

# Students' Reasoning With Logical Mathematical and Visual Spatial Intelligence in Geometry Problem Solving

Jafar A. Aziz<sup>1,\*</sup> Dwi Juniati<sup>1</sup> Pradnyo Wijayanti<sup>1</sup>

<sup>1</sup> Mathematics Education Postgraduate Program, Universitas Negeri Surabaya, Surabaya, Indonesia

\*Corresponding author. Email: jafar.18002@mhs.unesa.ac.id

## ABSTRACT

Reasoning in solving geometry problem used by students to recognize shapes, identify properties and analyze relationships between geometric objects. This study aimed at investigating the students' reasoning in geometric problem solving with predominance of different intelligence types. This research was qualitative. The research subjects were two male 9th grade students with equal mathematical abilities. First student dominant in logical mathematical intelligence (LM), other's in visual spatial intelligence (VS). The instrument was test to determine the dominance of students' intelligence type and geometry test. Geometry tests used to find out students' understanding and reasoning in solving problem about cuboid surface area. Interview used to find out more about the student's process completed the geometry test. Students geometry reasoning was explained based on Van Hiele's geometric thought model. The results, both students able to identify the shapes, and analyze the characteristics of objects as well. VS able to represent problem into image and visualize information to find the correct procedures. Both are able to providing good explanations about the relationship between geometry constructs, LM uses mathematical model, and VS uses a visual approach. In performing problem solving, LM is more thoughtful using mathematical procedures and calculations while VS had visual analytic abilities.

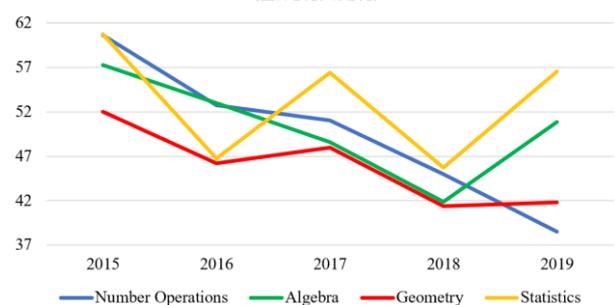
**Keywords:** Geometry, Intelligence, Logical Mathematical, Problem Solving, Reasoning, Visual Spatial

## 1. INTRODUCTION

Geometry is one of the main subjects in mathematics education, it's the oldest branch of mathematics. Geometry is taught in almost every education level. Even in preschool, children are begin introduced about figures and shapes. Geometry has many implementation in daily life. Mastery of geometry is an important competency for students, Cesaria & Herman have argued that learning geometry helps students understand about shapes and their relationships [1]. Geometry learning contributes to develop logical thinking skills, spatial intuition, and basic of learning other math topics.

As a fundamental subject of mathematics, geometry is one of the compulsory materials that tested in the national exam annually. If reviewing the mean score of junior high school students' national exams, geometry becomes one of the mathematics subjects with lowest score in a few years [2].

Graph of Junior High School Mathematics National Exam Mean Score since 2015 to 2019



**Figure 1** Mean Score Trends of Mathematics National Exam

One of the reasons is the students' lack of reasoning skills. Gunhan explained that in the problem-solving process, students have to be able to connect known concepts, learning experiences, and new concepts, then deciding the appropriate strategy to be applied. It much depends on reasoning abilities [3]. Therefore, reasoning has a positive correlation with problem solving ability.

Problem solving in mathematics not only a memorized procedure steps but also a systematic reasoning process [4]. Reasoning can be defined as a thought process to get logical conclusions based on arguments and relevant facts. Reasoning ability is critical to understand mathematics and make it meaningful [5]. Battista has argued that reasoning is the process of manipulating and analyzing objects, representations, diagrams, symbols, or statements to draw conclusions based on evidences or assumptions [6]. Further reasoning is treated as logical knowledge by Sumpter [7]. NCTM identifies several indicators of reasoning ability include; 1) observe patterns or regularity, 2) formulate generalization and conjecture that associated to regularity observed, 3) assess or test conjecture, 4) construct and examine mathematical argument, 5) describe/validate logical conclusion about some ideas and their relatedness [8].

