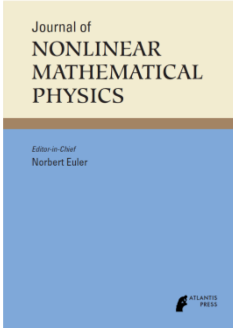

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Corrigendum: "Classification of 3D Consistent Quad-Equations"

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CORRIGENDUM
CLASSIFICATION OF 3D CONSISTENT QUAD-EQUATIONS

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Unfortunately, the list of type H^6 equations given in Sec. 2.2 on page 340 of the paper is incomplete: one quad-equation is missing and in another equation, there is one summand missing. The correct list is as follows:

Every quad-equation of type H^6 is equivalent modulo $(\text{Möb})^4$ to one of the following quad-equations characterized by the quadruples of discriminants:

- $(0,0,0,0)$:

$$Q = x_1 + x_2 + x_3 + x_4$$

- $\left(1,0,1,\delta^2\right)$:

$$Q = x_1 + x_3 + x_2\left(x_4 + \delta x_1\right)$$

- $(1,4x_2,1,1)$:

$$Q = x_2 + x_1x_3 + x_1x_4 + x_3x_4$$

- $\left(x_1^2, x_2^2 + 4\delta_1\delta_2\delta_3, x_3^2, x_4^2\right)$:

$$Q = x_1x_3 + x_2x_4 + \delta_1x_2x_3 + \delta_2x_3x_4 + \delta_3$$

As a consequence, in Theorems 3.9–3.11 several 3D consistent systems have to be replaced by new ones. In Theorem 3.9 on page 357 the systems (3.19)–(3.20) and

(3.21)–(3.22) can be unified into the following system:

- $\left(x^2+4\delta_1\delta_2\delta_3, x_1^2, x_{12}^2, x_2^2\right)$:

$$\begin{aligned} A\left(x, x_1, x_2, x_{12}; \delta_1, \delta_2\right) &= x x_{12}+x_1 x_2+\delta_1 x_1 x_{12}+\delta_2 x_2 x_{12}+\delta_3, \\ B\left(x, x_2, x_3, x_{23}; \delta_1, \delta_2, \alpha\right) &= \alpha\left(x x_2+x_3 x_{23}\right)-\left(x x_{23}+x_2 x_3\right) \\ &\quad+\left(\alpha^2-1\right)\left(\delta_2 x_2 x_{23}-\frac{\delta_1 \delta_3}{\alpha}\right), \end{aligned} \tag{3.19}$$

$$\begin{aligned} \bar{A} &= A\left(x_3, x_{13}, x_{23}, x_{123}; \delta_1, \delta_2\right), & \bar{B} &= B\left(x_1, x_{12}, x_{13}, x_{123}; 0, \delta_2, \alpha\right), \\ C &= B\left(x, x_1, x_3, x_{13}; \delta_2, \delta_1, \alpha^{-1}\right), & \bar{C} &= B\left(x_2, x_{12}, x_{23}, x_{123}; 0, \delta_1, \alpha^{-1}\right), \\ K &= A\left(x, x_{13}, x_{23}, x_{12}; \delta_1 \alpha^{-1}, \delta_2 \alpha\right), & \bar{K} &= A\left(x_3, x_1, x_2, x_{123}; \delta_1 \alpha^{-1}, \delta_2 \alpha\right), \end{aligned} \tag{3.20}$$

and there is one new system to be listed in Theorem 3.9:

- $(4x, 1, 1, 1)$:

$$\begin{aligned} B\left(x, x_2, x_3, x_{23}; \alpha\right) &= \left(x-x_3\right)\left(x_2-x_{23}\right)+\alpha\left(x+x_3-2 x_2 x_{23}\right) \\ &\quad-\alpha^2\left(x_2+x_{23}\right)-\alpha^3, \\ \bar{B}\left(x_1, x_{12}, x_{13}, x_{123}; \alpha\right) &= \left(x_1-x_{13}\right)\left(x_{12}-x_{123}\right)-\alpha\left(x_1+x_{12}+x_{13}+x_{123}\right)+\alpha^2, \\ K\left(x, x_{13}, x_{23}, x_{12}; \alpha\right) &= x+x_{13} x_{23}+x_{13} x_{12}+x_{23} x_{12}+\alpha\left(x_{13}-x_{23}\right)-\alpha^2, \end{aligned} \tag{3.21}$$

$$\begin{aligned} A &= K\left(x, x_1, x_2, x_{12}; 0\right), & \bar{A} &= K\left(x_3, x_{13}, x_{23}, x_{123}; 0\right), \\ C &= B\left(x, x_1, x_3, x_{13}; -\alpha\right), & \bar{C} &= \bar{B}\left(x_2, x_{12}, x_{23}, x_{123}; -\alpha\right), \\ \bar{K} &= K\left(x_3, x_1, x_2, x_{123}; \alpha\right). \end{aligned} \tag{3.22}$$

