The Nash Equilibrium Analysis of the Final Price Arbitration Model

Feng Jingjing

School of Intelligent Science and Information Engineering, Xi'an Peihua University, Xi'an,710125, China

*Corresponding author. Email: fengjingjing@163.com

ABSTRACT. Using the model of the last asking price arbitration mechanism, taking the welfare of employees as an example, the paper describes the welfare problem and analyzes the Nash equilibrium solution of the welfare problem of employees under the model of the last asking price arbitration. Finally, the Nash equilibrium solution of the model of the last asking price arbitration mechanism is given under the conditions of the probability distribution of Cauchy distribution, uniform distribution, Laplace distribution, Weibull distribution, exponential distribution, Rayleigh distribution, Pareto distribution. The preference scheme of arbiter can get better profits.

1. INTRODUCTION

When the participants in the economic life cannot reach an agreement on their own interests, the arbitration system can play a great role in addition to resorting to law. There are two common arbitration methods, one is traditional arbitration, which means that after the price is called by both parties, the arbitrators determine the final solution according to their own preferences. Generally, this kind of arbitration will form a most comprehensive compromise. The final result under the traditional arbitration system, the participants can expect the preference of the arbitrators, so that the participants will encourage more extreme prices. The other is the final price arbitration, which requires the arbitrators to only choose the price of one of the participants as the final result, so as to reduce the chilling effect of the agreement arbitration. Most of the articles[1-6] are based on the preference of the arbitrators. Under the condition of probability distribution of state distribution, the last asking arbitration mechanism model is applied to analyze traffic accidents, length of service buyout, power market and other problems, and the Nash equilibrium solution is obtained. However, in practical problems, the preference scheme of the arbitrator may also obey other probability distributions such as Cauchy distribution, exponential distribution and so on. In this paper, taking the employee welfare problem of the enterprise as an example, the final discussion is made. And the Nash equilibrium of the final bid arbitration mechanism model under Cauchy distribution and Laplacian distribution and other distributions are discussed.

It is necessary to have some knowledge of probability theory[7] to discuss the final bid arbitration mechanism model.

Let $S = \{e\}$ be the sample space of random experiment E. $X = X(e)$ is a real-valued singlevalued function defined in sample space S. For any real number x, $X = X(e)$ is random variable when the set $\{e \mid X(e) \leq x\}$ has definite probability. Let X be a random variable, $F(x) = P\{X \leq x\}$ is a distribution function of random variables for any real number x.

If the distribution function $F(x)$ of a random variable X can be expressed as an integral of a nonnegative integrable function $F(x) = \int_{-\infty}^{x} f(x)dx$, then X is called continuous random variable and $f(x)$ is the probability density function of random variable X.

2. Nash Equilibrium Solution of Final Request Arbitration Model

Taking the welfare treatment of enterprise employees as an example, this paper discusses the model of the final asking price arbitration mechanism. Assuming that there is a dispute between the enterprise and the labor union on the welfare treatment, the game is divided into two steps: first, the two sides negotiate on the welfare issue under the supervision of the arbitrators, that is, the enterprise and the labor union offer their own welfare at the same time. The benefits are represented by ξ_1 and ξ_2, respectively. if $\xi_1 \geq \xi_2$, there is no need for arbitration. Secondly, the arbitrator chooses one of them as the final solution.

Assuming that the arbitrator himself has his own reasonable plan for the price of the house with ϕ expressing this ideal value and further assumes that after observing the bids ξ_1 and ξ_2 of both parties, the arbitrator simply chooses X that the closest bid. Let $\xi_1 < \xi_2$, if $\xi < \dfrac{\xi_1 + \xi_2}{2}$, then thearbitrator will choose ξ_1. If $\xi > \dfrac{\xi_1 + \xi_2}{2}$, then the arbitrator will choose ξ_1. If $\xi = \dfrac{\xi_1 + \xi_2}{2}$, then the arbitrator tossed a coin to decide. The arbitrator knows the ideal value ϕ, but neither of the participants knows it. The participants consider ξ as a random variable whose distribution function is $F(x) = P\{X \leq x\}$.

