

Application of Rescaled Range and Wavelet Analysis on Climate Prediction: A Case Study in the Lower Yellow River Region

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Abstract—Rescaled range analysis is a new method to reveal the natural self-similarity. Wavelet analysis shows the frequency characteristics on whole frequency domain, also can give the overall characteristics on time domain. Fengqiu is selected as the representative study area of lower Yellow River. Predicting climatic change trend based on the non-linear mathematical method, analyzing time series lone-run memory effects and memory period by Hurst index, fractal dimension and non-period cycle average cycle length. Then, analyzing the multi-time scale characteristics of the temperature time series by the Morlet wavelet transformation method. Results show that the temperature trend is fluctuating upward, warming rate is 0.09°C/10a, 5a moving average curve has a peak value in 1990s; the Hurst value is 0.9503, the fractal dimension value is 1.0497; variation trend of temperature will inherit the past trend; the annual mean temperature mainly has three different scales of oscillation period, they are 3-5a, 17a and 32a respectively. The case study shows that the rescaled range analysis and wavelet analysis can get satisfactory results for revealing some multiscale, multilevel, and multiresolution problems.

Keywords—rescaled range analysis; wavelet analysis; climate prediction; application

I. INTRODUCTION

Fractal theory, dissipative structure and chaos theory are considered as three major discoveries in the history of science in the last century. Many natural phenomena are fickle and lack of rules, and can't be described by the traditional Euclidean geometry, but these phenomena are self-similarity [1-2]. Self-similarity and long-range dependence are two universal phenomenon in natural world. The founder of fractal geometry B.B. Mandelbrot first proposed the fractal concept, and expounded the basic idea of fractal theory, and argued that the object of fractal research is disordered system with self-similarity, the change of dimension is continuous. Fractal structure has two basic properties: self-similarity and scale invariance. Fractal theory makes people's cognitive methods develop from linear to nonlinear stages, reveal the multifaceted, multi-view, and multi-dimensional connection, it has

good application prospect in the earth science research [3-5]. Rescaled range analysis method is based on the fractal theory, Principle of rescaled range analysis is that changing the time scale of the researched object, and observing the statistical characters laws, and using small time scale regularity to recognize large time scale or used large time scale to recognize small time scale regularity. It has good effect to study time series, especially for climate prediction [6-7].

Wavelet analysis is developed from the Fourier transform method, is a useful signal analysis tool, and is called as mathematical microscope. Wavelet analysis shows the frequency characteristics on whole frequency domain, also can give the overall characteristics on time domain [8-9]. Wavelet analysis can get satisfactory results for some multiscale, multilevel, and multiresolution problems. Almost all weather and climatic phenomena on earth are associated with multiple time scales, especially for climate change, it has multiple time scales, including month, season, interannual, and several decades scales variation. Wavelet analysis can detect multiscale and multilevel structure characteristics in climate change [10-12]. Fengqiu is selected as the representative study area of lower reaches of Yellow River.

II. METHODOLOGY

A. Rescaled Range Analysis

First, confirm that you have the correct template for your paper size. For the time series which length is M , the time series X_i is divided into W continuous subsequences which length is n , every subsequence is F_a ($a=1,2,\dots,W$), every component of subsequence is $Q_{r,a}$ [13].

Calculating the mean value G_a of the subsequence F_a with following Eq. (1):

$$G_a = \frac{1}{n} \sum_{r=1}^n Q_{r,a} \quad (1)$$

The accumulated deviation $X_{t,a}$ of subsequence F_a is calculated with Eq. (2):

$$X_{t,a} = \sum_{r=1}^t (Q_{r,a} - G_a), \quad t=1,2,\dots,n \quad (2)$$

The range R_a of subsequence F_a is calculated by following Eq. (3):

$$R_a = \max_{1 \leq t \leq n} (X_{t,a}) - \min_{1 \leq t \leq n} (X_{t,a}) \quad (3)$$

The standard deviation of subsequence F_a is expressed as Eq. (4):

$$S_a = \left[\frac{1}{n} \sum_{r=1}^n (Q_{r,a} - G_a)^2 \right]^{1/2} \quad (4)$$

Range is R_a divided by the standard deviation S_a as Eq. (5):

$$(R/S)_a = R_a / S_a \quad (5)$$

For every subsequence, above steps are calculated, the rescaled range sequence $(R/S)_a$ and mean value of this sequence can be calculated by Eq. (6)

$$(R/S)_n = (1/W) \sum_{a=1}^W (R/S)_a \quad (6)$$

Hurst has established the relational expression as Eq. (7):

$$(R/S)_n = \rho n^H \quad (7)$$

Where R/S represents the rescaled range, and n is the length of time, ρ is constant, H is Hurst index.

$$\log(R/S)_n = H \log n + \log \rho \quad (8)$$

Where $\log(n)$ represents the explanatory variable, $\log(R/S)_n$ represents the explained variable.

