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The Calculation of the Sales Volumes Flow Based on the Game-Theoretic Model

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***Abstract*—In this article, we have developed a formal decision-making model for an enterprise to market a single product in the future. The model is built in the form of an antagonistic two-person game. The payoff function is the profit of the enterprise. We took into account the limitedness of the target segment and the supply volume of the enterprise competitors. Sales volumes were calculated by averaging the optimal risk-free and acceptable risk values. We used the forecasts of the probability distributions of competitive bids as information on the volume of such bids. As a result of the model application, we calculated the recommended sales volumes of the product and profit forecasts for the enterprise at several promising trading periods.**

***Index Terms*—decision making, antagonistic game, uncertainty, product sales, sales volume flow, sales forecasting**

I. Introduction

When marketing products a situation of uncertainty in forecasting economic results is typical. Researchers use different approaches to account for uncertainty when building a sales flow in the future [1], [2]. Research can be divided into two types: theoretical models and models of technical analysis. Theoretical models are based on the study of deterministic functions, differential equations and Markov processes. Technical analysis models use regression analysis, time series analysis, optimization methods, neural networks, simulation modeling.

The problem formulation of the firms's game in the duopoly model in the classical form is considered in the article [3]. In the study [4], the authors proposed a deterministic sales model for some innovative goods, taking into account the rate at which information about a new product arrives, the change in the purchasing power of potential consumers depending on the price and the interval of the time between the receipt of information about the product and its actual purchase. In the article [5] sales models based on Markov processes are built.

Traditional models of technical analysis are characterized by extrapolation of the statistical results of the study of the sales process history, such as in [6] research. The disadvantage of this approach is the absence of the model that reveals the mechanism of the sales factors interrelation. This is an obstacle to transparent decision making.

Recently, works have appeared in which the authors propose to take into account the influence of various marketing factors on its volume. For example, the authors of [7] proposed a model of non-differentiable optimization, which takes into account the price elasticity of demand and the impact of promotions. In [8], the authors studied the effect of promotions on sales of consumer goods. In the [9] article, the authors constructed a regression demand model, in which key explanatory variables are selected from a very large set of data, and including intra-category and cross-category advertising effects. In the studies [10] and [11], machine learning methods were used to build sales forecasting algorithms. The article [12] a developed mathematical model of basic forecasting of sales volumes in short-term periods, the algorithm of multiple branching is described in detail, which is able to implement forecast variants in three scenarios (optimistic, realistic and pessimistic).

In this study, we consider the formal sales process for products of the same type. The main task is to develop a framework for practical modeling of the future sales flow, taking into account the uncertainty of profit forecasting due to competition. Such a framework is required to support decision making by setting the predicted values of sales factors. This contributes to the transparency of the decision-making process due to the fact that the decision maker can realize his preferences through expert adjustment of the process of optimizing the criterion of preference.

As a rule, enterprises do not have accurate information on the cumulative distribution of the competitors' supply volumes for the future (for several trading periods in the future). Consequently, there is uncertainty for the enterprise in assessing future profit. Therefore, it is advisable to build a model of optimizing the distribution over time of the product sales volume by an enterprise for profit forecast, taking into account the available net risk. As a basis for building a model, we will naturally use the game approach, which allows taking into account the risk arising from the direct conflict of interests [13], [14]. Usually the Nash criterion is used to select the optimal solution of the game (for two participants – the Neumann criterion) [15]. At the same time, the forecast of the economic result for the enterprise may be underestimated due to the position of extreme caution. In such cases, one should take into account the decision maker's desire to take risks, taking into account the available information. Then, expert criteria are used to select the optimal solution for the game [15]. In this paper, when calculating the flow of the sales volumes, we will use an analogue of the Hodge-Lehmann criterion.

II. Methodology

Suppose, an enterprise produces and sells a single product P on the target consumer segment S. Implementation occurs in periods. Potential sales of P are equal V to each trading period.

Consumers entering S buy P under the following conditions:

1) the demand is regular;
2) during each trading period, not all consumers make a purchase, that is, there is always a potentially unsatisfied demand;
3) consumers' willingness to pay for goods decreases with increasing supply.

In the segment S, besides the enterprise, direct competitors offer goods P, influencing the selling price and the costs of the enterprise.

