

STEM-Oriented Mathematics Learning with GeoGebra

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Abstract— This paper presents a series of GeoGebra-based mathematical applet (*mathlets*) for science and math subjects, developed for elementary, middle, high school, and Calculus students. The *mathlets* were developed in order to fulfill requirements for qualified mathematics learning as expected by the K-13 curriculum. Almost all of the *mathlets* have been developed by using scientific phenomenon or real-life as their background. By doing this, it was expected that students will be more motivated in learning mathematics. By using scientific phenomenon as a background for the *mathlets*, we tried to show students an integrated STEM subject by implementing the scientific approach asked by the K-13 curriculum. The results showed that students' engagement and achievement are increased significantly, due to the fact that the *mathlets* have made students actively engaged during the learning process, doing exploration, and construction of the mathematical concepts.

Keywords—*GeoGebra mathlet; engagement; exploration; concepts construction; K-13 curriculum; STEM.*

I. INTRODUCTION

Movement in the improvement of Educational Quality has occurred in every country, including Indonesia. Some countries have succeeded in achieving this improvement, while some others have failed. One of the countries which is not successful in doing this improvement is Indonesia. We have been outperformed by Vietnam in the quality of mathematics education.

Our government has done many efforts in improving the quality of mathematics education. The last effort is the implementation of The Curriculum 2013 (K-13) which focuses on the employment of scientific approach. K-13 is essentially similar to KTSP in the sense that both curriculum use the core concept of scientific approach to deliver mathematics learning, namely the implementation of EEC (Exploration, Elaboration, and Confirmation).

Further development in education that recently taking much of our attention is the so called STEM Education. STEM Education is an abbreviation of Science, Technology, Engineering, and Mathematics Education. It is proposed because of the continuous decline in students' interest to these STEM subject. Many countries start to worry to this interest decline due to the fact that our life becomes more and more controlled by technology, and STEM subjects are the core

subjects that nurture this technology abilities. STEM and K-13, are quite similar. Both are trying to integrate some of STEM subjects, especially science and mathematics. K-13 uses the scientific approach to do the integration, while a lot of STEM proponents propose the process of engineering design to do the integration. However, the concern of this paper was more about the difficulties faced in the integration. The main cause of this was the inabilities of the teachers to do this integration. Teachers are not ready to do the integration, because they mainly teach about science and mathematics mono disciplinary (silo approach).

The teachers, in general, are not only unable to integrate STEM subjects, especially science and mathematics but also teach by using an old paradigm where they consider students as objects. Quite the contrary, the new paradigm considers students as subjects of learning with teachers are merely facilitators and motivators for the process of learning. Being a facilitator and motivator seem more difficult than being an instructor. In the new learning paradigm, students should construct the concepts of mathematics based on their own prior understanding in order to make the concept meaningful and understandable.

II. INTEGRATING MATHEMATICS WITH SCIENCE OR WITH REAL-LIFE PROBLEMS

In the last part of the previous section, it is stated that mathematics teachers still have difficulties in making their students able to construct mathematics concepts they learnt. In order to make students get interested in constructing the concepts they learnt, teachers should be able to contextualize the concepts, probably by using students' prior knowledge, other subjects, or using its real-life applications. In other words, to make qualified mathematics learning, teachers eventually need to integrate mathematics and other subjects, mainly sciences, which is something like STEM education.

There are several methods that can be used to integrate Mathematics and other STEM subjects. One method of integration is the use of Standard for Mathematical Practice, having their roots in mathematics, to bridge STEM-related practices [1]. According to this approach, mathematics teachers only need to teach the main purpose of learning

mathematics that is to acquire common habits of mind and use it to link mathematics and other STEM subjects. The Common Core State Standards for Mathematics (CCSS) and Standards for Mathematical Practice provides an ideal framework by which STEM teachers can infuse essential practices that promote reasoning, communication, problem solving, and using appropriate tools to support and justify thinking (NGA Center & CCSSO, 2010). Specifically the Standards for Mathematical Practice (SMP) are:

1. Making sense of problems and persevering in solving them,
2. Reasoning abstractly and quantitatively,
3. Constructing viable argument and critiquing the reasoning of others,
4. Modelling with mathematics,
5. Using appropriate tools strategically,
6. Attending to precision,
7. Looking for and making use of structure, and
8. Looking for and expressing regularity in repeated reasoning.