Reasoning in geometry problem solving involves the process of identify and choose geometric properties to infer other properties [9]. Van Hiele explained that aspects of geometry reasoning include identify constructs based on its entirety appearance form, identifying construct's name based on the properties embedded or related to it, determine properties of geometric shapes through direct observation, describe a construct based on its characteristic, conduct problem solving that engages recognizing construct characteristic, and explain the relationship between constructs [10].

Reasoning ability in solving geometry problems, certainly different according to the each student's intelligence. One of the abilities that plays a role in geometry problem solving is the spatial ability to analyze geometric objects [11]. Reasoning process in solving geometry problems also requires the ability to logically analyze problem situations, identify correct solution and use right mathematical procedures [12]. So that logical mathematical intelligence also plays a role in geometrical reasoning process.

Problem solving ability is positively correlated to intelligence, especially problem solving that requires higher order thinking skills [13]. Therefore, every student has various characteristics in problem solving depend to their intelligence. Several intelligence types such as logical mathematic and visual spatial intelligence is positively correlated to mathematics problem solving ability [14]. Asyrofi & Junaedi also argued that ability to solve mathematical problems is greatly affected by logical mathematical and visual spatial intelligence [15].

Logical mathematical intelligence is an intelligence that involves words and numbers processing skills, using logic to analyze the problems logically, finding formulas and carry

out scientific investigations. Students with this intelligence type are able to analyze problems and formulate solutions in a logical order. Logical mathematical intelligence can be related to numeracy and analytical skills as well as logical and scientific thinking [16].

Visual spatial intelligence concerned with the ability to perceive visual and spatial world. Students who are dominant in this intelligence type, able to use image of object or pattern through mental process and use it for finding the solutions of problems related to geometry [17]. This intelligent type includes the ability to imagine, illustrate, and describe the visual processes that occur in the mind [18]. Based on that, the purpose of this study is to describe students' reasoning with logical mathematic and visual spatial intelligence in geometry problem solving.

## 2. METHOD

This research was descriptive with qualitative approach. The research subjects were ninth grade students of MTs Bilingual NU Pucang, Sidoarjo, Indonesia. They were two male students with equivalent math abilities. The first student is dominant in the logical mathematical (LM) and the other in visual spatial intelligence type (VS). Students' mathematics ability was known from mathematics score during last semester. While, intelligence type domination was obtained through the result of numerical and spatial aptitude tests adapted from Philip Carter [19].

The instruments used in this research were a test of numerical and spatial aptitude, and geometric tests. Test of numerical and spatial aptitude consist of 20 items. It used to determine the dominance of intelligence type, logical mathematical or visual spatial. The geometry test designed to identify student' reasoning and problem-solving process about cuboid surface area. The topic considered able to identify geometric reasoning and visual spatial abilities, and it's often used in topic of mathematics national exam. Interviews used to explore information about the reasoning process in geometry problem solving. The interview that used was an unstructured, which the researcher didn't use interview guidelines that are structured systematically and completely for data collection.

Data analyzed through three stages, namely data reduction, data presentation, and draw conclusions. Reasoning in solving geometry problem was described based on NCTM reasoning indicators. Geometric reasoning aspects that assessed in this study refers to Van Hiele. These indicators and aspect can be seen in table 1 [10].

**Table 1.** Geometry Reasoning Aspect and Indicator

No	Reasoning Indicators	No	Geometry Reasoning Aspect
1.	Observing patterns and regularity.	1.	Identify constructs based on its entirety appearance form
		2.	Naming construct based on its properties
2.	Formulate generalization and conjecture related to regularity that observed	1.	Determine the geometric characteristics through on direct observation
		2.	Describe the construct based on its characteristics
3.	Asses/ examine the conjecture.		Perform problem solving that engages recognizing construct characteristics.
4.	Construct and assess mathematical arguments.		
5.	Describe/ validate logical conclusion about several ideas and their interrelations		Provide explanation about constructs relationship.

