Experience of Mathematical Modeling of River–Floodplain Groundwater Dynamics

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Abstract— the study of natural geographical zoning is a traditional and well-developed field of physical geography. Zonal landscapes of the planet are well-classified and have clear diagnostic features. The situation is extremely different from intrazonal landscapes. The criteria for their differentiation are not so obvious. Despite the objective differences in the landscapes of floodplains of forest and steppe natural zones, the principles of their differentiation and classification have not been developed yet. The most important factor in the functioning of floodplain landscapes is the seasonal dynamics of groundwater. The annual series of observations allows displaying graphically this dynamics in the form of combined transverse profiles of river-floodplains, reflecting the relief, the level of surface waters and the changing position of the upper boundary of groundwaters. Two-dimensional figures on the profiles are subjected to mathematical processing and allow developing mathematical models of annual groundwater dynamics. Such models can serve as a basis to classify the intrazonal landscapes of river-floodplains. The study aims to attempt to estimate objectively the current situation of natural processes in the river-floodplains of the arid zone, basing on precise quantitative characteristic features which are comparable to each other.

Keywords: groundwater level, floodplain, intrazonal landscape, arid zone, mathematical modelling

I. INTRODUCTION

The study of natural geographical zoning is a traditional and well-developed field of physical geography. Zonal landscapes of the planet are well-classified and have clear diagnostic features. The situation is extremely different from intrazonal landscapes. Even the very formulation of the question of any typology, classification of such landscapes in the scientific literature is highly rare. In the present research, we decided to deal with the river-floodplains, the most common type of intrazonal landscapes, as the object of the classification. It is due to there is no typology of floodplain landscapes based on objective quantitative criteria. Meanwhile, the floodplains of large rivers have a length of thousands of kilometres, and, of course, must be differentiated in some way.
of groundwater is reflected as a change in the area of the hatched figure in figure 1. Similarly, with the changing reservoir volumes, the figure can be called a prism of groundwater treatment. The cross-sectional area of this prism can be mathematically expressed as a double integral of functions approximating the curves of the maximum and minimum position of the LGW. Combining several LGW profiles for one test site and the interpolation in the geoinformation environment will allow obtaining groundwater surfaces in the period of high water and low water, calculation of the prism volume from the groundwater drawdowning is possible by subtracting these surfaces. This approach allows us to formalize the results of observations, to use mathematical tools for their processing and to obtain objective quantitative values [6]. The main purpose of the research was the selection of mathematical tools for modelling the annual dynamics of groundwater in river-floodplains.

![Fig. 1. The combined profile of the groundwater mirror relief. I - the position of the groundwater mirror in the period of high water, II - the position of the groundwater mirror in the low-water period](image)

II. MATERIALS AND METHODS (THE MODEL)

The main method of the field studies was the traditional hydrogeological profiling. In the area, there were shutters crossing the floodplain array which were perpendicular to the direction of the main streams. Elevation marks along the profile line were determined by the Sokkia GSR1700 CSX geodetic GPS/GLONASS system. The depth of groundwaters was determined by means of a mobile geophysical instrument - GPR OKO-2 with antenna unit 150/400 MHz. The main feature of the method consists in the emission of pulses of electromagnetic waves and registration of the signals reflected from the interface of the layers of the probed medium having different electro-physical properties. Such boundary levels in the studied media are, for example, the contacts between dry and moisture-saturated soils (groundwater level), the contacts between rocks of different lithological composition, between the rock and the material of artificial structures, between frozen and thawed soils, between bedrock and loose rocks.

GPR allows determining reliably the position of the groundwater mirror due to the large difference in the dielectric constant of the properties of dry and water-saturated soils. The depth of GPR scanning of the soil is 12-15 meters, which significantly exceeds the real depth of groundwaters in river-floodplains (up to 6-7 meters). Such profiling also allows determining the composition and stratigraphy of soils along the profile line. Along the line of each profile, 1-2 control wells were drilled, which are necessary for calibration of the device and sampling of groundwaters. Vegetation and soils were described along the profile line [7].

