Critical Thinking Disposition: The Persistence of Skeptical Students in Completing Mathematical Problems

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Abstract

The skeptic in dealing with a problem is not very useful if it is not followed by the persistence to find the correct answer and solve it. This research is qualitative research with an exploratory descriptive type. The results of this research indicated that skeptics could make students apathetic, pseudo apathetic, and persistent with the problems given. Components involved in the persistence in solving problems including goals, motivation, effort, and tools used to achieve the goal.

Keywords: persistent, skeptical, mathematical problems

1. Introduction

The skeptic is one of the important aspects of critical thinking both in critical thinking abilities and dispositions [1]–[6]. The skeptic is an attitude of doubt about something that is accepted, so skeptics tend to hesitate or not believe in something they receive [1]. Skepticism can encourage someone to reflect so that they produce a correct conclusion and can make the right decision [1]. There are two causes of skepticism on mathematical problems, namely (1) the existence of cognitive conflict (CC) and (2) the existence of two conflicting results (BK) [1]. Skepticism will be very useful in dealing with a large amount of information [7] in logical-mathematical problems [1]. However, skepticism in dealing with a problem is not very useful if it is not followed by persistence in finding the correctness and solving it.

Persistence is defined as an action that leads to a goal that continues voluntarily despite obstacles, difficulties, and despair [8]. According to Schunk, Pintrich, & Meece (2008), persistence is an action that leads to a goal and goes on continuously by requiring a certain amount of time, especially when facing difficulties. Mukhoiyaroh (2017) has stated that persistence is an attempt to keep working on tasks despite difficulties. Moreover, Chien (2012) states that children who have high scores on their persistence are less anxious and try to find solutions to difficult problems. Persistence is strongly associated with the use of certain time allocations [12].

Persistence is important to be owned by every element of society either public or a student in achieving a goal. In the context of education, students’ persistence is an important problem in achieving educational goals [2]. Students need to be persistent in solving problems, especially in dealing with mathematical problems. Mathematical problem solving is the thinking process of someone who tries to understand the problem situation by using the mathematical knowledge that he has [13]–[15]. Knowledge of mathematics cannot be applied directly to new situations in mathematical problems, so the problem solver requires a new perspective on the use of knowledge that is owned [16]. Furthermore, Mundy et al (2000) state that solving problems means finding ways or ways to...
achieve goals or solutions that do not easily become apparent. Based on these explanations, it can be concluded that mathematical problem solving is a person's thinking process to understand the problem situation by using the mathematical knowledge that is owned to find a solution to the problem.

Solving mathematical problems according to Nunokawa (2005) has two phases, namely (1) trying to apply their mathematical knowledge through exploring problem situations and (2) obtaining pieces of information about problem situations. If tension or ambiguity is not fully resolved, further exploration of information is obtained in a problem. When problem solver has identified several mathematical entities in the situation and found new information about these mathematical entities, the information obtained will sometimes be incorporated into students' mathematical knowledge such as theorems or formulas. In some cases, a reflection of the method used or the lack of effective methods can build new mathematical methods or ideas. Unlike Nunokawa, Polya (1973) divides problem-solving into four steps of completion, namely: (1) understanding the problem, (2) planning problem solving, (3) implementing the plan, and (4) re-examining [19].

Based on this information, it is clear that persistence is needed in solving mathematical problems in each phase. However, based on surveys conducted in 13 junior high schools or equivalent in North Lampung, the fact is that most students tend to be lazy in solving mathematical problems. Laziness is indicated by having complaints that arise when the teacher gives a math assignment and doing the task by copying the work of one of the students who are good at class. The lazy attitude of students is influenced by several things such as (1) ease in obtaining information so that students get used to things that are instant like looking for answers from the internet or copying friends' answers, and (2) online games. Lazy students will have difficulty in facing 21st Century learning, especially in achieving the expected goals. One of the goals of 21st Century learning is to require students to think critically in which students must be persistent in facing difficult problems. Due to its significance of persistence in resolving mathematical problems, research is required to be conducted on students' persistence in dealing with mathematical problems.