Although the above SMP is originally framed around NCTM’s Principle and Standard for School Mathematics (2000), it fundamentally extends beyond mathematics and can be used to bridge STEM-related disciplines.

The use of SMP as a guiding framework around which teachers can support STEM thinking skills, the need to have extensive content knowledge in numerous and specific STEM area can be avoided. However, the minimum basic knowledge in other subjects of Science, Technology, or Engineering for Mathematics teachers are still needed and are of great value in teaching mathematics.

Another way to integrate STEM subjects is due to [2]. In the following Table I, there are four types of STEM integration, which are ordered in increasing level. Based on the level of our teachers’ competence, it is quite difficult to ask them to implement the 4th type of integration, i.e. trans-disciplinary. The K-13 mainly asks teachers to apply a multidisciplinary approach, probably interdisciplinary as well. However, even these two approaches of basic integration are difficult to be carried out by our teachers, because of their lack of knowledge and skills. A lot of teachers have very limited knowledge and skills even for his/her specialized subject, and mainly do not have any concerns about other subjects differing from their own. So, asking them to apply mathematics to other subjects seems unlikely. We have to refresh and upgrade their knowledge, especially sciences and mathematics, showing how to integrate those STEM-related subjects, before asking them to integrate them.

TABLE I. INCREASING LEVEL OF INTEGRATION

Form of integration	Features
1. Disciplinary	Concepts and skills are learnt separately in each discipline.
2. Multidisciplinary	Concepts and skills are learnt separately in each discipline, but within a common theme. This idea is in the same line with K-13 where in many occasion endorsed a thematic approach.
3. Interdisciplinary	Closely linked concepts and skills are learnt from two or more disciplines with the aim of deepening knowledge and skills.
4. Trans-disciplinary	Knowledge and skills from two or more disciplines are applied to real world problems and projects, thus helping to shape the learning experience. This approach, in our case, is applied in the course “Developing mathematics learning media using ICT.”

The only feasible way our teachers capable of doing is by applying the method of integration proposed in [1], that is by using the CCSS for mathematics and SMP to bridge all or at least two of the STEM-related subjects.

III. EXPERIENCE IN INTEGRATING MATHEMATICS AND OTHER STEM-RELATED SUBJECTS (CASES TAKEN FROM STUDENTS OF MATHEMATICS DEPARTMENT OF UNDIKSHA)

At the Department of Mathematics Education of Undiksha, there is a subject called “ICT-Based Mathematics Learning Media.” The goal of this subject is to train students to be able to develop a learning media using GeoGebra as the main software. This course is really an integration of mathematics and other subjects, especially science. This subject used The Project Based Learning approach as its method of learning. The products applet in the form of GeoGebra applets that the students built were: (i) explorative applet of a box built from a square plate by cutting its four corner, (ii) explorative applet of minimum length of mirror needed so that we can see the whole of our body in it, (iii) explorative applet of a spring oscillations complete with its swing animation, and (iv) explorative applet to determine the height of an object (in this case the statue of Singaraja landmark) by using a triangle similarities.

On one occasion, the students were asked to solve the following problem using GeoGebra.

Given ΔABC with sides $AB = 4$, $BC = 5$, and $CA = 6$. A point P is located at side AB with $AP = 1$. Now, if the vertices Q and R of ΔPQR are consecutively at sides BC and CA , what is the minimum perimeter of ΔPQR ? Explain why your result is minimum.

