

An Improved VIKOR Model for Ballistic Missile Threat Assessment And Ranking

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Abstract—For the problem which the subjectivity of confirming weight is too strong and depending on accurate data in VIKOR, we propose a method of ballistic missile threat assessment based on improved VIKOR. In our method, VIKOR is improved at two aspects. On the one hand, the algorithm of confirming weight is improved, which adopts combination weight covering for expert decision-making; On the other hand, the computing model of VIKOR is improved also, which boosts up the ability of dealing with fuzzy problem. Experiments show that our method of ballistic missile threat assessment is reasonable.

Keywords—ballistic missile; threat assessment; ranking; VIKOR; combination weight

I. INTRODUCTION

Because of long range, high velocity, strong resistance to penetration and nuclear bomb, the ballistic missile is the first choice of space and air offensive weapons in many countries, which brings huge threat for air defense operation. Ballistic missile threat assessment and ranking is important for anti-missile operations, which provides intelligence and basis of decision to generating interception strategy and actualizing fire interception, and influences anti-missile operations efficiency directly.

Domestic and foreign scholars carry out exploratory study to threat assessment and achieve fruitful research results, which include multiple attribute decision making[1], fuzzy set method[2], intuitionistic fuzzy set method[3], bayesian network method[4], neural network method[5] and so on.

Aim at the characteristic of ballistic missile threat assessment and existent problems, taking into account the advantages and disadvantages of existing assessment methods, we select the ViseKriterijumska Optimizacija I Kompromisno Resenje(VIKOR)[6][7] and improve it. After that, we present a method of ballistic missile threat assessment based on improved VIKOR.

II. BALLISTIC MISSILE THREAT ASSESSMENT INDEX SYSTEM

The factor of influencing grade of ballistic missile threat could include shoot location, impact point, range, velocity in shut-time, velocity of reentry, warhead power, radar cross section, warhead category, penetration capability, maneuver capability, hit accuracy, remaining flight time, importance of protection target, number of missile and so on in complex battlefield environment. We select importance of protection

target, range, remaining flight time and velocity in shut-time as evaluation index.

A. Importance of Protection Target

The importance of protection target is that the influence grade in politics, military and economy when the protection target is destroyed.

In this paper, the priority of protection target includes 3 grades, in which the 1 grade is the highest priority. The smaller the value of priority grade of protection target, the more important the protection target. The important grade of protection target can be compute by the formula:

$$\omega(j)=1-0.1I_j, 1 \leq j \leq 3 \quad (1)$$

Here, I_j is the priority grade of the j -th protection target.

B. Range

The range is one of important tactics indexes of ballistic missile, which can be classified as short-range missile, intermediate-range missile, long-range missile and intercontinental missile.

In anti-missile operations, the acquisition of ballistic missile range information is relatively easy. Assuming that the early warning radar can detect the information of a ballistic missile target T_k at a certain time, the position of launch point $L(J_1, W_1)$ and the position of landing point $I(J_2, W_2)$ can be obtained through intelligence synthesis and ballistic prediction, the formula for calculating the target range L of the incoming ballistic missile is:

$$L = R_e \arccos[\sin W_1 \sin W_2 + \cos W_1 \cos W_2 \cos(J_1 - J_2)] \quad (2)$$

Here, R_e is equatorial radius of earth, $R_e=6378\text{km}$. J_1, J_2 are the transverse coordinates of the target launch point and landing point, respectively. W_1, W_2 are the lengthways coordinates of the target launch point and landing point, respectively.

After comprehensive ballistic missile range classification characteristics, the target range threat degree calculation formula is:

$$\omega(L) = \begin{cases} 0.4, & L \leq 300 \\ 0.4 + 0.2 \times \frac{L-300}{1000-300}, & 300 < L \leq 1000 \\ 0.6 + 0.2 \times \frac{L-1000}{3000-1000}, & 1000 < L \leq 3000 \\ 0.8 + 0.2 \times \frac{L-3000}{5500-3000}, & 3000 < L \leq 5500 \\ 1, & L > 5500 \end{cases} \quad (3)$$

C. Remaining Flight Time

The remaining flight time refers to the length of time from the time when the incoming ballistic missile target is discovered by the early warning radar to its landing time.