This article focused on the persistence of skeptical students in dealing with mathematical problems. The purposes of this article were to find out: (1) the impacts of skepticism on problem-solving, and (2) the components involved in students' persistence in solving problems

2. The Material and Method

2.1. Design

This research is qualitative research with an exploratory descriptive type. This belongs to descriptive research because the researcher conducted the analysis only on the level of description and situation analysis, which was to describe the students' skepticism in solving mathematical problems carried out by skeptical students. This is following the purpose of descriptive qualitative research, namely to describe, combine, analyze, and interpret a situation from an individual, institution, and group [20]. The subjects studied in this research were relatively limited to students who were skeptical of illogical mathematics problems, and therefore this study was a type of case study.

2.2. Materials

The main data of this research were written answers, field and oral records obtained from the use of assistive instruments. Written answers were obtained from test instruments, field notes were obtained from direct observations and results from video recordings, and oral data were obtained from interviews. The assistive instrument used consists of 4 mathematical problems consisting of 2 problems which are classified as very difficult for middle school students so that they cannot be solved correctly and 2 problems that are possible to be solved by middle school students. Field notes and the results of students’ answers were used as a basis in the interview process for the subjects. The results of the interview data were used to validate the data. Validation of data in this research used member checking which is by asking participants about the description of the response that has been made by conducting interviews [21].

Table 1. The indicator's of skeptical students of mathematical problems

<table>
<thead>
<tr>
<th>Skeptical caused by CC</th>
<th>Skeptical caused by BK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identifying problems</td>
<td>Identifying problems</td>
</tr>
<tr>
<td>Students realize that</td>
<td>Trying to solve the problems with one of the known formulas</td>
</tr>
<tr>
<td>there is conflicting</td>
<td>Evaluating the work that has been done in step 2</td>
</tr>
<tr>
<td>information in the</td>
<td>Trying to solve the problems with a different formula with step 2</td>
</tr>
<tr>
<td>problem given</td>
<td>Recognizing that there is conflicting information on the problem given</td>
</tr>
<tr>
<td>Refuting or questioning</td>
<td>Refuting or questioning the logic of the problems</td>
</tr>
<tr>
<td>the logic of the</td>
<td></td>
</tr>
<tr>
<td>problems</td>
<td></td>
</tr>
</tbody>
</table>

In the subjects searching process, this research also used test instruments and interviews as used by Nugroho et al. (2018) to determine the research subjects. The subjects of this research were skeptical students, both those caused by CC and BK. The indicators of skeptical students can be seen
Table 2. The indicators of students who are persistent in solving mathematical problems

<table>
<thead>
<tr>
<th>Definition</th>
<th>Indicator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efforts are made continuously to solve a problem with a certain time despite experiencing difficulties</td>
<td>To resolve mathematical problems well or keep trying to solve difficult problems until the determined time is over</td>
</tr>
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</table>

2.3. Participant

The subjects of this research consisted of 4 students. The students came from SMPIT Insan Robbani, SMP N 06 Kotabumi, and SMP N 4 Kotabumi. The subjects selection was done using a purposive sampling technique [22] which was based on the criteria: (1) being skeptical about illogical mathematics problems that have been given; (2) having obtained materials about the angles, triangles, circles, Pythagorean theorems; (3) having good communication skills. Skeptical students caused by CC in this research are called SCC and skeptical students caused by BK in this research are called SBK. The four students who were the subjects in this research consisted of 1 SCC student and 3 SBK students (named SBK1, SBK 2, and SBK3).

