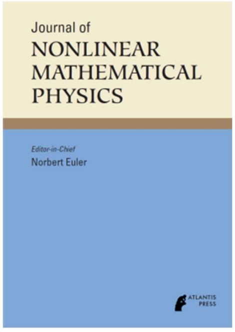

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On the Existence of Benthic Storms

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We study a model for the wind-induced current field of the Pacific ocean in order to demonstrate that currents in the surface layer are carried down to the deepest regions above the abyssal sea floor, which indicates the existence of the phenomenon of comparably strong currents in bottom regions as a result of wind-stress forces at the surface, also known as benthic storms.

Keywords: equatorial flows; eddy viscosity; stratification.

2000 Mathematics Subject Classification: 86A05

1. Introduction

A longtime assumption in oceanography was that the water in the benthic boundary layer (BBL) above the abyssal sea floor, which develops as a result of friction at the bed, would be rather still. Former theory of the BBL with focus on mean-flow properties relied on models assuming horizontally homogeneous stationary currents (see [3, 8]). The consideration of such models stemmed from the insufficient availability of empirical data for the hydrodynamics (but also the chemical and biological processes, etc. and their interactions) in the BBL, due to its inaccessibility in the open ocean. The perception changed fundamentally as a consequence of observations that have been made during long term projects such as the High Energy Benthic Boundary Layer Experiment (HEBBLE Program), which was primarily initiated for practical reasons such as the extraction of minerals from the see floor, anti submarine defense, navigation, the selection of waste burial sites, etc. It turned out that the BBL is not static at all. There are dramatic velocity increases in the benthic current, which occur periodically, the so-called benthic storms. Their importance springs from the fact that they stir up bottom sediments, which are then captured and transported over large distances by weaker but stable currents. Unlike the stationary weak currents, benthic storms seem to be controlled not only by forces resulting e.g. from thermohaline and tidal phenomena (also the Coriolis force plays a role in equatorial regions), but results from wind-stress at the surface layer, which is carried down to the BBL via mesoscale eddies having diameters up to 200km. This wind-driven mechanism as origin of benthic storms particularly applies to equatorial regions in the Pacific (see [9]); there are regions where wind plays no or merely a minor role in the generation of benthic storms (see [11]).

The purpose of these notes is to prove - on the basis of a simple (static) model - that wind-stress forces at the surface indeed propagate down to the sea floor and thus may generate benthic storms. Let us point out, that such a basic model is not capable to describe the complex dynamics of the currents in the BBL. However it tells us that currents at the surface layer influence the flow in deep regions directly above the bottom.

2. A note on currents in the equatorial region

Our considerations rely on a linear eddy viscosity model in [4] for the wind-induced current field of the Pacific ocean in the equatorial region, where stratification is greater than anywhere else in the ocean (see [6]): a sharp thermocline separates a shallow water layer of warm water from a deep layer of colder water with higher density. The fluid domain, sketched in figure 1, consists of the subsurface layer

$$U_1 := \{(x,z) \in \mathbb{R}^2 :\ -h < z < 0\}$$

with constant water density ρ between the water surface $\{(x,z) \in \mathbb{R}^2 : z = 0\}$ and the thermocline $\{(x,z) \in \mathbb{R}^2 : z = -h\}$ (with $h > 0$), and a deep layer

$$U_2 := \{(x,z) \in \mathbb{R}^2 :\ -d < z < -h\}$$

beneath the thermocline and above the flat bottom $\{(x,z) \in \mathbb{R}^2 : z = -d\}$ (with $d > h$) having a slightly higher density $\rho(1+r)$ for some fixed $r > 0$.