If the event A indicates that the amount of property given by the buyer is selected by the arbitrator, the event B indicates that the amount of property required by the seller is selected by the arbitrator. Then

$$P(A) = P(\xi < \frac{\xi_1 + \xi_2}{2}) + \frac{1}{2}P(\xi = \frac{\xi_1 + \xi_2}{2})$$

$$= P(\xi \leq \frac{\xi_1 + \xi_2}{2}) - P(\xi = \frac{\xi_1 + \xi_2}{2}) + \frac{1}{2}P(\xi = \frac{\xi_1 + \xi_2}{2}) = P(\xi \leq \frac{\xi_1 + \xi_2}{2}) - \frac{1}{2}P(\xi = \frac{\xi_1 + \xi_2}{2})$$

$$= F(\frac{\xi_1 + \xi_2}{2}) - \frac{1}{2}F(\frac{\xi_1 + \xi_2}{2}) + \frac{1}{2}F(\frac{\xi_1 + \xi_2}{2} - 0)$$

$$= \frac{1}{2}[F(\frac{\xi_1 + \xi_2}{2}) + F(\frac{\xi_1 + \xi_2}{2} - 0)] \qquad (1)$$

$$P(B) = 1 - \frac{1}{2}[F(\frac{\xi_1 + \xi_2}{2}) + F(\frac{\xi_1 + \xi_2}{2} - 0)] \qquad (2)$$

And then the expected amount of compensation is

$$W(\xi_1, \xi_2) = \xi_1 \cdot P(A) + \xi_2 \cdot P(B) = \xi_1 \cdot \frac{1}{2}[F(\frac{\xi_1 + \xi_2}{2}) + F(\frac{\xi_1 + \xi_2}{2} - 0)] +$$

$$\xi_2 \cdot \{1 - \frac{1}{2}[F(\frac{\xi_1 + \xi_2}{2}) + F(\frac{\xi_1 + \xi_2}{2} - 0)]\}$$

$$= \xi_2 + \frac{1}{2}(\xi_1 - \xi_2)[F(\frac{\xi_1 + \xi_2}{2}) + F(\frac{\xi_1 + \xi_2}{2} - 0)]. \qquad (3)$$

Assuming that the buyer's goal is to minimize the arbitration result of the amount of the property, the seller tries to maximize the amount of the property. If the bids (ξ_1^*, ξ_2^*) of both parties are Nash equilibrium of the game between the buyer and the seller, then ξ_1^* must satisfy:

$$\min_{X_1} W(\xi_1, \xi_2^*) \text{ and } \xi_2^* \text{ must satisfy:}$$

$$\max_{X_2} W(\xi_1^*, \xi_2).$$

Let the arbitrator's preference scheme ξ be a continuous random variable and the corresponding probability
Density function be $f(x)$. Then

$$P(A) = F(\frac{\xi_1 + \xi_2}{2}) \, , P(B) = 1 - F(\frac{\xi_1 + \xi_2}{2}) \, , \qquad (4)$$

and then $W(\xi_1, \xi_2) = \xi_1 \cdot P(A) + \xi_2 \cdot P(B)$

$$= \xi_1 \cdot F(\frac{\xi_1 + \xi_2}{2}) + \xi_2 \cdot [1 - F(\frac{\xi_1 + \xi_2}{2})] \, . \qquad (5)$$

The first-order condition of the above optimization problem must be satisfied by the price combination of the two parties (ξ_1^*, ξ_2^*) for the amount of real estate. That is

$$\begin{cases} \dfrac{\partial W(\xi_1, \xi_2^*)}{\partial \xi_1} = F(\dfrac{\xi_1^* + \xi_2^*}{2}) - \dfrac{1}{2}(\xi_1^* - \xi_2^*) f(\dfrac{\xi_1^* + \xi_2^*}{2}) \\ \dfrac{\partial W(\xi_1^*, \xi_2)}{\partial \xi_2} = 1 - F(\dfrac{\xi_1^* + \xi_2^*}{2}) - \dfrac{1}{2}(\xi_2^* - \xi_1^*) f(\dfrac{\xi_1^* + \xi_2^*}{2}) \end{cases} \qquad (6)$$

and then $F(\dfrac{\xi_1^* + \xi_2^*}{2}) = \dfrac{1}{2}, \xi_2^* - \xi_1^* = \dfrac{1}{f(\dfrac{\xi_1^* + \xi_2^*}{2})} \qquad (7)$

3. Analysis of Nash Equilibrium Under Multiple Probablity Distributions

In [8], Arbitrator's preferences subject to Normal distribution is given. In this Paper, we give abitrator's preferences subject to other distributions.

3.1 Arbitrator's Preferences Subject to Cauchy Distribution

Let the arbitrator's preference scheme obey the Cauchy distribution with parameters of θ and γ. That is, $\xi \sim C(\theta, \gamma)$. Its probability density is

$$f(x) = \frac{1}{\pi\gamma[1 + (\frac{x - \theta}{\gamma})^2]} = \frac{1}{\pi}[\frac{\gamma}{(x - \theta)^2 + \gamma^2}] \, , \qquad (8)$$

the distribution function is $F(x) = \frac{1}{\pi} \arctan \frac{x - \theta}{\gamma} + \frac{1}{2}$. we have $\frac{\xi_1^* + \xi_2^*}{2} = \theta, \xi_2^* - \xi_1^* = \pi\gamma$ .And then the Nash equilibrium bid of the game under Weibull distribution is

$$\xi_1^* = \theta - \frac{\pi\gamma}{2}, \xi_2^* = \theta + \frac{\pi\gamma}{2} \, . \qquad (9)$$

The equilibrium asking price of both parties is centrally symmetric θ, and the difference of the asking price increases with the increase of uncertainty γ of the arbitrator's preference scheme.