2.4. Procedure

The procedure of this research was conducted through several stages including (1) the selection of subjects. The selection of subjects in this research began with giving logical-mathematical problems to students. Ill-logical mathematics problem is a problem used by Nugroho et al. (2018). Furthermore, students who are skeptical of illogical-mathematics problems were chosen and categorized into SSC or SBK students; (2) the students who were skeptical of logical mathematics were given mathematical problems. The problem given consists of 4 items (see attachment) with each problem given time to complete and given on different days (see attachment); (3) observing the subjects directly, recording students’ activities with video recordings, and conducting interviews with the subjects using semi-structured interview guidelines; (4) analyzing everything related to students’ persistence in solving mathematical problems, (5) analyzing some interesting things, and (6) drawing conclusion of the results of the analysis.

2.5. Data Analysis

Valid data were used to find out (1) the impacts of skepticism on problem-solving, and (2) the components involved in students’ persistence in solving problems. The analysis process went through three stages, namely (1) data reduction by selecting the main things that we’re focused on the things needed and discarding unnecessary ones, (2) the presentation of data was the stage of grouping data that has been reduced into two groups, and (3) conclusion drawing on the findings that have been reduced.

3. Illustrations and Photographs

SBK1 and SBK 2 subjects

SBK1 and SBK2 were apathetic about the problems given. This happened because of the impact of students' skepticism on the logical mathematics problem. Therefore, the students did not want to solve the new problems given. Apathy is a behavior that describes the loss of motivation, poor persistence, and ignorance of something [23]. Apathy is shown by the utterances made by SBK1 and SBK2 as follows:

SBK1: “I bet the problem is definitely like the one we had yesterday. I do not want to do this. I have other tasks to do, sir.”

SBK2: “I’m sorry, sir. I do not think I can do it now. It is bad that I have put all my efforts to work on it and in the end, I find it incorrect.”

Both of these statements indicated a response to the rejection stated by SBK1 and SBK2 on the problems given. Based on this information, it seemed that SBK1 and SBK2 were apathetic to the problems due to their skepticism about a problem.

SBK3 student

Based on the video recordings and direct observations, it was seen that SBK3 students showed a persistent attitude in completing the 4 problems given. The student did not collect the work before the allotted time was over. In the third and fourth items, the students could complete them before time ran out, but the student used the rest of the time to review the answer to ensure the correctness of the results of the work. In contrast to problem-solving in point two, the students found it difficult to resolve the problem but the student insisted on resolving the problem so that when the time was almost finished the student answered the question carelessly by putting aside the rationality of the answer (rational intended in this research is an attitude that is carried out based on thoughts and considerations that match human common sense and following the rules of mathematics).

The irrational answer of the student to mathematical problems is shown in Figure 1 below.
Based on Figure 1, there were several errors, namely (1) the student used tangent rules without noting that the line PQ segment does not touch but intersects circle A, (2) an error occurred in the calculation process. The student did a mistake in reducing 100 - 60 and in doing the process of getting root value of 160, (3) the student used rules that were not clear even the student did not understand what was written. The students found it difficult to solve problems. The student made two points, namely points P and Q, then created a line connecting the two points. After making line PQ, the student was not able to connect the lines made with the questions in the problems and made mistakes in the calculation process. Therefore, the student was unable to determine the radius of circle B correctly. The student wrote an answer that the student did not understand, the correctness of the steps, by guessing. This was reinforced by the student’s statement at the interview that had been conducted. When interviewed about the reasons for the student’s written answers, the student answered, "I also don't know sir. The important thing is there is an answer. Hoho (smiling) "

After the in-depth interview, it turned out that the student had the determination to answer any problems even though the answer is written incorrect. Because the student remembered what the teacher said that every answer must be answered even though it is incorrect one would get a plus from the teacher. After being asked again why the student wanted to get a plus from the teacher, the student answered, "Yes, I have always tried to get a plus, so I feel ashamed when I have a blank answer sheet." 

Likewise in solving the problem of the first item, arcs AC and BC, respectively, have a center at B and A. If arc AC length is 12 cm, then determine the circumference of the circle that touches arc AC, arc BC, and line AB. The student found it difficult in solving it because the students could not identify the problem well as the answer shown in Figure 2.

