

Improving the Student's Numerical Reasoning by Using Mathematics Cognition-Based Mathematical Textbook Development at Elementary Schools

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ABSTRACT

The results of a preliminary study in 2019 showed that there were still many high-grade elementary school (S.D.) students who did not master arithmetic operations. In general, the elementary school students studied also did not have good numerical reasoning skills, even though they had studied mathematics for almost six years. One reason was that teachers in elementary schools (SD) were more likely to teach how to do mathematics, not how to think mathematically. This problem needed to be resolved from an early age so that students' perceptions did not form that learning mathematics was about remembering ways of doing something (how to add, multiply, divide, calculate area, volume, etc.). Therefore, through this research, a mathematical cognition based mathematics textbook was developed for low-grade elementary school students. The contents of the textbook were focused on developing students' numerical reasoning, especially those related to number sense, number relations, and number construction. The research method used was design research that consisted of three phases, namely the preliminary stage, the prototyping phase, and the assessment phase. Data collection methods used were observation, interviews, checklists, videotaping, analysis of student work results, and numerical reasoning tests for students. The collected data were analyzed by using descriptive statistics and inferential statistics. The result of the research was that mathematical cognition-based mathematics textbooks met the valid and practical criteria.

Keywords: *Mathematics Textbook, Mathematical Cognition, Numerical Reasoning.*

1. INTRODUCTION

1.1. Background

The main skills that students need to survive in the face of the challenges of the 21st century and the 4.0 industrial revolution are high-order thinking skills, one of which is the ability to reason. according to [1] reasoning abilities are the basis for building mathematical knowledge. reasoning and how the brain works are very closely related [2]. according to [3], people who are rational and think analytically will tend to recognize patterns, structures, or regularities both in the real world and in symbols. mathematics education in schools is intended so that students have good reasoning power, especially when solving problems in mathematics subjects. [4,5] found that one tendency causes students to fail to properly master the subjects in mathematics, namely students did not understand and

used good reasoning in solving the problems given. likewise with the opinion [6] which states that the lowest average percentage achieved by Indonesian students is in the cognitive domain at the reasoning level, namely 17%. the results of the TIMSS study showed that Indonesian students were still weak in the aspects of reasoning and problem solving [7, 8, 10]. the same condition was also found in the results of the PISA study which showed that Indonesia's ranking is always in the lowest 10%, and almost no Indonesian students reach the two highest levels (levels 5 and 6) [11,12,13].

The main cause of the emergence of the problem as described above is that reasoning ability has not been a concern of teachers in teaching mathematics in schools. [9] revealed that in learning in schools teachers tend to use questions in supporting books. The questions in the book were dominated by indicators of remembering and

understanding, and at least with indicators for analyzing, evaluating, and creating [13, 14].

To be able to develop students' reasoning, teachers need to have the competence to solve and design reasoning problems. Besides, developing reasoning requires the ability of teachers to plan and manage effective learning in teaching students to think logically [15, 16]. Teachers also need to improve the ability to create learning that encourages the development of student reasoning [17].

From the results of preliminary studies conducted in several elementary schools in West Sumatra, it is known that teachers have not been optimal in implementing mathematics learning which can stimulate student reasoning, especially numerical reasoning [42]. Through this research, a mathematics textbook based on mathematical cognition will be designed to improve numerical reasoning of elementary school students in lower grades. This textbook consists of Teacher's Book and Student Book, as well as learning media that supports mathematical activities in textbooks. The contents of the textbook will be focused on developing students' numerical reasoning, especially those related to number sense, number relations, and number construction. The formulation of the problem in this study is: what are the characteristics of a mathematical textbook based on mathematical cognition that is valid, practical, and effective to improve elementary students' numerical reasoning?

1.2. Special Purpose

This research aims to produce mathematical textbooks (Teacher's Book and Student Book) based on mathematical cognition that are valid and practical to improve the numerical reasoning abilities of elementary school students in low grades.

1.3. Research Urgency

Students' numerical reasoning needs to be built from an early age, because mastery of numbers and reasoning about numbers is the basis for mastery of almost all mathematical concepts. With the birth of this mathematical cognition-based mathematics textbook, it is hoped that it can change the paradigm of teacher teaching and student learning; that teaching mathematics is not teaching how to do mathematics, but teaching how to make students think mathematically.