Let n be the depth of profit forecast, that is, the number of standard trading periods under consideration. The flow of sales volumes of product P determine the dynamics of sales. We introduce the vector notation for them:

1) $x = colon(x_1, x_2, \ldots, x_n)$ – is the vector of the flow of sales of goods P for the enterprise ($x_i \geq 0$ – is the production and sales for the period with the number i, $i = \overline{1,n}$),
2) $y = colon(y_1, y_2, \ldots, y_n)$ – is the vector of the flow of sales of goods P for competitors ($y_i \geq 0$ – are cumulative sales for the period with a number i, $i = \overline{1,n}$).

Further, we consider all factors to be positive.

The selling price of goods P on the segment S in each period with the number i is directly proportional to the potentially unsatisfied demand: $a_i(V - y_i - x_i)$, where a_i – is the unit price of the unsatisfied demand. From a practical point of view, the coefficients a_i should be evaluated by statistical methods as the predicted characteristics of the segment S. In particular, for a stable demand, $a_i = a = const$.

The costs of the enterprise in each period with the number i are added from the values:

1) $b_i x_i$ – is the cost of production and marketing of its products, where b_i – is the projected cost of production;
2) $c_i(V - x_i)$ – is the cost of expanding the share of the market segment, c_i – is the planned expenditure rate;
3) $d_i y_i - e_i y_i^2$ – the costs of an enterprise to overcome competition with a relatively small supply of competitors are increasing, but with a significant competitive supply, they decrease.

For each trading period with a number i the coefficients c_i, d_i, e_i are set as expert estimates for the decision-making support process or are selected according to the optimization principle.

Enterprise profit for n standard trading periods is equal to

$$
\begin{aligned}
f(x,y) &= \sum_{i=1}^{n} f_i(x_i, y_i) = \sum_{i=1}^{n}(a_i(V - y_i - x_i)x_i) - \\
&- \sum_{i=1}^{n}(b_i x_i + c_i(V - x_i) + (d_i y_i - e_i y_i^2)) = \\
&= -x^T A x + y^T B y - y^T A x + v^T x - u^T y - q, \quad (1) \\
&A = diag(a_1, a_2, \ldots, a_n), \\
&B = diag(b_1, b_2, \ldots, b_n), \\
&u = colon(d_1, d_2, \ldots, d_n),\ q = V(c_1 + \ldots + c_n), \\
&v = colon(a_1 V - b_1 + c_1, \ldots, a_n V - b_n + c_n).
\end{aligned}
$$

By the formula (1) profit depends on the flow of competitive offers of uncertain volume. Therefore, when predicting the profit of the enterprise there is a risk.

To take the risk into account (in particular, for a guaranteed assessment of the profits of the enterprise) it is advisable to apply the game antagonistic model:

- player 1 – is the enterprise, his goal is maximum risk-free profit, pure strategies are vectors x;
- player 2 – is the competitors in the target segment S, acting as one enterprise, his goal is to minimize the profit of player 1, pure strategies are vectors y;
- the win function is given by equality (1).

Let us find out if the entered Neumann game is solvable. The quadratic function (1) is continuously differentiable in $\mathbf{R}^n \times \mathbf{R}^n$. We calculate the Hesse matrix by x and by y. Apply the Sylvester criterion to them (the inequality for the symmetric matrix coincides with the sign of the quadratic form defined by this matrix):

1) $f^{'}_{xx} = -2A < 0$,
2) $f^{'}_{yy} = 2B > 0$.

Hence, the function $f(x,y)$ has the properties:

1) for any y strictly concave in x,
2) for any x strictly convex in y.

Therefore, $f(x,y)$ on any convex compactum of $\mathbf{R}^n \times \mathbf{R}^n$ has a single saddle point [15].