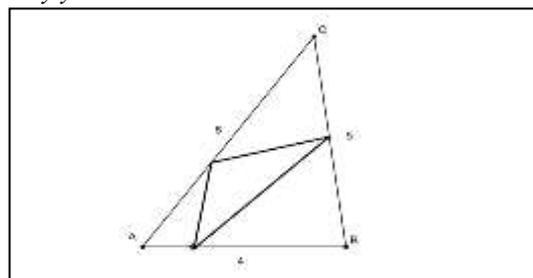


Fig.1. ΔABC , with $AB = 4$, $BC = 5$, $CA = 6$, P on AB with AP

To our surprise, students still did not succeed even in *constructing* the triangle with the specified dimension. This fact showed that students' understanding of mathematical concept was very limited, even for the basic one. Instead of using two circles to construct BC and CA with specified length of radii accurately, they drew segment BC with a length of 5, and segment AX with a length of 6, and then tried to make X and C coincide, and named it C. It is so obvious that to make X and C coincide was very difficult, and if it was achieved, it is very easy to be separated, since it was not "glued." This fact showed that students' understanding for these basic concepts is very low. One good student solved the problem of determining the $\triangle ABC$ analytically, by first determining all the angle sizes by using cosine rule. For the minimum perimeter, although they were able to determine the minimum value by using trial and error method, they were not able to explain why the result was correct. Apart from the unsuccessful effort in solving the problem, the problem was really rich. It connected a lot of concepts, from the most basic to quite advanced ones.

Before students developed their applets, the content material related to the applets which was going to be built were reviewed. For the problem of determining the minimum length of mirror needed to make us able to see the whole part of our body in it, there was no students could answer and explain it.

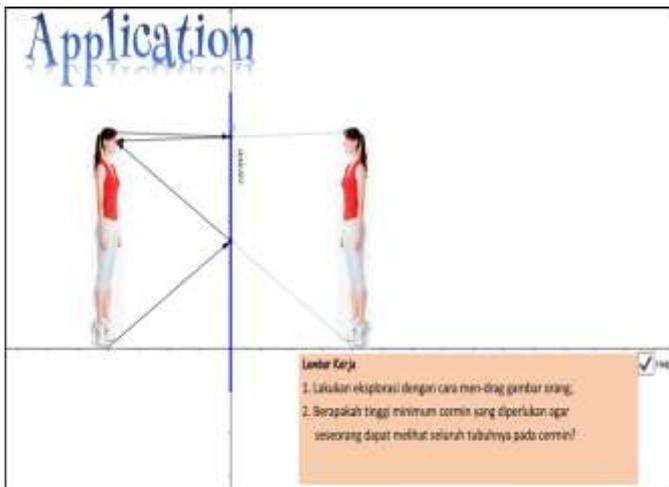


Fig. 2. Minimum Length of Mirror Needed.

Most students answered this question misconceptually, that is the length of mirror needed to see all part of our body in it is similar to the length of our body. This answer of course is incorrect. Although the students were able to guess the answer of the question but, they were not able to explain why the result was like that. The most interesting problem was how to construct a box from a square plate by cutting its corner. This problem had been used in Elementary school in Singapore [1]. Of course elementary students were not asked to determine the maximum box-volume that can be obtained by using this construction, because these students did not know yet about Calculus or graphics as a visual tool. But for secondary school students, it is possible to ask them what is the maximum box-volume obtained visually. This was the applet that our semester VI mathematics students was asked to build. In detail, the final applet of our semester VI mathematics students asked to build is as follows (see Fig. 3). By sliding the slider *a*, size of the cut can be changed and in turn will also change the box-volume. Simultaneously, this volume will be plotted on the third window (right-most window) in Fig. 3. By observing this simultaneous changes, students of secondary school will be able to determine for what value of *a* the box-volume will be maximum.