Ballistic missile flight time is generally between 3 and 42 min. In general, short-range ballistic missiles fly between 180 and 600s, medium-range ballistic missiles fly between 600 and 1000s, long-range ballistic missiles fly between 1000 and 1500s, and intercontinental ballistic missiles fly between 1500 and 2600s. The remaining flight time threat can be calculated according to the following formula:

$$\omega(t) = \begin{cases} 1, & t \leq 300 \\ 1 - 0.2 \times \frac{t-300}{600-300}, & 300 < t \leq 600 \\ 0.8 - 0.2 \times \frac{t-600}{1000-600}, & 600 < t \leq 1000 \\ 0.6 - 0.2 \times \frac{t-1000}{1500-1000}, & 1000 < t \leq 1500 \\ 0.4, & t > 1500 \end{cases} \quad (4)$$

D. Velocity in Shut-time

The speed of the shutdown point determines the re-entry speed and the attack power of the incoming ballistic missile target. According to the principle of ballistic missile flight, the range of ballistic missiles is different, and the corresponding shutdown point speed is also different. The formula for calculating the speed threat degree of the target shutdown point is:

$$\omega(v) = \begin{cases} 0, & v < 1 \\ 0.4 + 0.3 \times \frac{v-1}{3-1}, & 1 \leq v < 3 \\ 0.7 + 0.3 \times \frac{v-3}{6.5-3}, & 3 \leq v < 6.5 \\ 1, & v \geq 6.5 \end{cases} \quad (5)$$

III. COMBINATION WEIGHTING APPROACH

A. Determination of Subjective Weight Based on IAHP

IAHP is developed by AHP to solve the uncertainty of subjective judgments and decision attributes. IAHP is an extension of AHP in the interval number. Let $\tilde{\mathbf{A}} = (\tilde{a}_{ij})_{m \times n}$ be an interval Matrix, IE $\tilde{a}_{ij} = [a_{ij}^L, a_{ij}^U]$, note $\mathbf{A}^L = (a_{ij}^L)_{m \times n}$, $\mathbf{A}^U = (a_{ij}^U)_{m \times n}$, and note $\tilde{\mathbf{A}} = [\mathbf{A}^L, \mathbf{A}^U]$. Also for interval vectors $\tilde{x} = (x_1, x_2, \dots, x_n)^T$, IE $x_i = [x_i^L, x_i^U]$, note $x^L = (x_1^L, x_2^L, \dots, x_n^L)^T$, $x^U = (x_1^U, x_2^U, \dots, x_n^U)^T$, and note $\tilde{x} = [x^L, x^U]$. For a given interval number judgment matrix $\tilde{\mathbf{A}} = [\mathbf{A}^L, \mathbf{A}^U]$, IAHP

determines the weight of the indicator by the following calculation steps:

Step 1: The normalized feature vector x^L, x^U with positive components corresponding to the maximum eigenvalues of \mathbf{A}^L and \mathbf{A}^U are respectively calculated using the power method.

Step 2: By $\mathbf{A}^L = (a_{ij}^L)_{m \times n}$, $\mathbf{A}^U = (a_{ij}^U)_{m \times n}$, α and β are calculated according to the following formula.

$$\alpha = \left[\frac{\sum_{j=1}^n \frac{1}{\sum_{i=1}^n a_{ij}^U}}{n} \right]^{\frac{1}{2}} \quad (6)$$

$$\beta = \left[\frac{\sum_{j=1}^n \frac{1}{\sum_{i=1}^n a_{ij}^L}}{n} \right]^{\frac{1}{2}} \quad (7)$$

Step 3: Subjective weight vector $\omega_1 = [\alpha x^L, \beta x^U]$.

B. Objective Weight Determination Based on Entropy Method

The entropy method is a method to determine the weight of indicators under objective conditions. It has the characteristics of strong operability and objectivity, can reflect the implied information of data, and enhance the significance and difference of indicators. In order to avoid the analysis difficulties caused by too small differences in the selection of indicators, all types of information are fully reflected. The steps for entropy weight empowerment are generally as follows:

Step 1: Data standardization

Standardized processing of data for each indicator. For the problem of ballistic missile threat assessment, the decision matrix $\mathbf{A} = (a_{ij})_{n \times m}$ is constructed, where n is the number of

ballistic missiles, m is the number of threat indicators, and a_{ij} is the attribute value of the first target under the j-th threat indicator. After the matrix \mathbf{A} is standardized with formula(1)

to(5), a standardized matrix $\mathbf{R} = (Y_{ij})_{n \times m}$ is obtained.

Step 2: Find the information entropy of each indicator

According to the definition of information entropy in Informatics, the information entropy of a set of data is:

$$E_j = -\ln(n)^{-1} \sum_{i=1}^n p_{ij} \ln p_{ij} \quad (8)$$

Where

$$p_{ij} = Y_{ij} / \sum_{i=1}^n Y_{ij} \quad (9)$$

If $p_{ij} = 0$, define $\lim_{p_{ij} \rightarrow 0} p_{ij} \ln p_{ij} = 0$.