The relationship between Hurst index H and fractal box dimension D is expressed as Eq. (9)

$$D=2-H \quad (9)$$

Some studies show there is no correlation relationship between the past increment and the future increment while

$H=0.5$ and $D=1.5$; there is positive correlation relationship between the past increment and future increment while $0.5 < H < 1$ and $1 < D < 1.5$; there is negative correlation relationship between the past increment and future increment while $0 < H < 0.5$ and $1.5 < D < 2$ [14-16].

B. Wavelet Analysis

The wavelet function $\Psi(t)$, belong to the class of functions with oscillatory characteristics and can be quickly reduced to zero [8]. It can be defined as Eq. (10):

$$\int_{-\infty}^{+\infty} \Psi(t) dt = 0 \quad (10)$$

Morlet wavelet was selected in this study, following an expression as Eq. (11):

$$\Psi(t) = e^{ict} e^{-t^2/2} \quad (11)$$

where C is a constant, and i is the imaginary number. Morlet wavelet scaling a and cycle T have the following relation as Eq. (12):

$$T = \left(\frac{4\pi}{c + \sqrt{2+c^2}} \right) \times a \quad (12)$$

Integrating the square of wavelet coefficient in the time domain, the wavelet variance can be obtained as Eq. (13):

$$Var(a) = \int_{-\infty}^{+\infty} |W_f(a,b)|^2 db \quad (13)$$

The wavelet variogram can be used to confirm the primary period of time series.

III. RESULTS AND DISCUSSION

A. Rescaled Range Analysis and Non-periodic Cycle Analysis of Temperature Time Series

Analyzing the annual temperature trend during 50 years in Fengqiu, As shown in Figure I, the temperature trend is fluctuating upward, warming rate is $0.09^\circ\text{C}/10\text{a}$, 5a moving average curve has a peak, the peak value appeared at 1990s. Analyzing the Hurst value and fractal dimension (Figure II), the Hurst value is 0.9503, the fractal dimension value is 1.0497, while $0.5 < H < 1$, $1 < D < 1.5$, it means temperature time series has obvious fractal structure, there are positive correlation relationship between past and future trends. Variation trend of temperature will inherit the past trend, and increasing continually. Analyzing the variation curves of V statistic values vs $\log(N)$, As shown in Figure III, the curve is

tilted upwards and has an upward trend, so variation trend of temperature series has continuous characteristics, the first inflection point of $\log(N)$ equals 2.0794, the corresponding average cycle lengths of temperature series is 8 years. It means the past status will affect future status for 8 years, while more than the critical point with 8 years, the memory of the time series will gradually disappear, showing random independence characteristics.