Figure 2 indicates that the student tried to describe the problem by writing down any known information. The students drew a circle with the known two arcs namely arc AC and arc ABC. Then, the students made two circles with the center of each at points A and B. After trying to illustrate the information the form of images, the student found it difficult to represent "the circumference of the circle that touches arc AC", and therefore the student was unable to solve the problem. The student continued to try to solve the problem until the specified time was over.

Figure 1 and Figure 2 indicate that the student had an extraordinary persistence by trying to solve problems even though the student had difficulty understanding problems. Based on this information, it appears that when the student had a persistent attitude to solve problems, the student used cognitive agents to work rationally in processing existing information (see Figure 2), but when cognitive agents were unable to work rationally, the students would be stuck working with ignoring rationality from the answer (see Figure 1).

**SCC student**

In contrast to SBK3 students, the SCC student seemed indifferent when given a problem. It was shown by the student’s statement: "I bet it will be another incorrect answer". From the statement, the student was in doubt about the correctness of the problem given, but the student still solved the problem given. This condition, in research, referred to as pseudo apathy towards the problem given. Doubts arise because of the assumption that a researcher always gives problems that are not logical as given when choosing a subject. The persistence of SCC students could be seen from the efforts taken to solve the problems given even though the student said the student was lazy to finish.

When the student was given a problem that was very difficult and could not be solved, the student continued to think and try to choose a formula that can be used to solve it. When the students could not solve the problem with the correct mathematical rules, and therefore (1) the student used the intuition to solve the problem given. The student...
used the hunch in solving mathematical problems as stated by [24] that intuition involves hunches and is associated with the use of heuristics in problem-solving. When the student had no intuition (see figure 3), and then (2) the student only wrote the information known in the problem given (see figure 4).

Fig. 3.a The completion of the second item of SCC student

Fig. 3.b Translation of Figure 3.a

Based on Figure 3, in point 4, there was an error in the conclusion drawn. The student drew a conclusion based on the patterns that the student-created (based on the hunch) that each circle would have radius four times smaller than the larger circle. The student did not use the right mathematical rules in problem-solving, and therefore the student did not take rational actions in solving problems. The student used intuition to solve the problem. The student viewed the problem as a pattern so that the radius of the small circle is ¼ from the radius of the medium circle. It is clear that the student used the intuition as follows:

\[ r \text{ of small circle} = \frac{1}{4} \times r \text{ of medium circle} \]

\[ \text{So the radius of circle B is 0.5 cm} \]

Fig. 4.a The completion of the item question for SCC student

Fig. 4.b Translation of Figure 4.a

S: “I have no idea, sir. I just think I should be able to answer this question. When I was answering, I could not use the formula and then when I saw the picture, it seemed this is also a quarter of this (while showing picture 3)

The conversation snippet indicates that the student showed strong persistence in solving the problem given. This was caused by self-efficacy which indicated that the problem given was included in a problem that was easily solved. The self-efficacy arose because of having experiences in solving problems that were considered similar. However, after trying continuously until the specified time over the student still could not solve the problem. This can be seen from the following interview conversation snippet:

P: “Why did you use that method to answer the problem?”
S: “The problem seemed to be easy at first because apparently, I have got a problem similar to this. However, when I was trying to search, I didn’t find it. I just did what I could do.”

According to the information, it can be seen that if a person having a strong persistence faced with rational cognition cannot work in mathematical problem solving, the intuition will appear to act as a form of irrational action in solving problems. This is following the statement of [25] which states that when rational cognition agents are unable to work because of their limitations in managing information, the intuition will arise as an alternative action in solving problems.

According to this information, it can be seen that SSC student was pseudo apathetic towards the problems given by the researcher. Besides, when someone understands the problem and has a strong determination to resolve the problem, the students will try to act rationally in solving problems. If rational cognitive agents are not able to process information to solve problems, the intuition will appear as an alternative problem solving (based on Figure 3).