Step 3: Determination of weights for each indicator

According to the information entropy calculation formula, the information entropy of each indicator is calculated as E_1, E_2, \dots, E_k . The weight vector $\omega_2 = (W_1, W_2, \dots, W_k)$ of each indicator is calculated by information entropy.

Where

$$W_i = \frac{1 - E_i}{k - \sum E_i}, \quad (i = 1, 2, \dots, k) \quad (10)$$

C. Combination of Subjective And Objective Weights

The subjective and objective weights of the threat assessment indicators obtained by the IAHP and entropy methods are calculated by means of formula(6), and the combined weight $\tilde{\omega} = (\tilde{\omega}_1, \tilde{\omega}_2, \dots, \tilde{\omega}_n)$ obtained is taken as the weight of the threat assessment index. This combination weight not only takes into account the experience of experts. At the same time, objective factors are also considered to reflect the relative importance of threat assessment indicators.

$$\tilde{\omega} = \mu\omega_1 + \eta\omega_2 \quad (11)$$

Here, μ is the influence factor of the subjective weight, η is the influence factor of the objective weight, and $\mu + \eta = 1$. Its determination principle is: according to the integrity and credibility of the early warning radar detection data information, the degree of trust in the experts, and the combination of specific anti-missile warfare cases to determine the final subjective and objective weight of the influence factors.

IV. BALLISTIC MISSILE THREAT ASSESSMENT AND RANKING BASED ON IMPROVED VIKOR

The steps of the ballistic missile threat assessment algorithm based on the improved VIKOR algorithm include:

Step 1: Determine language variables and triangulated fuzzy numbers

Triangular fuzzy number is a method that converts fuzzy and uncertain language variables into determined values, which can better solve the problem that the characteristics of the evaluated object can not be accurately quantified and can only be qualitatively evaluated by using natural language. Its core idea is to transform the importance of the index and the triangular fuzzy number, and then formulate the corresponding relationship. The main collections used in this article are as follows: n incoming target set $T = \{T_1, T_2, \dots, T_n\}$; A set $I = \{I_1, I_2, \dots, I_m\}$ consisting of m evaluation indicators; Set $\tilde{\omega}$ of index weights.

Step 2: Construct standardized decision matrix

According to the relevant index information of incoming missiles, a decision matrix $A = (a_{ij})_{n \times m}$ is constructed. Through the threat assessment index system, the decision matrix is standardized and the standardized decision matrix $R = (Y_{ij})_{n \times m}$ is obtained.

Step 3: Subjective weight obtained by IAHP method

By seeking expert opinion to construct interval number judgment Matrix, the interval number weight of each threat evaluation index is obtained according to the calculation step of interval analytic hierarchy process.

Step 4: Objective weight obtained by entropy method

According to the standardized decision matrix, the objective weight of each threat assessment index is obtained according to the entropy method.

Step 5: Combination of subjective and objective weights into combined weights

The combination weight formula is used to combine the subjective and objective weights. Due to the uncertainty of the subjective and objective factors, the subjective and objective factors are assigned differently, so that the combined weights finally obtained are universal and the subjective influence brought by the influencing factors is reduced.

Step 6: Determine ideal scenario F^+ and worst scenario F^-

First, the positive ideal solution and the negative ideal solution of each index are obtained by formulas (12) and (13).

$$y_i^+ = [\max y_{ij} \mid i \in (1 \sim m), j \in (1 \sim n)] \quad (12)$$

$$y_i^- = [\min y_{ij} \mid i \in (1 \sim m), j \in (1 \sim n)] \quad (13)$$

The positive ideal solution of each indicator constitutes ideal scheme F^+ , and the negative ideal solution of each indicator constitutes the worst scheme F^- :

$$F^+ = [y_1^+, y_2^+, \dots, y_n^+] \quad (14)$$

$$F^- = [y_1^-, y_2^-, \dots, y_n^-] \quad (15)$$

Step 7: Calculate the group benefit values (S_i) and individual regret values (R_i) of each plan

The formula for calculating the group benefit values (S_i) and individual regret values (R_i) for each plan is as follows:

$$S_i = \left\{ \sum_{j=1}^n [\tilde{\omega}_j (y_j^+ - y_{ij}) / (y_j^+ - y_j^-)]^p \right\}^{1/p} \quad (16)$$

$$R_i = \left\{ \sum_{j=1}^n [\tilde{\omega}_j^L | y_j^+ - y_{ij} + \tilde{\omega}_j^U | y_j^+ - y_{ij} / (y_j^+ - y_j^-)]^p \right\}^{1/p} \quad (17)$$

$$S_i = \left\{ \sum_{j=1}^n [\tilde{\omega}_j | y_j^+ - y_j^- | + \tilde{\omega}_j | y_j^+ - y_j^- | / (y_j^+ - y_j^-)]^p \right\}^{1/p} \quad (18)$$