Consequently, if the restrictions on pure strategies are not taken into account, then the functions of the best response of the players $x = x(y) = \arg\max_{x \in \mathbf{R}^n} f(x,y)$ and $y = y(x) = \arg\max_{y \in \mathbf{R}^n} f(x,y)$ are determined by the equalities $f^{'}_x = 0_n$ and $f^{'}_y = 0_n$, respectively. Then the saddle point for $f(x,y)$ is determined by the necessary conditions for the extremum

$$
\begin{cases} f^{'}_x = -2Ax - Ay + v = 0_n, \\ f^{'}_y = -Ax + 2By - u = 0_n. \end{cases} \quad (2)
$$

System (2) is actually a set of linear systems for the sales volume of an enterprise and the totality of its competitors in each trading period:

$$
\begin{cases} -2a_i x_i - a_i y_i + (a_i V - b_i + c_i) = 0, \\ -a_i x_i + 2e_i y_i - d_i = 0, \end{cases} \qquad i = \overline{1,n}
$$

The solution $(\bar{x}, \bar{y}), \bar{x} = colon(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n)$, $\bar{y} = colon(\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_n)$ of system (2) has the form:

$$\bar{x}_i = \frac{2e_i(a_iV - b_i + c_i) - d_ia_i}{a_i(a_i + 4e_i)}, \qquad \bar{y}_i = \frac{a_i\bar{x}_i + d_i}{2e_i}, \quad (3)$$
$$i = \overline{1, n}.$$

Note that equalities (3) define the saddle point of the payoff function (1) without taking into account the constraint on the player 1's pure strategy and under $\bar{y}_i \in \left[0; \frac{d_i}{e_i}\right]$ (this inclusion determines one of the conditions for specifying coefficients d_i, e_i).

The profit $f_i(\bar{x}_i, \bar{y}_i)$ of the enterprise at (3) is a function of c_i, d_i , e_i. It means that we can estimate the recommended values of these parameters by the optimization principle $f_i(\bar{x}_i, \bar{y}_i) \to \max$. In this paper, we do not consider the process of such optimization in detail. We will illustrate only the idea of this process. The value $f_i(\bar{x}_i, \bar{y}_i)$ is a convex quadratic function of the parameter c_i. Inequality $\bar{c}_i = b_ie_i + +a_iV + \frac{a_i^2V + a_id_i}{2e_i} > 0$ is satisfied at the global minimum point, then we have $\min\limits_{c_i} f_i(\bar{x}_i, \bar{y}_i) = -\frac{(a_iV+d_i)^2 + 4b_ie_iV}{4e_i} < 0$. That is, to increase the profit forecast in the trading period i by optimizing the rate of expenditure for expanding the share of the market segment, you should choose either $c_i = 0$ or $c_i > \bar{c}_i$. The value c_i is selected from the conditions

$$\begin{cases} f_i(\bar{x}_i, \bar{y}_i) \to \max, \\ 2e_i(a_iV - b_i + c_i) - d_ia_i \geq 0. \end{cases}$$

If $\bar{x}_i \leq 0$ or $f_i(\bar{x}_i, \bar{y}_i) \leq 0$, then for the enterprise the sale of the product is not advisable. By the Neumann equilibrium condition $f_i(x_i, \bar{y}_i) \leq f_i(\bar{x}_i, \bar{y}_i) \leq 0$ for any $x_i \neq \bar{x}_i$. So, for the trading period i, from the position of extreme caution and for economic reasons, we choose the following estimates:

$$x_{0i} = \begin{cases} \bar{x}_i, & \bar{x}_i > 0 \wedge f_i(\bar{x}_i, \bar{y}_i) > 0 \\ 0, & \bar{x}_i \leq 0 \vee f_i(\bar{x}_i, \bar{y}_i) \leq 0 \end{cases}$$

are the recommended sales P for the enterprise;

$$f_{0i} = \begin{cases} f_i(\bar{x}_i, \bar{y}_i), & \bar{x}_i > 0 \wedge f_i(\bar{x}_i, \bar{y}_i) > 0 \\ 0, & \bar{x}_i \leq 0 \vee f_i(\bar{x}_i, \bar{y}_i) \leq 0 \end{cases}$$

is the assessment of the profit of the enterprise.

It is clear that the risk-free profit forecast obtained from the standpoint of extreme caution may be underestimated. Therefore, it is advisable to correct the recommendations received.

To simulate discreet optimism in calculating the flow of sales volumes, we assume that with probability R_i the company accepts the condition that the function $p_i(y_i)$ is known — it is the probability distribution density of the volume of competitive bid P in the trading period i. In this case, the parameter R_i is chosen expertly and is considered as an acceptable risk or as an indicator of optimism.

