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Application of Fishburn Sequences in Economic and Mathematical Modeling

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Abstract. The paper validates the fitness of taking a game-theoretic approach to modeling the process of executive decision-making in the economy in the context of the third information situation, when the probability values of the economic environment states are unknown and must be in line with the corresponding linear ordering relations. Using the problem of funds allocation between the assets of an enterprise as an example, the paper analyzes the changes in the decision depending on the selection of sequences, which determine the probability distribution and comply with a certain ordering relation. To estimate the probabilities, weighting systems were selected with regard to nonstrict and strengthened nonstrict ranking, which were built using the increasing generalized arithmetic Fishburn progression and the sequence generated by Fermat numbers, respectively. The paper accentuates the role of the decision-making body when selecting the estimate of distribution of the economic environment states.

1. Introduction

The special features of economic and social systems call for a development of such a game-theoretic approach to modeling the executive decision-making process in the economy, which would make it possible to account for the ambiguity, random nature and incompleteness of information. These features of the economy result in a situation where in the game-theoretic modeling of the economy not all exact and true values of the payoff matrix entries are known or they may be absolutely unknown, however, the law that can be applied to find them is established (or can be identified). In such circumstances, it is necessary to apply neoclassical zero-sum games. Furthermore, to extend the scope of zero-sum games application in economic and mathematical modeling, it is worth to use them in combination with statistical games, as well as with probability theory, mathematical statistics, and the theory of stochastic processes.

2. Relevance

In [1], the author goes into the details of the concept of combined application of statistical and zero-sum games for executive decision-making in the economy. This concept has widespread application in various spheres of executive decision-making, being used for the optimal allocation of available resources between different assets, evaluation of the investment projects implementation, etc. Among other things, both Russian and foreign scientists consider the application of the game-theoretic approach to assess information security risks. One way of looking at information protection is in terms of optimizing the volume of expenses associated with information security [2] (allocation of monetary resources), the other – is in terms of optimizing resources spent on maintaining health of the network attack protection system [3] (allocation of system resources – system power). In these and other studies, the authors consider game-theoretic models based on the application of an estimate of the economic environment probability values. A set of sequences assessing the distribution of the economic environment states will allow the decision maker to have greater flexibility when analyzing various economic systems. In the monograph [4], in-depth consideration is given to the application of Fishburn formulas and their generalizations (Fishburn sequences and, particularly, generalized Fishburn progressions) in the modern portfolio theory for a proper modeling of the task of selecting an efficient portfolio in the field of various information situations.

3. Problem statement

The game-theoretic approach to modeling the executive decision-making process in the economy should account for the ambiguity, random nature and incompleteness of information. In game-theoretic models, probability values of the economic environment (EE) probable states should be taken into account. Estimates of the unknown probability values can be acquired by expertise or proposed by the decision maker based on the available information, his or her preferences and goals. Correctness and adequacy of the probability values estimate impact the resulting model indicators, analysis quality and management process outcomes based on the application of such a model. To make an estimate of probability values, it is proposed to use Fishburn sequences [4] and second-order Fishburn sequences [5] complying with a simple and partially strengthened nonstrict linear ordering relation (e.g. see [4, pp. 96-97]) of factors. The purpose of the article is to consider the potential for using Fishburn sequences in game-theoretic models.

4. Theoretical

The first step to take prior to applying the games theory for executive decision-making in the economy is to construct a payoff matrix, which is generally the most time-consuming phase. Whereas in the game-theoretic modeling of the economy not all exact and true values of the payoff matrix entries are known or they may be absolutely unknown, the law that can be applied to find them is established (or can be identified). The process of finding the unknown members is based on applying the estimate of the economic environment probability values. Let us consider a case when nothing is known for the probability values of the economic environment states, except that they are governed by a certain type of an ordering relation. According to a classification of information situations (IS) by R.I. Trukhaev [6, p. 13], this is the case of a third IS. In the third IS field, the unknown probability values of the economic environment state must be in accord with the given set of constraints or a certain ordering relation. These ordering relations have been studied comprehensively by P. Fishburn [7-9] and are discussed, for example, in the monograph by R. I. Trukhaev [1-21]. R. I. Trukhaev presents the main point estimates of the prior probability distribution of the economic environment states in the third IS field. In his monograph, R.I. Trukhaev refers to these estimates of the prior probability distribution of the economic environment states as Fishburn point estimates. Fishburn formulas provide for a simple and natural way of defining the estimates of the probability values of the economic environment states, when any given priority vector is set for these probabilities, i.e. one or another ordering relation. The article [10] introduces a concept of Fishburn progressions: Fishburn arithmetic progression and Fishburn geometric progression; their properties were also discussed in [11], [12]. Fishburn progressions are often used in the literature on mathematical modeling of decision making under ambiguity, for example, in [13-20]. The concept and main properties of the generalized Fishburn progressions are stated, for example, in the monograph [1-21]. In [5], the concept of Fishburn sequences of the first and second orders is considered in depth. Let us consider the potential for using Fishburn sequences in game-theoretic models.