### 3. RESULT AND DISCUSSION

This research was conducted on students of MTs Bilingual Muslimat NU Pucang by taking population of 34 students from ninth grade students in the same class. After being given numerical and spatial aptitude test, and reviewing students' mathematics ability from score report of last semester, two subjects were taken. They're two male students with equivalent mathematics ability assumption. First subject dominant in the logical mathematical, while other in the visual spatial intelligence type.

The given geometry problem is as follows:

The wall of a bathroom will be covered by tiles. The dimension of the bathroom's floor is  $300 \times 150 \text{ cm}^2$  with 210 cm height. The size of a tile is  $30 \times 30 \text{ cm}^2$ . The cost of retail a tile is 4.000 rupiahs in shop A. Store B sells the tiles at wholesale prices if you buy a box of tiles containing 8 tiles for 31.200 rupiah per box. How is your strategy to buy tiles so that the cost of buying bathroom tiles is as economical as possible? Give your reason and calculation!

The results of geometric problem solving from each students can be seen in figure 2 and 3.

$$\begin{aligned}
 \text{Wall area} &= 2 \times 300 \times 210 + 2 \times 150 \times 210 \\
 &= 126000 + 63000 \\
 &= 189000 \text{ cm}^2 \\
 \text{Tile area} &= 30 \times 30 \\
 &= 900 \text{ cm}^2 \\
 \text{Tiles} &= \frac{189000}{900} \\
 &= 210 \\
 210 : 8 &= 26 \text{ box} + 2 \text{ tiles} \\
 \text{Cost} &= 26 \text{ box} \times 31200 + 2 \text{ tiles} \times 4000 \\
 &= 811200 + 8000 \\
 &= 819.200 \text{ rupiahs}
 \end{aligned}$$

**Figure 2** LM's problem solving

$$\begin{aligned}
 \text{tile} &= 30 \times 30 = 900 \text{ cm}^2 \\
 \text{The wall area} &= (150 \times 210) \times 2 = 63.000 \\
 &+ (300 \times 210) \times 2 = 126.000 + \\
 &= 189.000 \text{ cm}^2 \\
 \text{the number of tiles needed} &= 189.000 \div 900 = 210 \\
 \text{retail's cost} &= 210 \times 4000 = 840.000 \\
 \text{wholesale} &= 210 : 8 = 26,2 = 26 \times 31.200 = 842.400 \\
 &\text{i will choose the retail cause its cheaper}
 \end{aligned}$$

**Figure 3** VS's problem solving

#### 3.1. Logical Mathematical Student

In observing patterns or regularities, LM was able to identify constructs based on the figure described in the

problem. This can be observed from the results of the following interviews.

- P : After reading the question, what information did you get?
- LM : The bathroom walls will be covered by tiles. Then we will look for cheapest way to buy from shop A and B
- P : What's the shape of the bathroom? how do you know?
- LM : The shape is cuboid, because it has length, width, and height in different sizes.
- P : Where did you get the length and width?
- LM : From the floor that has 300 x 150 cm<sup>2</sup> size.

In formulate conjecture related observed regularity, LM was able to identify geometric characteristics of the figure in the problem and describe constructs based on it characteristics using geometry concepts that he already has to determine problem solving formula. It can be observed from the problem-solving result and the following student interviews.

