>	0.1760	0.5684	0.6768	0.4674	0.7884	1.0000	0.4674	0.7069	0.8951

## 4.2. Comparative Analysis and Advantages of the Proposed Approach

This section compares some existing case-retrieval methods with the proposed approach.

### 4.2.1. Comparative analysis

To illustrate the characteristics and effectiveness of the proposed method, we compare it with some existing case-retrieval methods including (1) the traditional case-retrieval method [12], namely CBR-F; (2) the case-retrieval method without considering decision makers' psychological behavior, namely CBR-NPB; and (3) the case-retrieval method based on PT [8] without considering the evaluation information, namely CBR-PT; and (4) the case-retrieval method based on PT considering the evaluation information, namely CBR-PTT. The most suitable historical cases based on the four case-retrieval methods for the above case study are detailed in Table 6. Figure 3 shows the ranking of historical cases by case-retrieval methods without considering the evaluation of decision makers, i.e., the proposed method, CBR-F, and CBR-PT. Figure 4 shows the ranking of similar historical cases by case-retrieval methods, i.e., the proposed method, CBR-NPB and CBR-PTT.

Based on Table 6 and Figure 3, we analyzed the four case-retrieval methods as follows:

First, CBR-F obtains the most suitable historical case based on case similarities. The ranking of the case similarities is  $C_3 > C_{10} > C_5 > C_9 > C_2 > C_8 > C_7 > C_4 > C_6 > C_1$ , and the most suitable historical case was  $C_3$ . However, in reality, the most suitable historical case should be selected while considering the case similarity and the evaluation criteria for similar historical cases [8]. In addition, decision makers are not completely rational during the decision-making process [13]. Therefore, selecting the most suitable historical case considering the evaluation criteria and the decision makers' psychological behavior is more appropriate in real-world decision-making scenarios.

**Table 6** | Comparison with different approaches.

Approach	The Most Suitable Historical Case
CBR-F	$C_3$
CBR-NPB	$C_3$
CBR-PT	$C_{10}$
The proposed method	$C_9$

Next, CBR-NPB does not consider the decision makers' psychological behavior when calculating case similarities and utilities. A set of similar historical cases is  $\{C_3, C_5, C_9, C_{10}\}$ . We calculate the utility of the evaluation criteria according to [20]. Subsequently, we calculate the comprehensive utilities of similar historical cases using Eq. (31). The ranking obtained using the CBR-NPB method is  $C_3 > C_9 > C_5 > C_{10}$ . This is different from the results of the proposed method because CBR-NPB does not consider the psychological behavior of decision makers. However, in reality, decision-making is affected by personal preferences and psychological behavior. Therefore, it is more reasonable to include psychological behavior in the model.

Furthermore, we compare the proposed method with CBR-PT. We set the parameters in PT as  $\alpha = 0.89$ ,  $\beta = 0.92$ , and  $\lambda = 2.25$ . The reference point for all attribute distances is set at 0.5. The ranking of the case similarities is  $C_{10} > C_9 > C_8 > C_3 > C_6 > C_7 > C_5 > C_2 > C_4 > C_1$ . Therefore, the most similar historical case is  $C_{10}$ . This result is the same as that of the proposed method when calculating the case similarity. However, CBR-PT does not consider the evaluation criteria, and the final result is different. In the selection of the most suitable case, it cannot be determined by the similarity alone, and the historical case with a slightly lower similarity may be more consistent with the target case. Therefore, it is necessary to determine the appropriate historical case through the evaluation of the decision maker, and the results would be more accurate.

Finally, the ranking of the similar historical cases obtained using the method based on CBR-PTT is  $C_9 > C_{10} > C_3 > C_5$ .

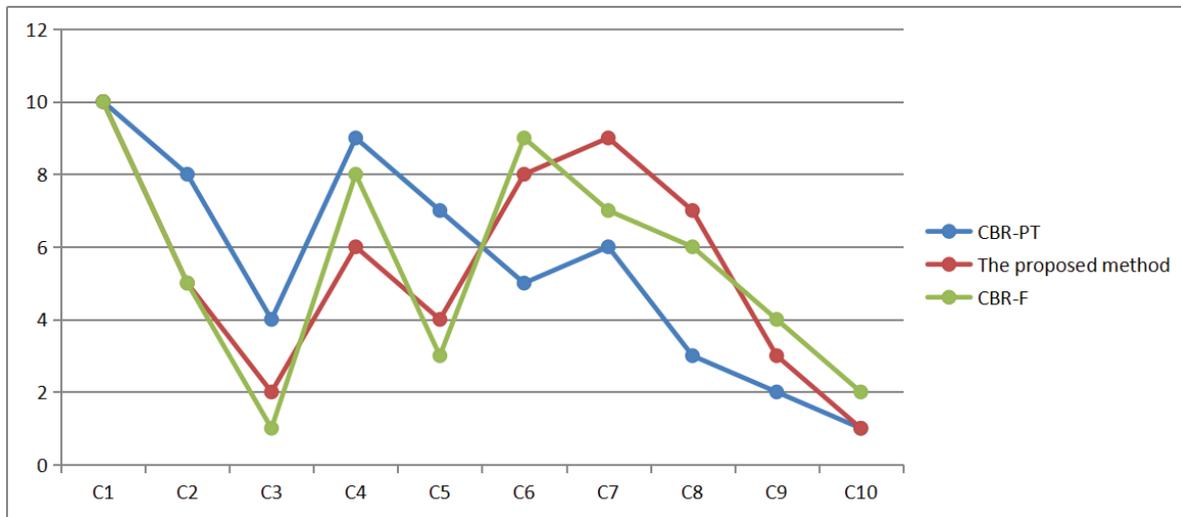


Figure 3 | The ranking of the historical cases.

