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Mental Structure Constructed by Field Dependent Student on ACE Learning

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Abstract——This study aimed to describe the mental structure constructed by field-dependent (FD) students in the concept of group theory on ACE learning. This study was a descriptive qualitative. We discuss our findings based on the genetic decomposition of the group. The participants of this study were three (3) undergraduate students. The data was collected by a written test, videotape, and interviews. Instrument validation was obtained through expert judgment for the test of the proof ability and experiment for the genetic decomposition of a group. The credibility of the data was done by using triangulation: test and depth interview. Data analysis used Miles and Huberman's model. The FD-1 participant constructed the mental structure Action for the axioms, Action for the set, and Process for binary operation. The FD-1 participant constructed the mental structure of Action for a group. For the FD-2 participant constructed the mental structure of Process for axioms, Object for a set, and Object for the binary operation. The FD-2 participant constructed the mental structure of Process for a group. The FD-3 participant constructed the mental structure of Schema for axioms, Object for a set, and Object for the binary operation. The FD-3 participant constructed the mental structure of Schema for a group.

Keywords: mental structure, field-dependent, ACE learning, genetic decomposition

I. INTRODUCTION

Miyazaki, Fujita, and Jones, (2017) stated that teaching and learning of deductive proof in mathematics is one of the most important goals in mathematics education. Meanwhile, Beilock, Gunderson, Ramirez, and Levine (2010) suggested that teachers are an important influence not only on the quality of individual mathematics learning but also on self- study. A prerequisite to show that proof is important for school mathematics is stated by Stylianides and Stylianides (2007) that teachers at all levels have a strong knowledge of proof. Therefore, prospective teacher students must be equipped to teach mathematics at school so students can achieve these abilities. This provision, among others, through the lecture material group theory whose characteristic is loaded with deductive proof.

Anton and Rorres (2015) stated that a proof is a convincing argument that justifies the truth of a mathematical statement. In line with this opinion, a proof is a convincing argument stated in a mathematical language that the statement is true (Solow, 2014). Someone who can write valid proof shows that the person has a thorough understanding of the problem. General representations such as mathematical symbols or quantified variables are often used in formal proof (Yopp & Ely, 2016).

The role of proof according to Sarah, Bleiler, and Jeffrey was to prove, explain, systematize, discover and communicate. Hanna and Hersh stated the three main roles of proof are to prove, explain, and convince (Lo & Crory, 2009).

The categories of the ability to prove have been done by Sowder and Harel, Weber, and Isnarto. Sowder and Harel (1998) classify based on the schema of proof into three categories, namely externally base schema of proof, empirically schema of proof, and analytically schema of proof. Weber (2004) classifies into three categories of procedural proof, syntactic proof, and semantic proof. Isnarto (2014) revealed six aspects that need to be considered in the construction of mathematical proofs namely the initial steps, the flow of proof, the related concepts, the arguments, the key expressions, and the language of proof.

Durbin (2009) states that almost everyone has difficulty in constructing proofs. Lo and Crory (2009) argued that the mathematics education community throughout the world faces challenges to improve students' ability to prove and reason mathematically at all levels.

Samkoff, Lai, and Weber (2012) and Wijayanti and Wiyanti (2016) report that the majority of mathematics students have difficulty in constructing, understanding, and validating proof. Samkoff, Lai, and Weber (2012) stated students have many surprising difficulties in the construction of proofs. Because proof forms the basis of mathematical structure, this inability presents a big problem. The ability to understand proofs is an ability that students must master to construct proof well. The results of a preliminary study of student difficulties (field-independent, field-neutral, and field-dependent) in the proof conducted by Wijayanti and Wiyanti (2016) include difficulties in identifying elements in a set, misconceptions related to the use of mathematical notation, identifying the known statements, using definitions to prove, even in writing definition.

To be able to construct proof properly requires the ability to select, understand, and process information. The preferred way of selecting, understanding, and processing new information by Witkin and Goodenough is called cognitive style (Cataloglu & Ates, 2013). Cognitive style can be divided into two, namely field-independent (FI) and field-dependent (FD). The instrument used to measure cognitive style is the Group Embedded Figure Test (GEFT) from Witkin.