The site in the southern part of the Volga-Akhtuba floodplain (Kharabalsinsky district of the Astrakhan region, Russia), which has the highest repeatability of observations - from April 2017 to April 2019, was selected as the object of mathematical modelling of groundwater dynamics. Observations were carried out in different phases of the hydrological year, which allows characterizing fully the dynamics of groundwater levels.

The nearest reservoirs are the arm of the Volga - the Akhtuba, and two canals - Tserkovny and Molochny, flowing from the Akhtuba. The Tserkovny canal restricts the duct lined with an array of the fields of the South, the Molochny canal - from the North.

The profile has a length of 3.4 km and it is located from the West to the East, crossing the floodplain array from the Akhtuba arm to a vast meadow lowland, flooded in high water. In 3 km to the North of the profile line, there is Khutor Gremuchy. These reservoirs, surrounding the site of observations, participate in the replenishment of groundwater in high water and drain the site in the low water. The profile crosses the natural levee with a height to 7.5 meters above the low-water level and the vast space of the central floodplain with the indicators of 4.5-4.8 meters above the low-water level. The vegetation in the near-forest zone is represented by communities of several species of tree willows (Salix sp.). The central floodplain is occupied by meadow grass vegetation.

The Volga-Akhtuba floodplain is well-studied in the landscape, hydrological and geocological terms [8-11]. This is the largest river-floodplain in Europe, its width in some places reaches 35 kilometres. Downstream the Volga River the floodplain extends from the city of Volgograd for 350 km, and in the lower it reaches the Volga Delta. Above Volgograd, the floodplain of the Volga is completely flooded during the construction of a cascade of hydroelectric power plant.

The capacity of the modern alluvium of the Volga-Akhtuba floodplain reaches more than 30 meters. A characteristic feature is the predominance in the section of the floodplain of the channel facies of alluvium and a slight development of the old and floodplain facies. Perhaps, this is due to that the area is located in the tectonically active zone and the floodplain facies are eroded. The power of the floodplain facies sometimes reaches 7-8 m, the old river-beds facies - up to 6 meters. The channel facies of alluvium are medium-grained sands with high filtration coefficient. In this regard, the groundwater level can change quite quickly when the surface water level rises or decreases [12].

Based on the results of the field studies in the Microsoft Excel program the profiles of the studied hypsometric shutters were built, which were further combined with the curves reflecting the depth of groundwater in different periods of the year. The obtained and combined graphs became the subject of mathematical processing by Verner Graphical Analysis software.
III. RESULTS AND DISCUSSION

In order to determine the area of the vertical cross-section of the prism of the groundwater treatment, it is necessary to approximate the LGW curves with mathematical functions and to determine the area bounded by the graphs of these functions. The area can be defined by an integral of the form:

\[
\int_{x_1}^{x_2} (f(x) - g(x)) \, dx
\]

where: \(f(x)\) - the curve of the LGW maximum, \(g(x)\) - the curve of the LGW minimum, \(x_1, x_2\) - the beginning and end of the profile (the section profile), respectively.

In addition to the determining of the cross-sectional area of the groundwater prism it is possible to analyze the dynamics of groundwater levels on the basis of formulas: both the spatial one, along the profile, and the temporal one, based on a comparison of functions describing the position of groundwater over a year or several years. Also, the presence of the function allows interpolating with more accuracy the intermediate values of LGW obtained both on the basis of geophysical methods and the wells and delves. Under conditions of the Volga-Akhtuba floodplain, the curves of groundwater levels are approximated by sigmoids.

In the Verner Graphical Analysis program, the coefficient values of this equation were selected by the actual LGW marks.