As probably can be guessed, no student was successful in building this rather complex applet. However, students had learnt a lot about how to build an explorative mathematical applet (mathlet). This can be seen from their successful effort in building other applets about maximum and minimum problems.

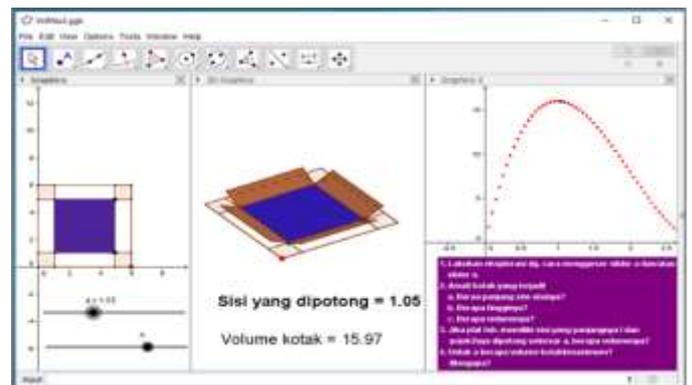


Fig. 3. A Box Constructed from a Plate

Mathlet that clearly relates science and mathematics is the mathlet about a spring oscillations. In this task, the students were asked to apply their knowledge of Differential Equation to model the oscillation of a spring and then to build a mathlet that can be animated to visualize the result. Prior to build this mathlet, students actually have known solutions of the second order ordinary differential equation. However, they do not know that the solutions are related to the oscillation of a spring bob. It is also clear from Fig. 4, that initial conditions for the spring can be changed freely and see its effects on the oscillation.

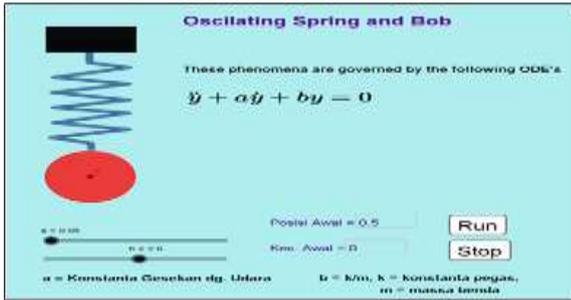


Fig. 4. Mathlet of A Spring Oscillations

Students really had difficulty in coping with this problem, both in building the mathlet and in interpreting the oscillation showed by the animation. Although they roughly remembered how to solve the ODEs, they did not know how to use the analytical results in the construction of the GeGebra mathlet, and finally made the construction oscillates. In interpreting the results (oscillations), they were not able to interpret meaningfully, why the mathlet oscillates forever if $a = 0$ and practically will stop if $a \neq 0$. Also, they were not able to make a relation between oscillation and the analytical solutions, so they were not able to explain why, for certain values of a and b , there is no oscillation at all. This results certainly show us that integrating science and math is really hard, even for students that just finished their course related to science.

IV. THE USE OF GEOGEBRA MATHLET IN LEARNING MATHEMATICS

Contrary to those unsuccessful stories in building mathematical applets, the use of applet in learning mathematics to increase their engagement and achievement both for secondary and college students have been successful. In secondary school, the use of mathlet has been successful in improving students' understanding about the concept of position of two lines. In the research by [3], the following mathlet (Fig. 5) had been used to improve students understanding about gradient of lines and their relationship to position of two lines.

By using the mathlet, students were easily able to predict what happens to the graph of the lines if their gradients are positives, negatives, or zeros. Conventionally, students would be forced un-meaningfully to memorize that if a line has a positive gradient then its graph will be increased up-rightly, while a line with negative gradients will have a graph direction of increasing up-left. But, by using the mathlet, students can easily conclude through explorations that the afore-mentioned results are correct. Of course, the conclusions should be derived by students, not given by teacher in order to be memorized by students. The mathlet was also had been used to explore conditions for two lines to be parallel, intersect to each other, and intersect perpendicularly. The mathlet was accompanied by a sign of perpendicular intersection, so that

students could see whether the lines have been intersecting perpendicularly or not during the explorations. Some prompting questions had been written on the left side of the mathlet to guide students' exploration. As students do their exploration by sliding the sliders, the product of gradients of those two lines are also changed accordingly. Through this simultaneous dynamic visualisation, i.e. continuous changing results of the product of gradients and the continuous changing results of the angle between the two lines, students will be able to see the condition that should be satisfied in order to make two lines intersect perpendicularly to each other.