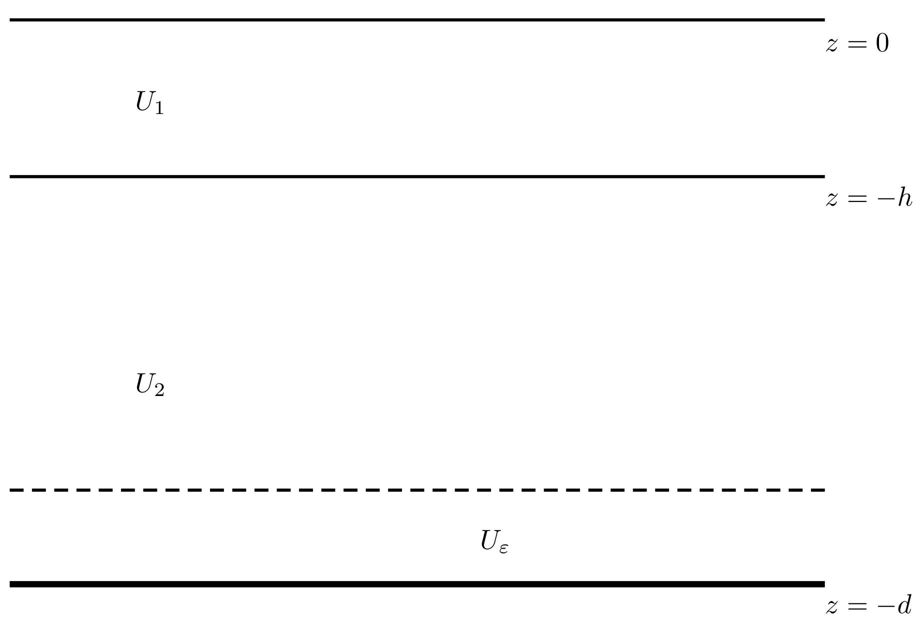

Fig. 1. The two layers are separated by the thermocline at a depth of roughly 120m. Most parts of the sea floor in the Pacific ocean are indeed flat, located at depths exceeding 3500m; so-called abyssal plains.

We consider the linearized equations for a steady state flow with a vanishing vertical-fluid-velocity component in the f-plane approximation for both frictional layers. That is, presume

$$0 = -\frac{1}{\rho} P_x + (\nu u_z)_z, \tag{2.1}$$

$$-2\Omega u = -\frac{1}{\rho} P_z - g, \tag{2.2}$$

$$u_x = 0 \tag{2.3}$$

throughout U_1, and accordingly we require

$$0 = -\frac{1}{\rho(1+r)} P_x + (\nu u_z)_z, \tag{2.4}$$

$$-2\Omega u = -\frac{1}{\rho(1+r)} P_z - g, \tag{2.5}$$

$$u_x = 0 \tag{2.6}$$

to hold within the deep layer U_2. The two unknowns - the velocity field u and the pressure P - are assumed to be smooth within $U_1 \cup U_2$. The depth dependent viscosity parameter $\nu = \nu(z)$ is given. It is smooth throughout

$$U := U_1 \cup U_2 \cup \{z = -h\},$$

positive and away from zero; in [5] it is suggested to take a suitable exponential decay to some small positive value as a good approximation for ν. The constants g and Ω denote the gravitational constant and the rotational speed of the earth around the polar axis toward the east.

In addition to equations (2.1)-(2.6) we consider the following boundary conditions. We impose a no-slip condition on the bottom, a vanishing vorticity of the velocity field on the thermocline and a constant atmospheric pressure on the surface. Furthermore we assume the velocity and the pressure to be continuous across the thermocline:

$$\begin{cases} u = 0 & \text{on} \quad \text{z=-d}, & (2.7) \\ u_z = 0 & \text{on} \quad \text{z=-h}, & (2.8) \\ P = P_{\text{atm}} & \text{on} \quad \text{z=0}, & (2.9) \\ u, P \in \mathscr{C}(\overline{U}). & & (2.10) \end{cases}$$

The proposition below tells us that an absence of currents in regions above the sea floor would imply zero current up to the water surface. Let us denote such a bottom region by

$$U_\varepsilon := \{(x,z) \in \mathbb{R} \colon -d \le z < -d + \varepsilon\}; \tag{2.11}$$

c.f. figure 1.