- P : How is your strategy to buy tiles so that the cost of buying bathroom tiles is cheapest as possible?
- LM : By comparing price from both shops separately and combine purchases, but firstly need to find how many tiles required
- P : How?
- LM : By calculate the area of the bathroom walls then divided by the area of a tile
- P : How to calculate the area of the walls?
- LM : Use formula  $2 \times \text{length} \times \text{height} + 2 \times \text{width} \times \text{height}$

LM was able to solve problem that involves recognizing construct characteristics through available information. He could provide explanation about relationship between constructs to determine how many tiles are needed to cover the entire bathroom wall. LM was able to analyze information on the question well. He could determine the formula to calculate only the vertical side area of cuboid that represents the bathroom walls. LM used more mathematical procedures than visual approach in problem solving. He was able to uses mathematical logic properly to determine how to buy tiles so that the cost of buying bathroom walls tiles is as economical as possible by combining retail and wholesale purchases.

### **3.2. Visual Spatial Student**

If reviewing VS reasoning in solving the geometry problem, he was able to identify figures construct described in the question. He drew cuboid as a visual representation of the figure on the problem. VS was able to name constructs based on their characteristics. It can be observed from the interview.

- P : What information do you get after reading the question?
- VS : The walls of a bathroom will be covered by tiles.
- P : What is the shape of the bathroom? What's your reason?
- VS : The shape is cuboid, because the base of bathroom is rectangular
- P : How do you know?
- VS : From the information, it's known that the floor size is 300 x 150 cm<sup>2</sup>, it represents the length and the width

VS was able to formulate conjecture related to observed figure that described on the question. He could determine the geometric characteristics in the problem using visualization by drawing cuboid that represent figure in the question, as well as describing construct based on its characteristics using the help of image that he made. It can also be observed from the following interviews:

- P : How did you determine the most economical way to buy tiles for bathroom?
- VS : By calculate the purchase price of the tiles needed for the walls in each shop.
- P : How?
- VS : Firstly, calculate the tiles needed to cover bathroom walls.
- P : How did you do it?
- VS : I calculated the area of the walls then divided by area of a tile.
- P : How did you calculate the area of the walls?
- VS : (explaining the problem-solving plan through image that he drew) From this picture, the vertical sides of cuboid represent bathroom walls. Therefore, we need to calculate the area of that four vertical sides with rectangle area formula.

Based on the results of problem solving, VS was able to solve problem that engages recognizing construct characteristics using image that he drew as a visual representation of the figure described in the problem. Through interviews, VS could explain the interrelated relationship between parts of the construct with the help of his visualization. He was able to draw logical conclusions about several ideas and their relationships. Student didn't just calculate the whole surface area of cuboid, but firstly he analyzed the image to determine which sides that represent the area of walls to calculate. VS also analyzed the cheaper price of whole tiles that needed in shop A and B if buying retail and wholesale. He was not in hurry to determine the cheapest way to buy even though that price of some tiles cheaper if buy wholesale than retail. However, VS hasn't used mathematical logic optimally to determine purchasing strategy so the cost of the tiles that needed to cover the bathroom walls isn't as economical as possible. He just compared the cost of whole tiles needed to covers bathroom walls for each shop separately.

#### 4. CONCLUSION

The research results on reasoning of LM and VS students in solving geometry problem related to surface area of cuboid showed that LM and VS were able to identify the constructs in the question, both of them were also able to naming the constructs based on its characteristics. They could determine geometric constructs through the information in the question. VS was able to describe construct through a visual representation of the geometric figure that he made. Furthermore, VS need image visualization to help problem solving that involve recognizing construct characteristics. Without visual representation, LM was able to use logic and his analysis to understand the relationship between the parts of geometric figure that described in the question. Both students were able to apply the geometry concepts and formulas that obtained using information on the question to solve problem that engages recognizing construct characteristics. LM can use logical mathematical thinking better than VS to finding the relationship between quantities to find the final solution of the problem.

Researchers who are interested in exploring students' reasoning in solving geometry problems can continue on subjects with other intelligence type's domination. This research can also be continued with the use of broader geometry problems. It can be HOTS problems, using daily problems or other contextual geometry subjects.