The study of the ability of FI students and FD students has been carried out by Umaru and Tukur and Dowlatabadi and Mehraganfar. Umaru and Tukur's research findings show that students of the FI cognitive style significantly achieve higher Math Achievement Test results than students of the FD cognitive style (Umaru & Tukur, 2013). Dowlatabadi and Mehraganfar (2014) found that FD students tended to use social strategies more than FI students, while FI students used cognitive and metacognitive strategies more often than FD students.

According to Piaget, there are three kinds of knowledge namely social knowledge, physical knowledge, and logico-mathematical knowledge. Logico-mathematical knowledge is knowledge constructed in the minds of learners, knowledge of relationships (Asimow, 2013; Cousins, 2010). Piaget argues reflective abstraction in mathematics is a mental mechanism by which all logico-mathematical structures are obtained (Arnon et al, 2014). Dubinsky put forward the types of mental mechanism namely interiorization, reversal, coordination, encapsulation, de-encapsulation, thematization, generalization. These mental mechanisms lead to the mental structure of Action, Process, Object, and Schema (APOS).

Action is a transformation from an externally directed Object(s) that is marked by the ability to apply certain rules, can express expression explicitly, and can do calculations for special things. The Process is an action that occurs entirely in the mind. It is characterized by being able to carry out Actions mentally in the mind, can transform Processes through reversal, can coordinate Processes, can apply language and symbols to construct internal Processes. The Object is an entity in which Actions can be made in the mind. The student indicator has constructed the mental structure of the Object that can carry out Actions and Processes on the entity (encapsulation) and can de-encapsulate the entity back to the Process that produced the entity. The Schema for mathematical concepts is a coherent collection of mental structures (actions, Processes, Objects, and other Schemes) and the relationships between these structures to form a framework in the minds of students that might be brought to the problem situation associated with the concept. Mental structure construction Schemes are characterized by being able to perform Actions or Processes on cognitive Objects, can build a relationship between mental structures that support a concept, and can apply existing cognitive objects into a broader context.

Genetic decomposition is a hypothetical model that describes the mental structures and mechanisms that students need to learn certain mathematical concepts. It is possible to ascertain whether students have constructed mental objects based on the way they say and write about a concept (Tall, Thomas, Davis, Gray, & Simpson, 1999). Studies related to genetic decomposition have been carried out by Trigueros and Martinez-Planell (2010) and Roa-Fuentes and Oktac. Trigueros &Martinez-Planell (2010) conducted a study of the function of two variables by designing interviews to obtain information about the proposed genetic decomposition components. Roa-Fuentes and Oktac conducted interviews to obtain preliminary genetic decomposition which they proposed for the concept of a linear transformation (Arnon et al, 2014).

The ACE learning cycle was developed by Ed Dubinsky. The ACE learning cycle includes stages (A) activities, (C) classroom discussion, and (E) exercises. In this study, for the activities phase, students are grouped by 3-4 people. Students work on student worksheets in a group. At the classroom discussion stage, the lecturer presents the material with questions and answers and guides the discussion about the problem that arises at the activities stage. Next, in the stage of the exercises, students work on exercises as homework. A discussion about the homework needed by students held at the next meeting.

ACE learning is preceded by compiling genetic decomposition. In this study, genetic decomposition was arranged according to the research material, namely group. The genetic decomposition of a group using in this study refer to Wijayanti (2017).

This study aimed to describe the mental structure constructed by FD students in the concept of group theory on ACE learning.

II. METHOD

This study was a descriptive qualitative. We discuss our findings based on the genetic decomposition of the group refer to Wijayanti (2017)

The participants of this study were three (3) undergraduate students who were taking Introduction of Algebraic Structure 1, which included a group at Universitas Negeri Semarang. Firstly, the students took GEFT to determine the type of their cognitive style. There were three (3) FD students as participants of this study. Secondly, we conducted ACE Learning. After completion of the learning for the topic of a group, then we held a written test of the ability to prove. Analysis of this test was conducted to describe the mental structure of FD students.