Similarly, in Theorem 3.10 on page 359 the systems (3.29)–(3.30), (3.31)–(3.32) and (3.33)–(3.34) can be unified into the following system:

- $\left(x^2+4\delta_1\delta_2\delta_3, x_1^2, x_2^2, x_{12}^2\right)$:

$$\begin{aligned} A\left(x, x_1, x_2, x_{12}; \delta_1, \delta_2\right) &= x x_1+x_2 x_{12}+\delta_1 x_1 x_{12}+\delta_2 x_1 x_2+\delta_3, \\ B\left(x, x_2, x_3, x_{23}; \delta_1, \delta_2, \alpha\right) &= \alpha\left(x x_2+x_3 x_{23}\right)-\left(x x_3+x_2 x_{23}\right) \\ &\quad+\left(\alpha^2-1\right)\left(\delta_2 x_2 x_3-\frac{\delta_1 \delta_3}{\alpha}\right), \end{aligned} \tag{3.29}$$

$$\begin{aligned} \bar{A} &= A(x_{23}, x_{123}, x_3, x_{13}; \delta_1, \delta_2), & \bar{B} &= B(x_{12}, x_1, x_{123}, x_{13}; 0, \delta_2, \alpha), \\ C &= A(x, x_1, x_3, x_{13}; \delta_1 \alpha^{-1}, \delta_2 \alpha), & \bar{C} &= A(x_{23}, x_{123}, x_2, x_{12}; \delta_1 \alpha^{-1}, \delta_2 \alpha), \\ K &= B(x, x_{12}, x_{13}, x_{23}; \delta_2, \delta_1, \alpha^{-1}), & \bar{K} &= B(x_2, x_1, x_{123}, x_3; 0, \delta_1, \alpha^{-1}), \end{aligned} \tag{3.30}$$

and there is one new system to be listed here:

- $(4x, 1, 1, 1)$:

$$\begin{aligned} B(x, x_2, x_3, x_{23}; \alpha) &= (x - x_{23})(x_2 - x_3) + \alpha(x + x_{23} - 2x_2x_3) \\ &\quad - \alpha^2(x_2 + x_3) - \alpha^3, \\ \bar{B}(x_1, x_{12}, x_{13}, x_{123}; \alpha) &= (x_1 - x_{123})(x_{12} - x_{13}) \\ &\quad - \alpha(x_1 + x_{12} + x_{13} + x_{123}) + \alpha^2, \\ C(x, x_1, x_3, x_{13}; \alpha) &= x + x_1x_3 + x_1x_{13} + x_3x_{13} - \alpha(x_3 - x_{13}) - \alpha^2 \end{aligned} \tag{3.31}$$

$$\begin{aligned} &A = C(x, x_1, x_2, x_{12}; 0), && \bar{A} = C(x_{23}, x_{123}, x_3, x_{13}; 0), \\ &\bar{C} = C(x_{23}, x_{123}, x_2, x_{12}; \alpha), && K = B(x, x_{12}, x_{13}, x_{23}; -\alpha), \\ &\bar{K} = \bar{B}(x_1, x_2, x_3, x_{123}; -\alpha). \end{aligned} \tag{3.32}$$

Finally, in Theorem 3.11 on page 360 the systems (3.41)–(3.42) and (3.43)–(3.44) can be unified into the following system:

- $(x^2 + 4\delta_1\delta_2\delta_3\delta_4, x_1^2, x_2^2, x_{12}^2)$:

$$\begin{aligned} A &= xx_{12} + x_1x_2 + \delta_3\delta_4x_1x_{12} + \delta_2x_2x_{12} + \delta_1, \\ \bar{A} &= x_3x_{123} - x_{13}x_{23} + \delta_2x_{23}x_{123} + \delta_1, \\ B &= xx_{23} + x_2x_3 + \delta_2x_2x_{23} + \delta_4x_3x_{23} + \delta_1\delta_3, \\ \bar{B} &= x_1x_{123} - x_{12}x_{13} + \delta_4x_1x_{12} + \delta_2x_{12}x_{123}, \\ C &= xx_1 - x_3x_{13} + \delta_4x_1x_3 + \delta_3x_1x_{13} - \delta_1\delta_2, \\ \bar{C} &= x_2x_{12} + x_{23}x_{123} + \delta_4x_{12}x_{23} - \delta_3x_{12}x_{123}, \\ K &= xx_{12} - x_{13}x_{23} + \delta_3x_{12}x_{13} - \delta_2\delta_4x_{12}x_{23} + \delta_1, \\ \bar{K} &= x_1x_2 + x_3x_{123} - \delta_3x_1x_{123} + \delta_1. \end{aligned} \tag{3.41}$$

The complete proofs of these theorems are now published in my PhD thesis "Classification and Lagrangian structure of 3D consistent quad-equations" (see http://opus.kobv.de/tuberlin/volltexte/2012/3628/).