It is advisable to choose the function $p_i(y_i)$ according to the extremum principle of informational entropy, taking into account the restrictions on the considered volume $y_i \in Y_i$. Adherence to this principle serves as a compromise between awareness, generating optimism, and accepting the condition of complete uncertainty, leading to a position of extreme caution. Therefore, this approach can serve as a model of restrained optimism. It is natural to consider the following two cases.

Case 1. If it is assumed that $y_i \in Y_i = [y_i^{\min}, y_i^{\max}] \subset [0, \frac{d_i}{e_i}]$, then the uniform distribution over the segment has the greatest entropy. Then

$$p_i(y_i) = \begin{cases} \frac{1}{y_i^{\max} - y_i^{\min}}, & y_i \in [y_i^{\min}, y_i^{\max}] \\ 0, & y_i \notin [y_i^{\min}, y_i^{\max}] \end{cases}$$

Case 2. If $y_i \in Y_i = (0, \infty)$ and m_i is the forecast of the average value y_i then the exponential distribution has the greatest entropy. Then

$$p_i(y_i) = \begin{cases} 0, & y_i \leq 0 \\ \frac{1}{m_i}\exp(-\frac{y_i}{m_i}), & y_i > 0 \end{cases}$$

Taking into account the chosen distribution, it is possible to calculate the average profit of an enterprise

$$F_i(x_i) = \int\limits_{Y_i} f_i(x_i, y_i)p_i(y_i)dy_i$$

in the trading period i. Since,

$$\frac{\partial^2 F_i(x_i)}{\partial x^2} = \int\limits_{Y_i} \frac{\partial^2 f_i(x_i, y_i)}{\partial x^2} p_i(y_i)dy_i =$$
$$= \int\limits_{Y_i} (-2a_i)p_i(y_i)dy_i = -2a_i < 0$$

the function $F_i(x_i)$ has a single global maximum at the point $\bar{\bar{x}}_i$.

So, for the trading period i from the standpoint of restrained optimism in the presence of information and for economic reasons, we choose the following estimates:

$$x_{1i} = \begin{cases} \bar{\bar{x}}_i, & \bar{\bar{x}}_i > 0 \wedge F_i(\bar{\bar{x}}_i) > 0 \\ 0, & \bar{\bar{x}}_i \leq 0 \vee F_i(\bar{\bar{x}}_i) \leq 0 \end{cases}$$

are the recommended sales P for the enterprise;

$$f_{1i} = \begin{cases} F_i(\bar{\bar{x}}_i), & \bar{\bar{x}}_i > 0 \wedge F_i(\bar{\bar{x}}_i) > 0 \\ 0, & \bar{\bar{x}}_i \leq 0 \vee F_i(\bar{\bar{x}}_i) \leq 0 \end{cases}$$

is the assessment of the profit of the enterprise.

Finally, for decision making by an enterprise in the trading period i, the following estimates are recommended:

- $x_i^* = (1 - R_i)x_{0i} + R_ix_{1i}$ – is sales volume P for the enterprise,
- $f_i^* = F_i(x_i^*)$ – is the assessment of the profit of the enterprise.

Then the recommended flow of sales for the enterprise $x^* = colon(x_1^*, \ldots, x_n^*)$, the profit estimate $f^* = \sum\limits_{i=1}^{n} f_i^*$.

Table I
THE CALCULATION OF THE FLOW OF SALES FROM THE POSITION OF EXTREME CAUTION

Number i of the trading period	Parameters determining the profit		Saddle point of the profit function		Optimal price of the game	Optimal risk-free profit of the enterprise	Cautious recommendation of the sales volume for the enterprise
i	a_i	b_i	$\bar{x}_i$	$\bar{y}_i$	$f_i(\bar{x}_i, \bar{y}_i)$	f_{0i}	x_{0i}
1	10	60	65.33	363.33	16040.00	**16040,00**	**65.33**
2	11	62	61.97	370.42	20331.52	**20331.52**	**61.97**
3	12	64	58.93	376.80	24194.13	**24194.13**	**58.93**
4	13	66	56.18	382.57	27689.05	**27689.05**	**56.18**
5	14	68	53.66	387.82	30866.39	**30866.39**	**53.66**
6	15	70	51.36	392.61	33767.54	**33767.54**	**51.36**
7	14	72	53.61	387.64	30651.84	**30651.84**	**53.61**
8	13	74	56.06	382.19	27240.12	**27240.12**	**56.06**
9	12	76	58.73	376.20	23488.13	**23488.13**	**58.73**
10	11	78	61.67	369.58	19342.43	**19342.43**	**61.67**
Total						**253611.16**	

III. EXPERIMENTAL SETUP

The given economic and mathematical substantiation of the model shows its applicability for computer simulation. We will demonstrate how to calculate the flow of sales volume by the proposed method on abstract test data in the Excel package.