The data was collected by a written test, videotape and interviews. Instrument validation was obtained through expert judgment for the test of the ability to prove and experiment for the genetic decomposition of a group. The credibility of the data was done by using triangulation: test and depth interview. Data analysis used Miles and Huberman's model. The steps of analysis were data reduction, data display, and conclusion drawing/verification.

III. RESULTS AND DISCUSSION

The characteristics of FD students prefer externally defined goals and reinforcements, and clear definitions of desired outcomes, extrinsically motivated, less structured, less autonomous. We discuss our findings based on the indicators of each mental structure for the concept of groups. If students constructed the mental structure of Objects for the set and binary operation and can well-performed coordination mental mechanism between the mental structure of Process for the set, binary operation, and the axioms then students constructed the mental structure of the scheme of the group.

The main problem to be solved was (a) state the definition of a group; (b) give an example of a group and prove it. With this problem, students may only need the mental structure of Action for the set. Therefore, additional questions are needed to measure the mental structure constructed for the set. The additional questions state the definition of a subset and show that if H and K are subsets of G then the intersection of H and K is a subset of G. The interesting one about the answer is all participants give the set of all integers under addition as the example.

A. FD-1 Participant

The FD-1 participant can state the definition of a group using language correctly The FD-1 participant can not yet state the axiom using mathematical symbols precisely. The FD-1 participant can provide an example of a group correctly but has not been able to prove an example is a group. In demonstrating the existence of an identity element, it appears that the FD-1 participant does not yet understand the meaning of the identity element and also the inverse element so the procedures performed are not correct. The FD-1 participant uses the properties that will be shown as proof arguments. In this case, The FD-1 participant is said to construct mental structures of Action for axioms.

The use of language and symbols can not be done well, especially on the use of symbols. Therefore it is said that the FD-1 participant constructed the mental structure of Action for language and symbols. This is in line with the result of Thompson, Senk, and Johnson’s (2012) research which stated that students lack knowledge about concepts, definitions, and notations in the field of study.

The membership of the set of all integers cannot be well recognized by the FD-1 participant, which is marked by the FD-1 participant, yet it can not reveal the inverse element for addition on the set of all integers. The FD-1 participant requires an external instructions guide. This shows that the FD-1 participant constructs the mental structure of Action for the set.

The FD-1 participant has not constructed the mental structure of the Object for the binary operation addition on the set of integers with indicators not yet able to check the binary operation addition satisfies the properties of associative, the existence of identity element, and the existence of an inverse element. The FD-1 participant can take elements on the set and can act on those elements according to the defined rules of addition. Therefore it can be said that the FD-1 participant constructs the mental structure of the Process for the binary operation. According to Ebrahimi, Zeynali, and Dodman (2013), field-dependent participant prefers the inductive method. They have difficulty with logic and methods of proof (Blanton & Stylianou, 2014).

The coordination among the Processes of the set of integers, the binary operation addition, and group axioms has not been carried out correctly by FD-1 participant. The group Scheme is not coherent even though it can build an example of a group, but it cannot yet prove an example is a group. It also shows that the group schema has not yet been thematized.

In the next section, the FD-1 participant does not understand the definition of a subset so well that it has not been able to show a set is a subset of another set. The FD-1 participant was able to prove by an external guide. Thus, it is said that the FD-1 participant constructed the mental structure of Action for the set. This results in line with the characteristics of FD students that are extrinsically motivated and less structured (Oh & Lim, 2005).

B. FD-2 Participant

The FD-2 participant states the definition of a group by using language and mathematical symbols with a redundant requirement, namely closed property that is included in the terms of the binary operation. It appears that the FD-2 participant had difficulty in stating the definition. This is in line with the finding of Guerrier, Boero, Douek, Epp, and Tanguay (2012) that students have a problem working with definitions.

The FD-2 participant can give an example of a group and can prove it with the right steps. The FD-2 participant has not been consistent in the use of symbols. For example, the FD-2 participant states $<\mathbb{Z},+>$ is associative, $<\mathbb{Z},+>$ is closed, and $<\mathbb{Z},+>$ has an identity element. It can be concluded that the FD-2 participant constructs the mental structure of Action for language and symbols.