$$R_i = \max \{ \tilde{\omega}_j (y_j^+ - y_j^-) / (y_j^+ - y_j^-), \quad j \in [1, n] \} \quad (19)$$

$$R_i = \max \{ (\tilde{\omega}_j^L | y_j^+ - y_j^- | + \tilde{\omega}_j^U | y_j^+ - y_j^- |) / (y_j^+ - y_j^-), \quad j \in [1, n] \} \quad (20)$$

$$R_i = \max \{ (\tilde{\omega}_j | y_j^+ - y_j^- | + \tilde{\omega}_j | y_j^+ - y_j^- |) / (y_j^+ - y_j^-), \quad j \in [1, n] \} \quad (21)$$

Here, i is the number of the incoming missile; j is number for evaluation indicators; y_{ij} indicates the j -th value of the i -th missile; y_j^+ and y_j^- represent the positive and negative ideal solutions of each evaluation index, respectively, which are standardized values; The p is a function parameter (usually 1, 2 or ∞ , take 1 in this article); n is total number of indicators; $\tilde{\omega}_j$ is the weight of the indicator J_j ; S_i represents the group benefit of the i -th incoming missile, the smaller the value, the greater the group benefit; R_i represents the individual regret of the i -th incoming missile. The smaller the value, the smaller the individual regret.

Among them, (17), (20) are improved formulas used to deal with uncertainties under fuzzy weights, (18) and (21) are improved formulas used to deal with uncertainties when incoming missile information.

Step 8: Calculate the benefit ratio of each scheme to Q_i and rank the threats

The benefit ratio Q_i calculation formula is as follows:

$$Q_i = v \frac{(S_i - S^-)}{(S^+ - S^-)} + (1-v) \frac{(R_i - R^-)}{(R^+ - R^-)}, \quad \forall i \quad (22)$$

Here, $S^+ = \max S_i$, $S^- = \min S_i$, $R^+ = \max R_i$, $R^- = \min R_i$; v represents the decision-making mechanism coefficient. If $v > 0.5$, it means that the decision will be mainly based on maximizing the group benefit value; If $v = 0.5$, it means that the decision is made according to a balanced compromise; If $v < 0.5$, it means that decisions will be made mainly based on minimizing individual regret values. In this paper, set $v = 0.5$, which is a balanced compromise to maximize group utility while minimizing negative effects.

Finally, according to the Q value of each scheme, the degree of target threat is sorted. The smaller the value, the higher the degree of attacking target threat.

V. RESULTS AND ANALYSIS

In this section, a ballistic missile defense case is designed, and the threat assessment and ranking of ballistic missile threat assessment methods are applied, and the results are compared with those of rough set methods and Bayesian network methods.

A. Tactical Thinking

One day, A decided to launch an offensive against B. A country purchased a large number of advanced weapons, including ballistic missiles, and was ready to launch large-scale joint operations against B countries. The A country's authorities planned to launch joint fire strikes against B countries and defeat the combat system. Significantly weaken the war potential. Joint fire strike forces are mainly composed of long-range fire strike forces such as missile combat groups and air combat groups. When the first joint fire strike was conducted, the missile combat group first attacked key targets such as the key command center of the country B, important air defense weapon positions, and power hubs. Then the air combat group launched operations, focusing on key targets such as attacking airports.

In the face of a large number of ballistic missile fire strikes by the A Army, based on the incoming ballistic missile information detected by the early warning radar in real time, the ballistic missile targets that fall into the air defense area are screened, and the targets are threatened and the targets are selected according to the ranking results. In order to crush the enemy's attempt to suppress fire through ballistic missiles.

B. Threat Assessment Based on Improved VIKOR Algorithm

Using the ballistic missile threat assessment method proposed in this paper, the S , R , and Q values of each incoming missile target under different impact factors are computed, and the final ranking results are 2, 1, 3, 4, and 5.

VI. CONCLUSION

Ballistic missile threat assessment is of great significance in anti-missile operations and is an important basis for early warning, detection, interception and other combat missions. In view of this problem, this paper proposes a method for ballistic missile threat assessment based on improved VIKOR. In this method, the VIKOR method has been improved. It mainly includes two aspects: First, the weight determination method has been improved, and a single expert decision-making method has been replaced by a combination of subjective and objective weights, taking into account both expert experience and objective information. More scientific rationality; Secondly, the VIKOR calculation model is improved, which enhances the ability to deal with fuzzy issues. The improved model can carry out threat assessment of incoming ballistic missiles when the data is incomplete or uncertain. Experiments show that our ballistic missile threat assessment method is scientific and reasonable.

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