Figure 2 presents the difference between the actual (or markers) and expected groundwater levels for different observation periods (Fig. 2). Actual (or markers) and expected groundwater levels for different observation periods

The curve of the correlation between the maximum and minimum of the LGW marks along the profile line during the entire observation period is of great interest (Fig. 3). Maximum fluctuations in the groundwater levels are observed at a distance of up to 200 m from the river's encroachment line. The amplitude of groundwater level fluctuations will be about 6.5-7 m. Due to this, the riverside is occupied mainly by Willow trees, as Ashes and American maple species are not adapted to the large fluctuations and the high LGW.

The vertical cross-sectional area of the groundwater treatment prism is shown in figure 4. In the floodplain flood zone (up to the distance of 230 m from the beginning of the profile) the cross-sectional area will be determined as the integral of the difference between the relief curves and the minimum LGW (the dark and grey area), and further along the profile - the difference between the maximum and minimum groundwater levels (the light and grey area).

The points of intersection of the relief curve with the curves of the maximum and minimum levels of GW are determined based on equations:

\[
h(x) = f(x) \text{ and } h(x) = g(x)
\]

where \(h(x)\) - the relief curve, \(f(x)\) - the maximum groundwater level curve, and \(g(x)\) - the minimum groundwater level curve.

IV. CONCLUSION

Under modern conditions, with a significant increase in the resources used and the impact on the environment, with huge information which must be taken into account, the traditional and empirical methods of decision-making prove to be insufficient. The development of science and economy should be based on the development of new management methods and the introduction of new technologies and the use of effective research methods. Such effective methods should include the mathematization of research.

Mathematization of research involves, first of all, obtaining a mathematical model which describes with the accuracy and adequacy this study process. Upon the availability of a model, it is possible to replace the further study of the process with the analysis of its mathematical model in order to obtain solutions to particular tasks [13]. In this case, the developed models serve as a basis for forecasting natural processes.

There are other applications for mathematical models. The results obtained in the modelling of the natural process can serve as a basis for classification of objects. This
approach is close to the application of quantitative cladistics methods in biological taxonomy [14]. In our opinion, this approach can find application in the development of typology of intrazonal landscapes. This problem is difficult to be solved by using traditional methods of physical and geographical zoning. In the landscape formation of the river-floodplains, one of the most significant factors (in this case, it is the abundance of moisture) smooths the influence of other factors of landscape formation [15]. Therefore, it is very difficult to differentiate the landscapes of river-floodplains, despite their considerable length. There are no obvious criteria for defining natural boundaries (for example, the presence or absence of forests). For river-floodplains, the criterion of landscape differentiation can serve as indicators of groundwater dynamics, in particular, mathematical models of the prism of groundwater treatment during a year.

At the first stage of the research, the experimental fields were selected to obtain a representative profile of the relief and the position of the groundwater mirror during a year. In accordance with generally accepted requirements, the influence of anthropogenic factors of moisture, in particular, irrigation of crops, was excluded from the fields [16]. Two-year observations of the groundwaters have provided a comprehensive picture of their dynamics and relationship to the surface water regime. The results of this research were reflected graphically in the form of profiles. At the next stage, with the help of the Verner Graphical Analysis program, the adequate mathematical models describing the process were selected. Verification of the models confirmed a high degree of their reliability (the correlation coefficient of the expected and actual indicators was 0.88-0.98). Such adequacy of the model is quite sufficient for modelling hydrogeological processes [17]. An important result of the study was the possibility of differentiation of the prism shape into 2 zones – the riverside and the inner floodplain (Fig. 4). It gives the possibility to analyze in details the shapes of prism treatment.

The next stage of the study must be the comparative characteristics of river-floodplains of different natural zones and regions, based on the proposed approach. In combination with other criteria (for example, humus content in soils, species composition of tree and shrub vegetation), this will make it possible to differentiate the landscapes of river-floodplains and to develop schemes for intrazonal landscapes division.

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REFERENCES