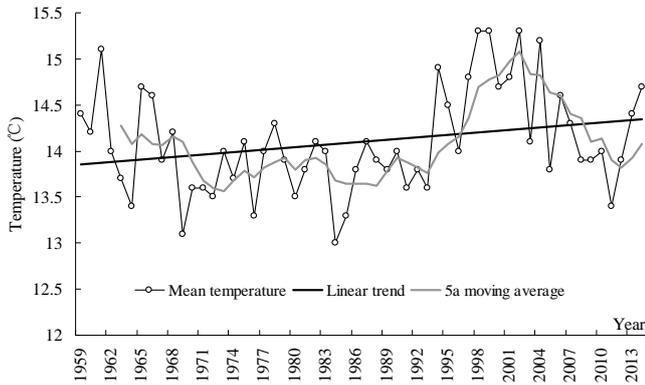


FIGURE I. VARIATION OF ANNUAL MEAN TEMPERATURE IN FENGQIU

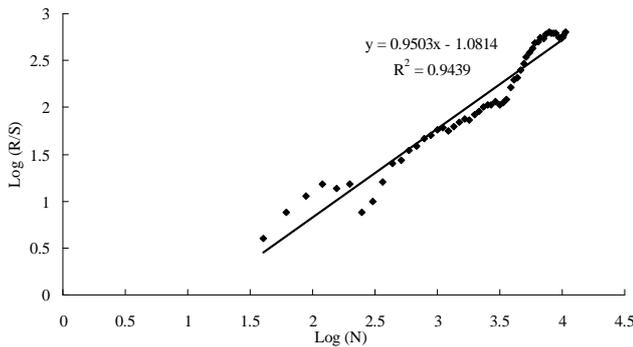


FIGURE II. RESCALED RANGE ANALYSIS OF ANNUAL MEAN TEMPERATURE IN FENGQIU

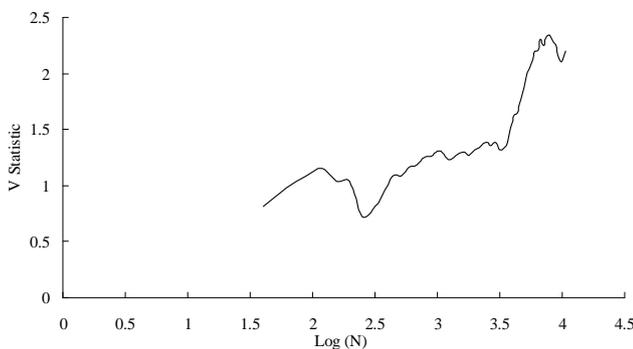


FIGURE III. VARIATION CURVES OF V STATISTIC VS LOG(N) BASED ON RESCALED RANGE ANALYSIS OF TEMPERATURE SERIES

B. Multi-scale Analysis of Temperature Variation

The continuous Morlet wavelet transformation method is used to analyze the multi-scale structure characteristics of the temperature variation in Fengqiu region, the distribution and phase of periodic variation on different time scales is as shown Figure IV. The annual mean temperature mainly has three different scales of oscillation periods, they are 3-5a, 17a and 32a respectively. The oscillation signal is the strongest in the 3-5a scales, there are 9 cold and warm cycles, it is very obvious in 1960s-1990s. There are 6 cold and warm cycles in the 17a scales, based on this rules, the next warmer period may appear after 2015. There are 4 cold and warm cycles in the 32a scales. The wavelet variogram can reflect the primary period of climate change in 50 years, as shown in Figure V, there are 2 obvious peak value, the first primary period is 17a with maximum peak value, the second primary period is 4a.

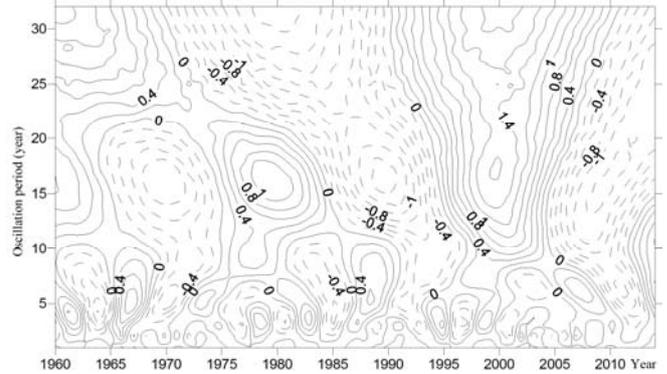


FIGURE IV. TIME-FREQUENCY DISTRIBUTION OF ANNUAL MEAN TEMPERATURE SERIES IN FENGQIU REGION

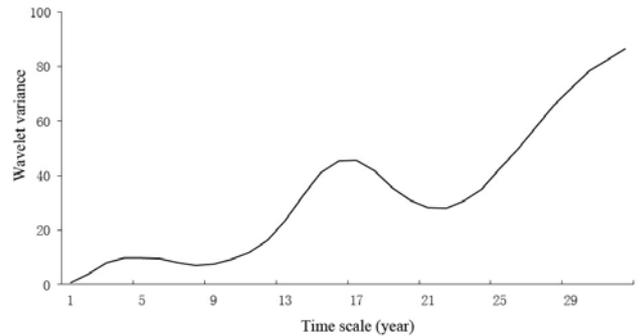


FIGURE V. THE WAVELET VARIOGRAM OF ANNUAL MEAN TEMPERATURE SERIES IN FENGQIU REGION

IV. CONCLUSION

Rescaled range analysis is a new method for revealing the natural self-similarity. Wavelet analysis can discover the time-frequency distribution characteristics of time series. Fengqiu is selected as the representative study area of the lower Yellow River.

1) Predicting climatic change trend based on the non-linear mathematical method, analyzing lone-run memory effects and memory periods of the time series by Hurst index, fractal

dimension and the non-period cycle length. The continuous Morlet wavelet transformation method is used to analyze the multi-scale structure characteristics of the temperature variation in Fengqiu region. Rescaled range analysis and Wavelet analysis can get satisfactory results for some multiscale, multilevel, and multiresolution problems.

2) The temperature trend is fluctuating upward, warming rate is $0.09^{\circ}\text{C}/10\text{a}$, 5a moving average curve has a peak, the peak value appeared at 1990s. the Hurst value is 0.9503, the fractal dimension value is 1.0497. Variation trend of temperature will inherit the past trend, and increasing continually. The curves of V statistic values vs $\log(N)$ is tilted upwards and has an upward trend, the first inflection point of $\log(N)$ equals 2.0794, the corresponding average cycle lengths of temperature series is 8 years.

3) The annual mean temperature has three scales oscillation periods, they are 3-5a, 17a and 32a respectively. The oscillation signal is the strongest in the 3-5a scales, there are 9 cold and warm cycles; there are 6 cold and warm cycles in the 17a scales; there are 4 cold and warm cycles in the 32a scales. The wavelet variogram reveals the primary periods of climate change in past 50 years, there are two obvious peak values in the wavelet variogram, the first and second primary periods are 17a, 4a respectively.

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