3.2 Arbitrator's Preferences Subject to Exponential Distribution

Let the arbitrator's preference scheme obey the exponential distribution with parameters of λ. Its probability density is $f(x) = \begin{cases} \lambda e^{-\lambda x}, x > 0 \\ 0, x \le 0 \end{cases}$, the distribution function is $F(x) = \begin{cases} 1 - e^{-\lambda x}, x \ge 0 \\ 0, x < 0 \end{cases}$. Then the Nash equilibrium bid of the game under exponential distribution is

$$\xi_1^* = \frac{\ln 2 - 1}{\lambda}, \xi_2^* = \frac{\ln 2 + 1}{\lambda} \, . \qquad (10)$$

If the arbiter's preference scheme obeys the exponential distribution with subordinate parameter λ, then the equilibrium asking price of both sides is centrosymmetric with the expected value $\frac{1}{\lambda}$ of the arbiter's preference scheme $\ln 2$ times, and the difference of asking price increases with the increase of uncertainty $\frac{1}{\lambda^2}$ of both sides to the arbiter's preference scheme.

3.3 Arbitrator's Preferences Subject to Uniform Distribution

Let the arbitrator's preference scheme obey the uniform distribution on (a,b). Its probability density

is $f(x) = \begin{cases} \dfrac{1}{b-a}, x \in (a,b) \\ 0, x \notin (a,b) \end{cases}$, the distribution function is $F(x) = \begin{cases} 0, x < a \\ \dfrac{x-a}{b-a}, a \le x < b \\ 1, x \ge b \end{cases}$. Then the Nash

equilibrium bid of the game under exponential distribution is

$$\frac{\xi_1^* + \xi_2^*}{2} = \frac{a+b}{2}, \xi_2^* - \xi_1^* = \frac{1}{f(\frac{a+b}{2})} = b - a \cdot \qquad (11)$$

If the arbitrator's preference scheme obeys the uniform distribution on (a,b), then the equilibrium asking price of both sides is symmetric with the expectation value $\frac{a+b}{2}$ of the arbitrator's preference scheme, and the difference of asking price increases with the increase of uncertainty $\frac{(b-a)^2}{12}$ of the two sides to the arbitrator's preference scheme.

3.4 Arbitrator's Preferences Subject to Laplace Distribution

Let the arbitrator's preference scheme obey the Laplace distribution with parameters of μ and λ. That is,

$\xi \sim L(\mu,\lambda)$. Its probability density is $f(x) = \dfrac{1}{2\lambda} e^{-\frac{|x-\mu|}{\lambda}}$,

the distribution function is

$$F(x) = \frac{1}{2}[1 + \text{sgn}(x-\mu)(1 - e^{-\frac{|x-\mu|}{\lambda}})]. \text{ We have}$$

$$\frac{\xi_1^* + \xi_2^*}{2} = \mu, \xi_2^* - \xi_1^* = \frac{1}{f(\frac{\xi_1^* + \xi_2^*}{2})} = 2\lambda \cdot \qquad (12)$$

And then the Nash equilibrium bid of the game under Weibull distribution is $\xi_1^* = \mu - \lambda$, $\xi_2^* = \mu + \lambda$.

The equilibrium bid of both parties is centrally symmetric with the expectation value μ of arbitrator's preference scheme, and the difference of bid increases with the increase of uncertainty λ of arbitrator's preference scheme.

3.5 Arbitrator's Preferences Subject to Weibull Distribution

Let the arbitrator's preference scheme obey the Weibull distribution with parameters of λ and k.

Its probability density is $f(x) = \begin{cases} \dfrac{k}{\lambda}(\dfrac{x}{\lambda})^{k-1} e^{-(\frac{x}{\lambda})^k}, x \ge 0 \\ 0, x < 0 \end{cases}$,the distribution function

is $F(x) = \begin{cases} 1 - e^{-(\frac{x}{\lambda})^k}, x \ge 0 \\ 0, x < 0 \end{cases}$ .we have

$$\frac{\xi_1^* + \xi_2^*}{2} = \lambda\sqrt[k]{\ln 2}, \xi_2^* - \xi_1^* = \frac{2\lambda}{k}(\ln 2)^{-\frac{k-1}{k}}. \qquad (13)$$

And then the Nash equilibrium bid of the game under Weibull distribution is

$$\xi_1^* = \lambda\sqrt[k]{\ln 2}[1 - \frac{1}{k}(\ln 2)^{1-k}], \xi_2^* = \lambda\sqrt[k]{\ln 2}[1 + \frac{1}{k}(\ln 2)^{1-k}] \cdot$$

The equilibrium asking price of both parties is centrally symmetric $\lambda\sqrt[k]{\ln 2}$, and the difference of the asking price increases with the increase of uncertainty of the arbitrator's preference scheme.