Osterholm found that undergraduate students better understand the introduction to group theory when it is written in words rather than words and symbols (Inglis & Alcock, 2012).

The FD-2 participant can examine axioms of associative, the existence of identity element, the existence of inverse element in the set of all integers under addition. But then the FD-2 participant adds another axiom for a group axioms. Thus, the FD-2 participant constructs the mental structure of Process for the axioms.

The membership of the set of integer numbers can be well recognized by the FD-2 participant which is characterized by being able to identify the identity element and the inverse element for addition on the set of all integers. This shows that the FD-2 participant has constructed the mental structure of Action for the set.

The FD-2 participant has constructed the mental structure of the Object for the binary operation addition on the set of all integers. This is characterized by the ability to check the binary operation addition meeting the properties of associative, identity element, and inverse element.

The coordination among the Processes of the set of all integers, the binary operation addition, and the axioms of a group has been done well by the FD-2 participant. Unfortunately, the use of axioms for a group was not precise. The Schema of a group has not been thematized even though the FD-2 participant can check the axioms of a group. The ability to coordinate the Processes of the set, the binary operation and the axioms of a group lead the FD-2 participant to construct the mental structure of Process for a group.

In the next section, the FD-2 participant understands the notion of a subset well and can show that a set is a subset of another set. Thus, it is said that the FD-2 participant constructs the mental structure of Object for the set.

C. *FD-3 Participant*

The FD-3 participant stated the definition of a group in the language without the symbol. The FD-3 participant can give an example of a group correctly. The FD-3 participant can show axioms of associative, the existence of the identity element, and the existence of the inverse element so that it is said that the FD-3 participant has constructed the mental structure of the Schema for axioms. In proving the existence of the identity element and the existence of the inverse element, the Subject FD-3 draws a conclusion using the symbols but not given its meaning. The FD-3 participant still uses symbols that are not quite right in the proof process. It appears that the FD-3 participant is still experiencing difficulties in the use of language and symbols. This is in line with the result of the research of Canadas, Molina, and Rio (2018) which states that students may have difficulty giving meaning to algebraic symbolism. Therefore, it is said that the FD-3 participant constructed Action for language and symbols.

The membership of the set of integer numbers can be well recognized by the FD-3 participant which is characterized by being able to identify the identity element and the inverse element for addition on the set of all integer numbers. This shows that the FD-3 participant has constructed the mental structure of Action for the set of all integer numbers.

The FD-3 participant has constructed the mental structure of Object for the binary operation addition on the set of all integers. This is indicated by the ability to check the binary operation addition satisfies properties of associative, identity element, and inverse element.

The coordination among the Processes of the set of all integers, the binary operation addition, and the axioms of a group have been carried out by the FD-3 participant. It shows that the Schema of a group has been thematized.

In the next section, the FD-3 participant understands the notion of a subset and can show that a set is a subset of another set. Thus, it is said that the FD-3 participant constructs the mental structure of Object for the set.

IV. CONCLUSIONS

This study concludes that the FD-1 participant constructed the mental structure of Action for the axioms, Action for the set, and Process for binary operation. The FD-1 participant could not coordinate the Processes of the set, the binary operation, and the axioms of a group. As a result, the FD-1 participant fails to construct the mental structure of Schema for a group. The FD-1 participant constructed the mental structure of Action for a group.

For the FD-2 participant constructed the mental structure of Process for axioms, Object for a set, and Object for the binary operation. The FD-2 participant was able to coordinate the Processes of the set, the binary operation, and the axioms of a group. By adding another axiom, it was said that the FD-2 participant constructed the mental structure of Process for a group.

The FD-3 participant constructed the mental structure of Schema for axioms, Object for a set, and Object for the binary operation. The coordination among the Processes of the set, the binary operation, and the axioms of a group was well done. Therefore it was called the FD-3 participant constructed the mental structure of Schema for a group.

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