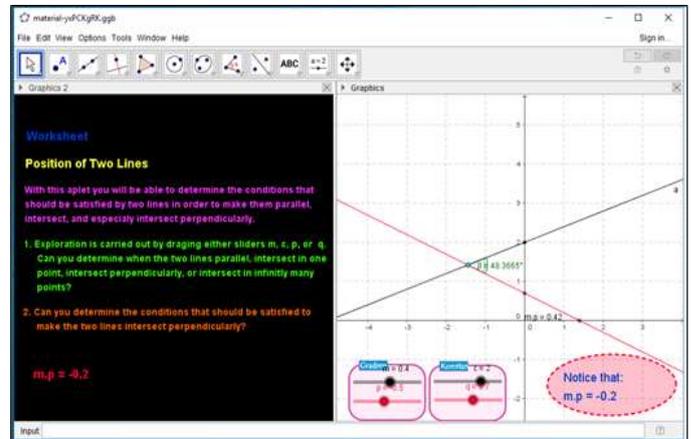


Fig. 5. Position of Two Lines.

Another use of mathlet is in learning of triangle similarity. As a context for this concept, we brought a problem of determining the height of Singaraja statue, which is a landmark of Singaraja and it is situated at the centre of Singaraja city. Hence, this is a Problem Based Learning using a problem of the week approach (PoW). This problem is supposed to be solved within the week, and during that week all concepts which is required to solve the problem are discussed. This problem is not only related to the concept of triangle similarity, but also the concept of scale, because bringing real situations to the class will certainly involve scaling. For example, the distance between the foot of the statue and the place where somebody standing up determine the angle of elevation of the statue, is certainly measured in meters. But bringing that distance (in meters) to a picture on a piece of paper will certainly need some knowledge of scaling.

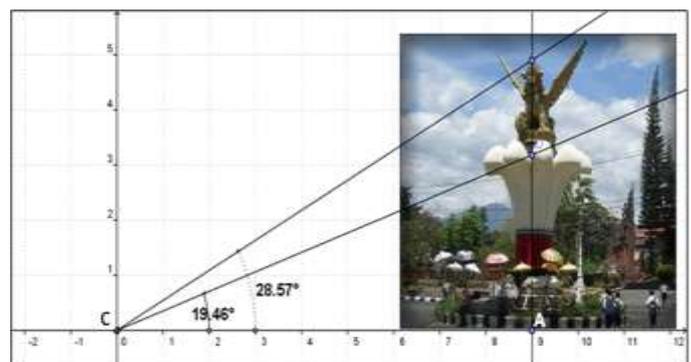


Figure 6. The City Landmark of Singaraja

In essence, this problem is a quite complex problem for students of secondary school, but it is very interesting because it shows real application, challenging enough, and within the grasp of secondary students.

This mathlet has been successful in making mathematics interesting and increase students' engagement. This is reflected on the questioner results taken at the end of the experimental study.

Using this kind of mathlet, we try to show that mathematics is useful, because it can be used to solve real problem, related to other subject, for instance technology since it involves computer, and also science [4]. This is really the core of STEM Education goals.

On the college level, mathlet also has been used to improve learning quality of the pre-service teacher of the third semester of mathematics students. We used a mathlet during Analytic Geometry lecture. This mathlet is intended to improve the understanding of the concept of locus of points satisfying a certain set rule [5]. So far, the learning of locus point has been mainly verbal. Students rarely have been shown whether or not a mathematical equation that they derived as a locus of points is really depicting a curve intended. For example, below is the definition of parabola taken from a book of Analytic Geometry.