2. METHOD

This type of research is design research. Design Research is carried out to develop and produce a product as a solution to problems related to education. This research activity was carried out with the Plomp model design [57]. This research design consisted of

three phases, namely: (a) a preliminary research phase; (b) the prototype development phase (development / prototyping phase); and (c) the assessment phase.

3. RESULT AND DISCUSSION

The results of the analysis of the problem of learning to count in elementary school showed that in general teachers still have the principle that teaching arithmetic is teaching how to perform mechanistic arithmetic operations. When the teacher is asked to do the following questions:

Calculate $127 + 398$ without going down the order. All teachers (60 people) give answers by mentioning the units, tens, and hundreds of both numbers. This means that they still use the downward tiered method, as seen from the answers of the following three teachers.

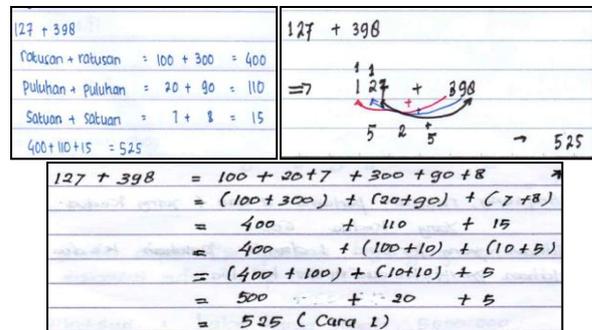


Figure 1 Answers of some teachers The three answers above basically still use the following methods

$$\begin{array}{r} 127 \\ 398 + \\ \hline 525 \end{array}$$

None of the teachers looked at each number they added up (for example thinking about: how big is the number, which number is close to?), So they can answer one of the following answers.

- a. $127 + 398 = 127 + 400 - 2 = 527 - 2 = 525$
- b. $127 + 398 = 130 + 400 - 3 - 2 = 530 - 5 = 525$
- c. $127 + 398 = 120 + 7 + 400 - 2 = 520 + 7 - 2$
- d. $= 520 + 5 = 525$
- e. $127 + 398 = 120 + 7 + 390 + 8 = 120 + 390 + 7 + 8 = 510 + 15 = 525$

Furthermore, the teachers were given the following questions.

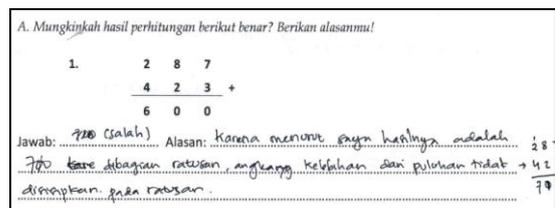


Figure 2 The Teacher's Answer to Question 1

As expected, all teachers have the perception that teaching numeracy is teaching a series of procedures or ways of working on problems. This is illustrated by the answers given by the teacher. The teacher's answer above shows that he is more interested in giving "only" judging whether the answer written on the question is "wrong" or "correct" (even though the questions do not want to be). What is wanted is that the teacher gives an assessment, is it possible that the sum of 287 and 423 is 600? If not, what is the reason? If possible, what is the reason?

What is interesting to say, all the teachers gave similar reasons for answering this question. All of them said that there was an error in the procedure to solve the problem, as seen in the reasons given below.

Jawab: Salah Alasan: 7+3 hasilnya 10, akan di simpan pada angka 8 tetapi pada menjumlahkan 8+2 = tidak dihitung angka yang di simpan maka hasilnya akan salah.

Figure 3 Another Teacher's Answer to Question Number 1

In other words, all the teachers saw the above questions as "only" as a procedure of addition, and there were mistakes in the procedure due to negligence. Neither teacher tries to look carefully at the two numbers added up, how big each number is, where it is in the number order, or the universe of numbers (number space). If the teachers want to pay attention that a first number is a number greater than 200 and a second number is a number greater than 400, then it is impossible for the sum of the two numbers to be only 600 (it should be more than 600).

The same thing was found when the teachers worked on question number 2, as can be seen from one of the following teacher answers.