First, we define V for the volume of the segment.

To calculate the flow from the position of extreme caution in each trading period i, we will set the predicted or expert estimates of the parameters a_i, b_i, c_i, d_i, e_i (the meaning of the parameters is defined in paragraph 2). After that, in each trading period i:

- using formulas (3) without taking into account economic constraints, we calculate the optimal solution of the antagonistic game $\bar{x}_i$, $\bar{y}_i$, $f_i(\bar{x}_i, \bar{y}_i)$;
- calculate for the company the recommended values of sales x_{1i} and a cautious assessment of profit f_{1i}.

As a result, we will calculate the total profit for all considered trading periods.

As an example, consider the data and results from Table I. Here $V = 500$. The company does not spend money to expand the share of the market segment ($c_i = 0$, $i = \overline{1;10}$). The predicted unit values of unmet demand a_i first grow and then decrease. The projected cost of production b_i is increasing. Expert estimates of the parameters d_i, e_i that determine the model of enterprise expenditures for overcoming competition are stable $d_i = 800$, $e_i = 0$, $i = \overline{1;10}$. In Table I, the constant parameters are not listed.

This example illustrates the complex dynamics that the recommended flow of sales volumes can demonstrate. In the considered case, the profit estimate is clearly correlated with the price of unmet demand. At the same time, the flow of sales volumes shows a reverse trend and is not associated with the dynamics of cost.

To correct the results from a position of restrained optimism depending on the position of the decision maker, we use two approaches. In both cases, we use data for a careful forecast (V, a_i, b_i, c_i, d_i, e_i, $i = \overline{1;n}$).

In the first case, we will set for each trading period i expert estimates $y_i^{\min}$, $y_i^{\max}$ of the boundaries of the value of competitive bidding on a segment and the value of acceptable risk R_i (for example, the Harrington scale can be used). We assume that the distribution of the competitive offer is uniform. After that, in each trading period i:

- calculate the maximum point of the average profit and derive optimistic forecasts of sales volume and average profit for the enterprise;
- we calculate for the enterprise, taking into account the pessimistic and optimistic positions, the weighted average of sales volume x_i^* and average profit f_i^*.

As a result, we calculate the total average profit.

As an example, consider the data and results from Table II. We used the same parameters of the profit function (1) as in Table I. The boundaries of the volume of competitive offers are stable: $y_i^{\min} = 200$, $y_i^{\max} = 400$. The value of acceptable risk decreases with time.

In this example, the dynamics of an optimistic profit forecast is similar to the dynamics of cost (see Table I), and the dynamics of the adjusted flow is similar to the dynamics of a careful assessment of the flow.

In the second case, for each trading period i, we will set an expert estimate m_i of the average value of the competitive offer on a segment and the value of acceptable risk R_i. We assume that the distribution of competitive offers is indicative. After that, in each trading period i:

- calculate the maximum point of the average profit and derive optimistic forecasts of sales volume and average profit for the enterprise;
- we will calculate the weighted average of the sales volume x_i^* and the average profit f_i^* for the company, taking into account the pessimistic and optimistic positions.

As a result, we calculate the total average profit.