Parabola is a locus of points having the same distance from a (fixed) point, called focus, and a (fixed) line, called directrix.

Analytically, with the help from lecturer, it is not very difficult to derive an equation that satisfies the definition given above. For example, this derivation will be obtained:

1. Let's the focus has a coordinates of $F(k, 0)$,
2. Let's the directrix line has an equation of $d: x = -k$,
3. Then an arbitrary point $P(x, y)$ will satisfies (according to the definition):

$$\begin{aligned} \sqrt{(x - k)^2 + y^2} &= \sqrt{(x + k)^2} \\ \Leftrightarrow x^2 - 2kx + y^2 + k^2 &= x^2 + 2kx + k^2 \\ \Leftrightarrow y^2 &= 4kx \end{aligned}$$

Equation of $y^2 = 4kx$ is equations of a set of family of parabola. In Fig.7. some of the members of this family have been drawn for various values of k .

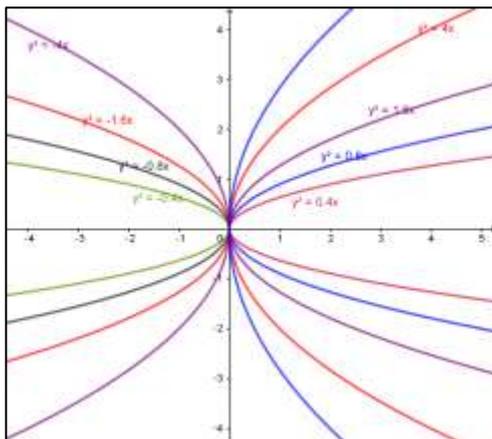


Fig. 7. $y^2 = 4kx$ for various valued of k .

But, the conditions given on the derivation for the focus F and the directrix d have been very restricted, namely the focus F has a coordinates of $F(k, 0)$ while the directrix d has an equation of $x = -k$. These taken values of course are very restricted. What about other values? Or using the *mathematical habit of mind* question: *What if the values of F and d are different with those given above?*

This is a kind of question we are trying to explore by using a GeoGebra mathlet. Besides, students of mathematics, or in general mathematics teachers, have been very familiar with only one type of mathematical thinking, namely *algebraic thinking*. Even some of them may not know that there are other kinds of thinking in mathematics, i.e. *visual thinking*. This visual type of thinking have been endorsed by some educator, since human being have many kinds of intelligences, and each child has his/her own strength in some of this intelligences. Because of this, a teacher should accommodate this strength in order to make his/her teaching understood by his/her students.

It is due to this rational, we try to use mathlet to visualize whether or not the definition of the parabola really produce a parabola. We asked students to develop a GeoGebra mathlet to show that the definition indeed produces a parabola. It turn out that this task is quite difficult for them. They do not know how to translate the definition of parabola into the GeoGebra process of construction. This is probably a kind of new question to them. The process is quite easy: (i) fix a focus F , fix a direction line d , and try to construct a point P that satisfies the definition. After that, move P and record its traces. This traces will form a curve of parabola.

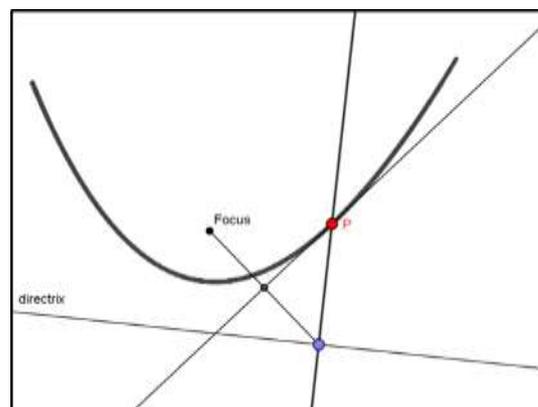


Fig.8. The Parabola.