Meanwhile, if someone does not understand the problem, no matter how strong the determination to solve the problem, the students not able to act rationally in solving problems and even the intuition will not appear as an alternative problem solving (based on Figure 4).

In the third and fourth items, the student could complete before the time ran out properly. In the fourth item, the student used the intuition to solve the problem given. The answer can be seen in Figure 5 below.

Figure 5 indicates that the student simplified the problem given by changing the shaded image to a square with a side length of 7 cm. The activity was carried out by the student using the intuition. It was seen in the student’s statement in the interview, “Yes, this is my hunch, sir. It looks like the picture in question is the same as the square.” SCC students used hunches to solve mathematical problems and produced correct answers.

4. Discussion

Based on the results of the research, it appears that skepticism could make students apathetic, pseudo apathetic (persistent but indifferent), and persistent with the problems given. There are four main components involved in persistence in solving mathematical problems, namely: (1) objective, (2) motivation, (3) business, and (4) tool. A persistent person has clear goals to achieve. Clear goals to be achieved encourage someone to work on the right track in achieving those goals. Furthermore, the motivation referred to in the main component of persistence is a strong drive to achieve goals. This is following Kotaman’s view (2018) which says that motivation works to initiate goal-directed behavior. A person's motivation can arise from internal and external factors. External factors are the challenges (mathematical problems) of people who are considered important to them such as a teacher. Teachers can increase the level of students’ motivation [27] in solving mathematical problems. Besides, Koka & Hein (2003) reveal that teachers must provide students and assign tasks that are optimally challenging. Meanwhile, the internal factors in the form of embarrassment if they cannot achieve the goal (prestige) or in the form of a large amount of self-efficacy to solve the problem. This is similar to Schunk (1995) statement that self-efficacy has an impact on one's motivation as stated in Figure 6.

Self-efficacy in completing tasks can influence persistence and achievement [28]–[31] (Bandura, 1977; Biddle, 1999; Rhodes & Fletcher, 2014; Dale H Schunk, 1995). Furthermore, Kotaman (2018) states that self-efficacy in getting things done can increase motivation and persistence. Self-efficacy can arise from personal experiences which according to students they have had similar problems.

Strong desire (motivation) will not be tangible without any effort being made to achieve that goal. Kotaman (2018) states that motivation determines and requires efforts to achieve goals. Efforts can be made in various ways such as
(1) thinking involving cognitive agents to work rationally, using intuition, and guessing in solving mathematical problems. Rational thinking is the main basis in making decisions to solve problems, but rational cognition agents are unable to work because of their limitations in managing information [25]. The limitations of rational cognition agents in managing information can lead to intuition as an alternative action in solving problems [25]. Intuition is an important component of decision making that needs to be developed [32], [33], (2) looking for relevant information, (3) using several kinds of problem-solving strategies, and (4) determining formulas/theorems that can process information to solve problems.

In addition to goal, motivation, and effort, a component that is also important is the tool used to achieve the goal. The tools include formulas, available information, cognitive structures, time, and so on. As Kotaman (2018) states that several efforts and time allocated by someone to achieve goals are a form of persistence.

In general, the scheme of students’ skeptical persistence in solving problems can be seen in Figure 7 below.

![Diagram](image)

**Information:**
- : The Process is done
- : The Process is not done

**Fig. 7.** The scheme of students' skeptical persistence in solving problems

### 5. Conclusion

Based on the results and discussion, the conclusion of this article is as follows: (1) skeptics can make students apathetic, pseudo apathetic (persistent but indifferent), and persistent with the problems given, and (2) components involved in persistence in solving problems, namely goal, motivation, business, and tool used to achieve the goal.

Based on research conducted by the researcher, several questions have not been answered. The question will be possible as further research. The question is “How does the process of intuition arise in skeptical students of mathematical problems?”

### References


no. 11, 2012.