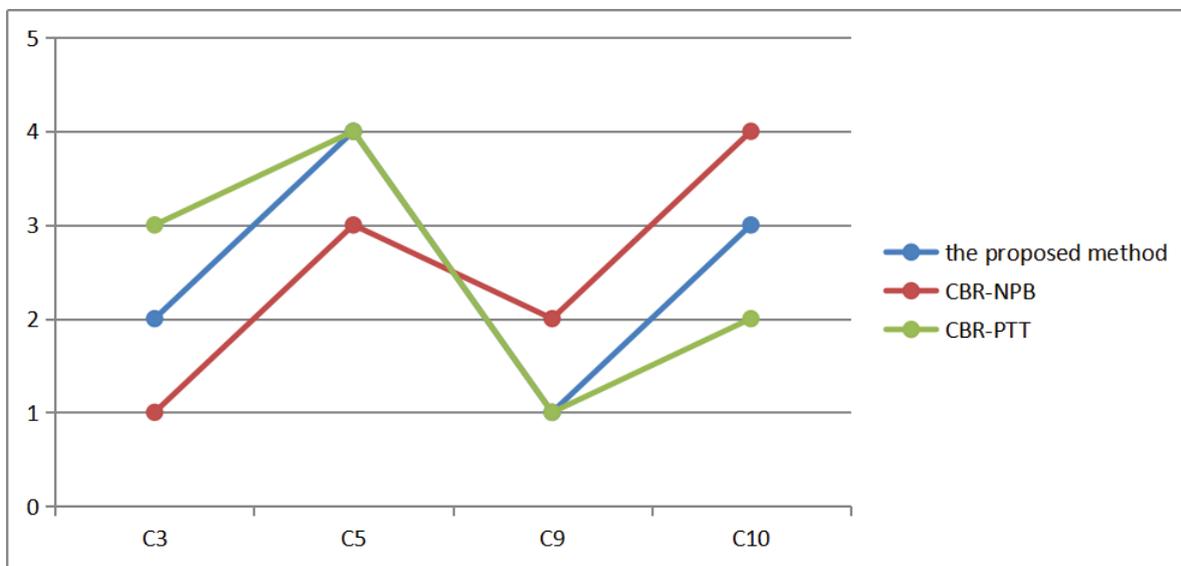


Figure 4 | The ranking of similar historical cases.

The most suitable historical case is  $C_9$ , which is the same as that obtained using the proposed method. However, first, PT requires the determination of the reference point. Furthermore, there are several parameters that need to be determined. However, RT determines fewer parameters. The determination of these parameters is generally done by decision makers. With the increase in the number of parameters, the subjective decision of decision maker will lead to great fluctuation of the results, so that the difference between the calculated results will be large, which will lead to the reduction of the accuracy of the results. Therefore, the case retrieval based on RT is more suitable for the method based on PT.

In addition, the weights of case attributes and evaluated attributes are determined by LINMAP, which sets up a mathematical programming model, considering the case information and incomplete multidimensional preference information to determine the weights. Compared with the methods that construct models to determine weights [13,20], this method considers not only objective

information, but also the subjective preferences of decision makers. Therefore, it is more suitable for the real-world decision-making process.

#### 4.2.2. Advantages of the proposed approach

In our comparison with the existing case-retrieval methods, based on case retrieval, we identified the following advantages of the proposed approach:

- (1) The existing case-retrieval methods seldom consider psychological behavior in the calculation of case similarity. Although PT-based case retrieval considers psychological behavior, many parameters should be set in the calculation process. Moreover, a change in the parameters changes the results. In contrast, the proposed method not only considers the psychological behavior, but also has fewer parameters and simpler calculations.

- (2) When selecting the most suitable historical case, the proposed method considers an evaluation of alternatives from historical cases similar to the target case.
- (3) We use the LINMAP method to determine the incomplete weights in case attributes and evaluated attributes. This approach can provide more objective and accurate results when calculating case similarities and comprehensive utilities.

## 5. CONCLUSION

In this paper, we propose an RT-based case-retrieval method based on case retrieval and RT, to retrieve the most suitable historical case in scenarios with multiple heterogeneous attributes and incomplete weight information. The proposed method has the following three characteristics: (1) Heterogeneous multi-attribute information is considered in uncertain cases. (2) Case similarities and comprehensive utilities are calculated considering the psychological behavior of decision makers, which is a more realistic scenario. (3) To improve the accuracy of the results, the LINMAP method is used to determine the weights of case attributes and evaluated attributes. Furthermore, LINMAP aggregates three types of information: case similarity or evaluated utility, incomplete weight information, and multidimensional preference information.

The proposed method helps decision makers to select the most suitable historical case. However, some limitations also exist. For example, the process of case retrieval only considers one state, but, in reality, there may be constant changes in emergencies. Therefore, future research can focus on a case-retrieval method for scenarios with a dynamic evolution of emergencies.

## CONFLICTS OF INTEREST

The authors declare no conflicts of interest regarding the publication for the paper.

## AUTHORS' CONTRIBUTIONS

Zhang Kai and Wang Ying-Ming proposed the methodology, Zhang Kai and Zheng Jing conducted the validation and formal analysis, Zhang Kai wrote the original draft preparation, Wang Ying-Ming reviewed the writing, Zhang Kai edited the writing. All authors read and approved the final manuscript.

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