Different from those derived previously, the focus F and the directrix d in this construction are really very general; there are no restrictions for the F and the d . This generality is also depicted on the graph of the parabola $y^2 = 4kx$ and the graph constructed directly from the definition. Comparing

Fig. 7 and Fig. 8, one certainly can see obviously that the parabola in Fig. 8 is more general than the parabola in Fig. 7.

So, with the help of GeoGebra mathlet, the definition really produces a parabola, and in fact a very general parabola. Without the help of the computer, in this case, GeoGebra mathlet, it is unlikely we can plot Fig. 8 since it needs a very extensive calculation and drawing. It was this dynamic geometry we wanted students to see by their selves. Although the process of constructing the applet was quite complex, this kind of learning approach (again it is a problem-based learning approach) made the concept of parabola becomes problematic, interesting, rich, and connected to other concepts.

V. CONCLUSIONS

It has been argued in this paper that the learning process required to be implemented in the K-13 curriculum, which is the scientific approach, is in line with STEM Education. K-13 requires teachers to use a multidisciplinary approach for his/her learning. This was the second method of integration in STEM education proposed by [2]. In elementary school, this type of integration was done by developing a theme for a set of subjects. For example there a theme called Environment, covering the subject of Natural Sciences, Mathematics, and Social Sciences. In secondary school, the integration process can be done by using the scientific approach asked by the K-13 curriculum. The scientific approach in the K-13 curriculum consists of observing, questioning, experimenting, analyzing, and communicating. These components are very similar to those endorsed in STEM Education, namely the engineering process.

For example, in the process of delivering the concept of triangle similarity, we have used a STEM approach to teach it. We started by bringing a real phenomenon to students, that is the height of Singaraja statue. From that, the students were asked to think about how to solve the problem. During the process of solving the problem, students learning the concepts needed, whether from the book, the teacher or the internet. We also introduce the GeoGebra mathlet to be explored. The process of exploration essentially involves some of the scientific processes. And finally, in the end, we asked students to present their findings; this is a communication process and also a kind of publication process.

The difficulty teachers faced in implementing the K-13 curriculum was basically in the process of developing all the learning tools needed and in the process of developing a learning scenario to make the learning meaningful. This difficulty was due to the lack of teachers' knowledge about an appropriate natural phenomenon suited for the concept being

taught (S in STEM), and about how to integrate them using technology (E and T in STEM).

Due to these obstacles, [1] proposed to use the Standard Mathematical Practice (SMP) as a shared structure for STEM learning. This structure is originally framed around NCTM's Principle and Standard for School Mathematics, but now it goes beyond Mathematics and becomes a common standard that is used to bridge the STEM discipline. The standard mathematical practice is basically a standard of a good process for learning mathematics which is also quite similar to the 5 processes in the K-13 curriculum, namely *observing, questioning, experimenting, analyzing, and communicating*. By using the SMP approach for integrating mathematics and other STEM subjects, or by using the scientific approach of K-13, mathematics teachers do not need to have extensive knowledge about other STEM subjects because they only need to do a qualified process of teaching mathematics. So, in short, what we need to improve the quality of students' mathematics learning similar to that of STEM, is to improve the capability of our teachers to deliver what is asked by the K-13 curriculum, since the approach taken by the K-13 curriculum is quite similar to the process of STEM integration. One feasible way to help the integration process is to use mathlets during the learning process. By using the explorative GeoGebra mathlets, a mathematical concept is no longer just a series of formulae that should be memorized by students, but they become goals that should be achieved and constructed by students. The used of GeoGebra mathlets certainly familiarize students the scientific process, the engineering process, and improve the learning process, so that students' proficiencies and students' interest in STEM-related subjects and career can be improved.

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