2.
$$\begin{array}{r} 905 \\ - 286 \\ \hline 729 \end{array}$$

 Jawab: Salah Alasan: karena angka yang telah di pinjam tidak ditukarkan maka hasilnya akan salah

2.
$$\begin{array}{r} 905 \\ - 286 \\ \hline 729 \end{array}$$

 Jawab: Salah Alasan: karena seharusnya hasil dari pengurangan adalah 619 namun pada hasil pengurangan puluhan salah dan pada ratusan juga salah

Figure 4 The Teacher's Answer to Question Number 2

The teachers only focus on the subtraction procedure for two numbers and are not interested in observing the two numbers that are operated. If teachers have number sense, they will see that the first number is close to 900 and the second number is close to 300, so it is impossible to subtract more than 700.

From the results of curriculum analysis and concept analysis presented in mathematics textbooks, it was found that the presentation of number concepts and number operations in grade 1 elementary school focused more on remembering the basic facts of numbers and number operations, while activities to stimulate students' numerical reasoning were minimal. Therefore, in the designed textbook:

- a. The introduction of number symbols is carried out after the students
 - 1) Has a number sense of quantum 0 – 9
 - 2) Activities about number relations for quantities 0 - 9
- b. The initial numbers introduced were 0 - 9 (not 1 - 10)

To develop students' numerical reasoning the following teaching aids were used by Inge Schwank.

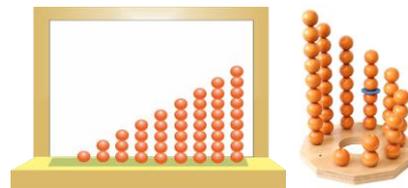


Figure 5 Props Created by Inge Schwank

Both of these tools (or presenting arithmetic problems using representations of these two tools) are believed to be able to stimulate students' numerical reasoning. For example, using circular media, students are asked.

- a. What attracted students' attention when they saw the picture above?
- b. When students have answered about the blue divider) Where else can the blue border be placed? Why?
- c. (When a student has answered on a pole whose balls are 2, 4, and 8 and their reasons (the blue divider divides the many balls above and below it equally), the question can be continued: Where else?
- d. This series of activities will lead students to say that the blue divider can also be placed at 0 (which will indirectly lead students to the concept: 0 is an even number or a number that is divisible by 2)

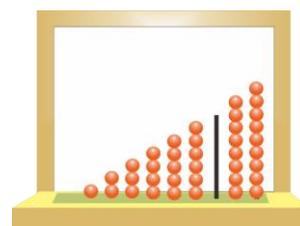
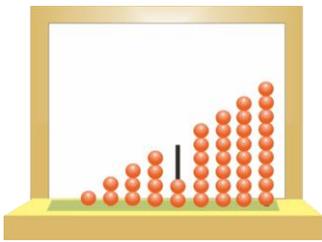


Figure 6 Problems Used in the Developed Book

For the picture above, the students are asked how many balls should be placed on the empty pole? Why?. This activity encourages students to come up with answers with different reasoning, for example:

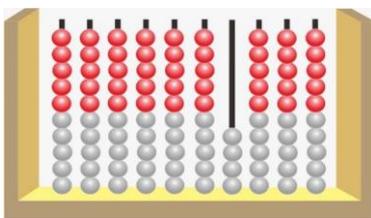
- a. the number of balls is 7 because the post is after the pole with the 6 balls;
- b. the number of balls is 7 because the post to the right contains 8 balls;
- c. after counting from 1, then the empty pole is the 7th pole, so the number of balls is 7;
- d. and others.

From the possible answers, it appears that students will use reasoning related to number relations (relationships between numbers). Meanwhile, with activities as shown in the following two figures, students will be facilitated to use reasoning related to number construction.



How many balls do you need to place on the black pole above? Why?

(a)



How many balls do you need to place on the 7th pole in the picture above? What color balls? Why?

(b)

Figure 7 The Problems Used in the Developed Book

4. CONCLUSION

Based on the data collection and analysis results with descriptive statistics and inferential statistics, the research results were obtained in the form of a mathematical cognition-based mathematical textbook that met the valid and practical criteria.

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