Proposition 2.1. *Assume that there is some region U_ε above the bottom where the velocity field u vanishes. Then u is identically zero from the bottom to the surface.*

Proof. We have that $(\nu u_z)_{zz} = 0$ in $U_1 \cup U_2$. In order to see this, we differentiate (2.2) with respect to x first, and use (2.3) to infer that $P_{zx} = 0$ in U_1. The claim follows by differentiating (2.1) with

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respect to z and exploiting the smoothness of P in U_1:

$$0 = P_{zx} = P_{xz} = (\nu u_z)_{zz} \quad \text{in } U_1.$$

Analogously we get that $(\nu u_z)_{zz} = 0$ in U_2. Therefore, $\nu u_z = A_2 z + B$ for $A, B \in \mathbb{R}$, and by (2.8) we find that

$$\nu u_z = A_2(z+h) \quad \text{in } U_2.$$

Since $u \equiv 0$ in U_ε, we find that $A_2 = 0$, hence $u_z = 0$ in U_2 and from (2.7) and (2.10) we infer that

$$u \equiv 0 \quad \text{in } \overline{U_2}. \tag{2.12}$$

We may now infer from (2.4) and (2.5) that $P_x \equiv 0$ and $P_z = -g\rho(1+r)$, thus

$$P(x,z) = -\rho(1+r)gz + C_2$$

in U_2 for some constant of integration $C_2 \in \mathbb{R}$.

Similarly we find that

$$\nu u_z = A_1(z+h) \quad \text{in } U_1 \tag{2.13}$$

for some $A_1 \in \mathbb{R}$. Therefore (2.9), (2.10) and (2.12) imply that

$$u(z) = \int_{-h}^{z} \frac{A_1(\xi+h)}{\nu(\xi)} \,\mathrm{d}\xi$$

for $-h \leq z \leq 0$.

We obtain from (2.1) and (2.13) that $P_x = \rho A_1$ in U_1 and from (2.2) that $P_z = \rho(2\Omega u - g)$ in U_1, thus

$$P(x,z) = -\rho g z + 2\rho\Omega \int_{-h}^{z} u(\xi)\,\mathrm{d}\xi + \rho A_1 x + C_1$$

in U_1 for some constant of integration $C_1 \in \mathbb{R}$. Due to (2.10), the following relation has to be fulfilled for all $x \in \mathbb{R}$:

$$\lim_{z \nearrow -h} P(x,z) = -\rho(1+r)gz + C_2 = -\rho g z + \rho A_1 x + C_1 = \lim_{z \searrow -h} P(x,z).$$

This is only possible, if $A_1 = 0$ is satisfied. We deduce from (2.9) that $C_2 = C_1 - \rho r g h$, and $C_1 = P_{\mathrm{atm}}$. Hence the pressure field equals the hydrostatic pressure:

$$P(x,z) = \begin{cases} P_{\mathrm{atm}} - \rho g z & \text{in } U_1 \cup \{z=0\} \\ P_{\mathrm{atm}} - \rho(1+r)gz - \rho r g h & \text{in } U_2 \cup \{z=-h\} \cup \{z=-d\}. \end{cases}$$

Moreover, knowing that $A_1 = 0$, we obtain from (2.13) that $u_z = 0$ throughout U_1, hence u is constant within U_2. Therefore by (2.12) and the continuity assumption (2.10) we conclude that $u \equiv 0$ throughout $\overline{U}$. □

Remark 2.1. Let us point out, that the region U_ε does not particularly represent the BBL mentioned earlier in the introduction. It stands for some - theoretically arbitrary small - region directly above the bottom.

The continuity assumption (2.10) can not be meaningfully strengthened in the sense of requiring u and P to be continuously differentiable or even smooth across the thermocline. We have seen in the proof of Proposition 2.1 that the hydrostatic pressure is only piecewise smooth; its z-derivative has a jump at the thermocline. Furthermore, requiring a smooth current u throughout U, would trivialize the model. We would then already obtain that $(\nu u_z)_z \equiv 0$ in U without imposing the additional condition of Proposition 2.1 (on some region U_ε). Hence we would get $u_z \equiv 0$ in U by (2.8) and therefore the bottom condition (2.7) tells us that the only smooth solution of (2.1)-(2.3) and (2.4)-(2.6) with boundary conditions (2.7)-(2.9) is the trivial one.

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