As an example, consider the data and results from Table III. We used the same parameters of the profit function (1) as

Table II
CALCULATION OF THE FLOW OF SALES FROM THE POSITION OF RESTRAINED OPTIMISM, WHEN EXPERT ESTIMATES OF THE SIZE OF THE COMPETITIVE OFFER ON THE SEGMENT AND THE AMOUNT OF ACCEPTABLE RISK ARE GIVEN

Number i of the trading period	Cautious forecast for the enterprise		Optimistic forecast for the enterprise		Acceptable risk	Adjusted forecast for the enterprise	
	profit	sales volume	profit	sales volume		profit	sales volume
i	f_{0i}	x_{0i}	f_{1i}	x_{1i}	R_i	f_i^*	x_i^*
1	16040.00	65.33	33466.67	70.00	0.80	**32953.96**	**69.07**
2	20331.52	61.97	41817.67	69.00	0.75	**40694.25**	**67.24**
3	24194.13	58.93	50026.67	68.00	0.70	**48023.01**	**65.28**
4	27689.05	56.18	58087.67	67.00	0.65	**54900.62**	**63.21**
5	30866.39	53.66	65994.67	66.00	0.60	**61291.10**	**61.06**
6	33767.54	51.36	73741.67	65.00	0.55	**67162.52**	**58.86**
7	30651.84	53.61	63914.67	64.00	0.50	**58674.52**	**58.81**
8	27240.12	56.06	54207.67	63.00	0.45	**50628.01**	**59.18**
9	23488.13	58.73	44426.67	62.00	0.40	**42942.01**	**60.04**
10	19342.43	61.67	35177.67	61.00	0.35	**35512.80**	**61.43**
Total	253611.16		521061.67			**492782.80**	

Table III
CALCULATION OF THE FLOW OF SALES FROM THE POSITION OF RESTRAINED OPTIMISM, WHEN EXPERT EVALUATION OF THE AVERAGE VALUE OF THE COMPETITIVE OFFER IN THE SEGMENT AND THE AMOUNT OF ACCEPTABLE RISK ARE GIVEN

Number i of the trading period	Cautious forecast for the enterprise		Optimistic forecast for the enterprise		Acceptable risk	Adjusted forecast for the enterprise	
	profit	sales volume	profit	sales volume		profit	sales volume
i	f_{0i}	x_{0i}	f_{1i}	x_{1i}	R_i	f_i^*	x_i^*
1	16040.00	65.33	332602.50	49.50	0.80	**332502.22**	**52.67**
2	20331.52	61.97	335251.11	49.68	0.75	**335147.28**	**52.75**
3	24194.13	58.93	337900.33	49.83	0.70	**337810.90**	**52.56**
4	27689.05	56.18	340550.02	49.96	0.65	**340488.52**	**52.14**
5	30866.39	53.66	343200.07	50.07	0.60	**343171.19**	**51.51**
6	33767.54	51.36	345850.42	50.17	0.55	**345846.07**	**50.70**
7	30651.84	53.61	343000.07	49.93	0.50	**342952.63**	**51.77**
8	27240.12	56.06	340151.56	49.65	0.45	**339990.24**	**53.18**
9	23488.13	58.73	337305.33	49.33	0.40	**336923.62**	**54.97**
10	19342.43	61.67	334462.02	48.95	0.35	**333711.18**	**57.22**
Total	253611.16		3390273.44			**3388544**	

in Table III. Estimates of the average competitive offer are stable: $m_i = 395$, $i = \overline{1;10}$. The value of the acceptable risk decreases with time, as in Table II.

In this example, averaging over exponential distribution led to a more moderate profit forecast than when averaged over a uniform distribution of the volume of competitive offers. Here, the mathematical expectation of the volume of the competitive bid was set close to the estimate of the upper limit of the competitive bid.

Thus, calculations on abstract test data show the potential practical effectiveness of the application of the proposed game model.

IV. CONCLUSIONS

The proposed formal model for calculating the flow of sales volumes of a single product allows you to organize a transparent decision support procedure. This is facilitated by the following circumstances. The task of the objective function (total profit) makes it possible to explicitly link the economic parameters that are naturally taken into account in the situation under consideration. Uncertainty of forecasting is taken into account by using the game approach to optimize the target criterion. Accounting for the preferences of the decision maker is implemented by setting expert estimates of the parameters defining the objective function, as well as by setting the level of caution in forecasting profits.

Test examples showed the practical effectiveness of the proposed procedure for calculating the flow of sales volumes, the possibility of flexible adjustment of the procedure for optimizing profits and modeling various user profit forecast scenarios.

Thus, this model can be used as a theoretical tool of economic and mathematical modeling to support decision making, as well as using real data as a practical tool for optimizing the sales flow of a single product.

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