5. Practical significance

We shall exemplify the proposed method application by establishing the information security budget of an enterprise. Today, information systems for both business and state-owned enterprises are an important component of the entire management system, therefore, the enterprise makes certain investments to ensure efficient protection against network attacks depending on the security level estimate. To identify any deficiencies in the information protection system, a cyber-security (CS) audit is carried out, e.g. an expert security audit based on the expertise of the involved auditors. An information system is comprised of several subsystems, either of which is meant to perform its function: maintenance support (computers; data collection and storage devices, data processors and communication units; office equipment), software support (systemic and applied), information support (databases, data classification systems, and documentation systems), organizational and legal support. The enterprise seeks to mitigate any damage while maximizing the probability of blocking an attack. Suppose that four assets (subsystems) are in danger of an attack against the information system and the enterprise plans to allocate 40 MU to ensure cyber security. The probability of the attack causing damage can be represented as a product of the attack probability by the probability that the attack was successful and impaired the enterprise. When calculating the attack probability values and the probability of damage the attack caused, statistical methods and peer reviews are used. Statistical methods rely on the analysis of historical data on actual information security incidents. Due to the lack of statistical data, statistical methods are not always practicable. Assume that during the cyber security audit, the probability values of damage due to a network-level attack are obtained based upon statistical data. Expert opinions were the same about the ranking of attacks on subsystems, though, were different about the number of probable attacks. In view of the decision to estimate the attack probability, weighting systems were selected for four subsystems with regard to nonstrict and strengthened nonstrict ranking, which were built using the increasing generalized arithmetic Fishburn progression and the sequence generated by Fermat numbers, respectively. Depending on the expert opinion, the following data for the probability of the attack causing damage were acquired (table 1).

Table 1. Probability of the Attack Causing Damage to the Enterprise Subsystems According to the Opinion of Two Expert Groups.

		Enterprise Subsystems			
		1	2	3	4
Damage Probability		0.1	0.05	0.03	0.1
Attack Probability	1st gr. of experts	3/8	2/8	2/8	1/8
	2nd gr. of experts	17/30	5/30	5/30	3/30
Probability of the Attack Causing Damage	1st gr. of experts	0.0375	0.0125	0.0075	0.0125
	2nd gr. of experts	0.0567	0.0083	0.005	0.01

The possible allocation of funds for secondary protection of the information system can be done using the strategies provided in Table 2. The probability of blocking an attack on the *j*-th subsystem, which received the r_{ij} funds can be taken as equal (e.g. see [2]):

$$a_{ij} = 1 - \left(1 - q_j\right)^{r_{ij}}, \quad i = \overline{1, k}, \quad j = \overline{1, n} \tag{1}$$

In this case, it was hypothesized that the higher the probability of the attack causing damage, the more advanced the used standardized safety devices, therefore, the subsystem requires less resources to remove security deficiencies.

Using a formula (1), two probability matrices of the subsystem cyber security maintenance can be obtained, according to the expert opinions. By applying the game theory, we can find a saddle point. Should there be none (as in this case), we proceed to finding mixed strategies.

Table 2. Allocation of Funds per Enterprise Subsystems.

Strategy No.		Enterprise Subsystems 1	2	3	4
1		5	10	5	20
2		5	7	10	18
3		10	13	13	4
4		12	5	10	13
5		15	10	10	5
Probability of the Attack Causing Damage	1st gr. of experts	0.0375	0.0125	0.0075	0.0125
	2nd gr. of experts	0.0567	0.0083	0.005	0.01

To find the mixed strategies, we should solve linear programming problems; the coefficients at variables are defined using a formula (1), respectively for the two expert opinions that we consider.

Solution of games in mixed strategies yields the following results for the first and the second case, respectively: $\mathbf{p}_1^* = (0; 0.2534; 0.7466; 0; 0)$ and $\mathbf{p}_2^* = (0; 0.1688; 0.8312; 0; 0)$. At that, the recommended amounts of subsystems financing are: $(8.7329; 11.4795; 12.2397; 7.5479)$ and $(9.1559; 11.9871; 12.4936; 6.3634)$.

6. Conclusion

The process of building a payoff matrix is one of the most important and complex phases of game-theoretic modeling. The problem of information incompleteness makes it possible, in some ways, to take into account and overcome the use of neoclassical zero-sum games, which are two-person zero-sum games defined by the partially known payoff matrices. The eventual outcomes obviously depend on the selected estimate of distribution of the economic environment states. The decision should be made by the decision-making body based on the available information, experience, competence and professional intuition.

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