#### REFERENCES

- [1] A. Cesaria & T. Herman, "Mathematical reasoning in geometry learning using Information and Communication Technology (ICT)," *J. Phys. Conf. Series PAPER*, 2019.
- [2] Kemdikbud. Hasil Ujian Nasional 2019. Retrieved from <https://hasilun.puspendik.kemdikbud.go.id/#2019>
- [3] B. C. Gunhan, "A case study on the investigation of reasoning skills in geometry," *South Afr. J. Educ.*, vol. 34, no. 2, p. 1–19, 2014.
- [4] I. Palengka, D. Juniati, & Abadi, "Creative Mathematical Reasoning of Prospective Teachers in Solving Problems Reviewed Based on Working Memory Capacity," *J. Phys.: Conf. Series*, 2019.
- [5] K., Rosidah, Budayasa, & D. Juniati, "An Analysis of Statistical Reasoning Process of High School Students in Solving the Statistical Problem," *J. Phys.: Conf. Series*, 2018.
- [6] M. T Battista, "Mathematical reasoning and sense making," In *Reasoning and sense making in the mathematics classroom: grades 6-8*, 2017.
- [7] Sumpter, "Themes and interplay of beliefs in mathematical reasoning," *Int. J. Sci. Math. Educ.*, vol. 11, p. 1115–1135, 2013.
- [8] Rohana, "The enhancement of student's teacher mathematical reasoning ability through reflective learning," *J. Educ. Pract.*, vol. 6, no. 20, p. 108–115, 2015.
- [9] A. R. A Berenger. "The development of geometric reasoning: middle years students' understanding of 2-dimensional shapes," *Royal Melbourne Institute Technology*, 2017.
- [10] W. Ayuningtyas, Mardiyana, & I. Pramudya, "Analysis of student's geometry reasoning ability at senior high," *J. Phys.: Conf. Series PAPER*, 2019.
- [11] U.Hanifah, D.Juniati, T.Y.E. Siswono, "Students' spatial performance: cognitive style and sex differences," *J. Phys.: Conf. Series PAPER*, 2018.
- [12] N.Karamikabir, "Gardner's multiple intelligence and mathematics education," *Procedia – Soc. Behav. Sci.*, vol. 31, p. 778–781, 2012.
- [13] V. Rajoo, *An investigation on form 4 students' intelligent quotient (iq) and higher order thinking skills (HOTS) ability on their performances in additional mathematics in selected secondary school in Selangor*. Open University Malaysia, 2019.
- [14] F. Rahbarnia, S. Hamedian, & F. Radmehr, "A study on the relationship between multiple intelligences and mathematical problem solving based on revised bloom taxonomy," *J. Interdis. Math.*, vol. 17, no. 2, p. 109–134, 2014.
- [15] M. Asyrofi, & I. Junaedi, "Mathematical representation ability viewed from multiple intelligence on constructivism-based hybrid learning," *Unnes J. Math. Educ. Res.*, vol. 5, vol. 1, p. 32–39, 2016.
- [16] J. A. Azinar, S. Munzir, & Bahrn, "Students' logical-mathematical intelligence through the problem-solving approach," *J. Phys.: Conf. Series*, vol. 1460, no. 1, p. 0–6, 2020.
- [17] S. C. Haas, "Algebra for Gifterd Visual-Spatial Learners," *Gifted Education Communicator (Spring)*, vol. 34, no. 1, p. 30–31, 2003.
- [18] R. A. Wahab, A. H. B. Abdullah, M. S. B. Abu, M. B. Mokhtar, & N. A. Atan, "A case study on visual spatial skills and level of geometric thinking in learning 3D geometry among high achievers," *Man in India*, vol. 96, no. 1–2, p. 489–499, 2016.
- [19] P. Carter, *Test and Assess Your Brain Quotient*. London: Kogan Page London and Philadelphia, 2009.