3.6 Arbitrator's Preference Scheme Subjects to Rayleigh Distribution

Let arbitrator's preference scheme obey Rayleigh distribution with parameter σ, and its probability density is $f(x)=\dfrac{x}{\sigma^2}e^{-\frac{x^2}{2\sigma^2}}, x>0$, the distribution function is

$$F(x)=\begin{cases}1-e^{-\frac{x^2}{2\sigma^2}}, x>0\\ 0, x\leq 0\end{cases}$$ We have $\dfrac{\xi_1^*+\xi_2^*}{2}=\sigma\sqrt{2\ln 2}$,

$\xi_2^*-\xi_1^*=\sqrt{\dfrac{2}{\ln 2}}\sigma$. And then the Nash equilibrium bid of the game under Rayleigh distribution is

$$\xi_1^*=\sigma\sqrt{2\ln 2}(1-\dfrac{1}{2\ln 2}),\ \xi_2^*=\sigma\sqrt{2\ln 2}(1+\dfrac{1}{2\ln 2}).\ (14)$$

3.7 Arbitrator's Preference Scheme Subjects to Pareto Distribution

Let the arbitrator's preference scheme obey Pareto distribution with parameter k, and its probability density is $f(x)=\begin{cases}\dfrac{kC^k}{x^{k+1}}, x\geq C\\ 0, x<C\end{cases}$, the distribution function is

$$F(x)=1-(\dfrac{C}{x})^k.$$ We have $\dfrac{\xi_1^*+\xi_2^*}{2}=C\sqrt[k]{2}$, $\xi_2^*-\xi_1^*=\dfrac{C2^{1+\frac{1}{k}}}{k}$. And then the Nash equilibrium bid of the game under Pareto Distribution is

$$\xi_1^*=C\sqrt[k]{2}(1-\dfrac{1}{k}),\ \xi_2^*=C\sqrt[k]{2}(1+\dfrac{1}{k}).\qquad (15)$$

4. Conclusion

Taking the issue of employee welfare as an example, this paper establishes the arbitration model of the final asking price, and discusses the Nash equilibrium analysis of the game under the seven probability distribution conditions. It can be seen that the arbitration mechanism model of the final asking price breaks the deadlock between the enterprise and the trade union in the interest competition, and urges both sides of the game to ask the price more seriously. The higher the trade union's asking price, the lower the enterprise's asking price. A higher asking price for a worker, a lower offer for a business will generate a higher return once it is selected by an arbitrator, but the possibility of the asking price being selected will be greatly reduced. Finally, the price arbitration gives participants greater uncertainty through the arbitrator's preference scheme, which embodies the principle of coexistence of high risk and high income. Therefore, both parties to the dispute can get better income by rationally judging the arbitrator's preference scheme.

Acknowledgment

References

[1] Yuan Zhiyong, Chen Tieying, Luo Yunfeng. The analysis of bidding strategy based on Nash Equilibrium in the electricity market[J]. Journal Huazhong University of Science and

Technonly (Nature Science Edition),2005, pp. 52-54.DOI: https://doi.org/10.13245/j. hust.2005.01.018

[2] Zhang Shengcui. On the perfection of revocation system of arbitral awards[J]. Journal of Shanghai University of Finance and Economics.pp.39-44.DOI: //https://doi.org/10.16538/j.cnki.jsufe.2012.01.003

[3] Xie Shiyu. Economic game theory, Fudan University Press,2002.

[4] Zhou Jixiang,Wang Yong. Research on a bargaining problem between a disadvantaged wholesaler and a supplier under asymmetric information[J].Journal of Systems Engineering.pp:481-493.DOI:https://doi.org /10.13383/j.cnki.jse.2016.04.006.

[5] Wang Chuanhui, Gong Weifeng, Fang Zhigeng. The Research of the Bargaining Game Model between Developers and Investors of Second Hand House[J]. Chinese Journal of Management Science.pp: 242-246.DOI: https://doi.org/10.16381/j.cnki.issn.1003-207X.2012.sl.039.

[6] Ai Xingzheng, Tang Xiaowo. Study about the Performance of Competing Channel Structure Under Bargaining Power[J]. Journal of Industrial Engineering Management.pp:123-125+133.DOI: https://doi.org/10.13587/j.cnki.jieem.2007.02.026.

[7] Zhao Yanhui. Probability theory and mathematical statistics, Northwest University Press,2014.

[8] Robert Gibbons.Game theory, China Science Press,1999.