SIMULATION-BASED FORECASTING OF THE REAL EFFECTIVE EXCHANGE RATE OF THE RUBLE

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Abstract
The real exchange rate of national currency is known to be one of the most important macroeconomic indicators. In this paper, we explore the opportunities of the Monte Carlo simulation technique combined with the polynomial residuals model for a medium-term forecasting the index of the real effective exchange rate for the ruble. The idea of the proposed approach is based on the successive (variable) differences method, designed for smoothing time series characterized by trend component and irregular component. The approach is illustrated numerically with the simulation-based forecast for the ruble real effective exchange rate, the values of the index in September-December 2017 being used to assess the forecast quality. The parameters in the corresponding polynomial residues models were calculated using the Nelder–Mead algorithm. The results of simulations at different "estimate depth" values and MAPE and RSME values indicate that the proposed approach allows constructing a relatively accurate medium-term forecast for the ruble effective exchange rate.

Keywords: real effective exchange rate, short-term forecasting, Monte Carlo simulation, polynomial trend approximation

JEL code: E170, C630

Introduction
The need for forecasting is in recent times increasing. Analyzing the modern economy, we need to re-consider the problems of forecasting the dynamics of its indicators. The results of the economy development are affected by various conditions and factors that must be considered in the forecasting process. It requires modification or transformation of corresponding models in order to adequately reflect the real economic processes in their formulation. When forecasting the economic development it is necessary to take into account the new trends and development directions of the modern economy.

In this paper, we explore the opportunities of the simulation-based medium-term forecasting (for 3-4 months in advance) the index of the real effective exchange rate (REER) for the ruble. REER is an indicator characterizing the dynamics of an exchange rate. It is not a factor that directly affects the economy future prospects. It can be treated as an indicator which determines the domestic current macroeconomic situation related to exchange rates in comparison with the countries that are the main trade partners. Its fluctuations in some sense allow us to judge how the competitiveness of the country under consideration results from the ongoing monetary and credit policy. For example, a high real effective exchange rate of a foreign currency means the relative weakness of the domestic currency, which stimulates exports of goods and services, reduces imports and decreases consumer welfare, primarily due to the relatively high import prices.

The standard technique of calculating the exchange rate for a particular currency is based on the consumer price indices ratio within the country and abroad, weighted in proportion to the share of every country in external turnover. These indices are usually calculated as the
weighted sum of the real exchange rate indices for the period in question relative to the reference value. The weights are derived from manufacturing trade flows and trace both bilateral trade and third-market competition by a double-weighting procedure (see Turner and Van’t dack, 1993).

REER indexes serve various purposes, in particular as components of monetary/financial conditions analysis and a measure of international competitiveness. Therefore, accurate forecasts of REERs are important for both market participants and policymakers.

General problems of analyzing and evaluating effective real exchange rates were considered earlier (see Chinn, 2006; Maeso-Fernandez et al., 2002; Anderson et al., 1987). In some papers the authors study the impact of the exchange rates under consideration on the prospects for economic growth (Dosse, 2007; Eichengreen, 2007), consider the analysis of exchange rates in terms of models of an open economy, given world prices for raw materials (Mahzur, 2018) including oil (Volkov, Yuhn, 2016); it is proposed to take into account characteristics of the monetary policy (Dąbrowski, Papież, Śmiech, 2018), including the banking sector (Wan, 2017); to analyze the impact of these courses on accumulation (Spilimbergo and Vamvakidis, 2003), economic activity of Russia (Evdokimova et al., 2013) and the development of countries with transitional economies (Maeso-Fernandez et al., 2005). When analyzing and forecasting the effective exchange rates themselves or their indices, charts are usually shown over a sufficiently long period (see Panfilov, 2009; Trunin et al., 2010).

For the forecasting of exchange rates it is proposed to use ARMA models (Rout, Majhi B., Majhi R, Panda, 2014), including continuous ARMA models (Arratia, Cabaña, A., Cabaña, E., 2016), GARCH models and their modifications (Gupta, Kashyap, 2016; Barunik, Krehlik, Vacha, 2016). There are proposals to link the use of ARIMA models with chaos algorithms (Yonghong, Zhiyong, Mingye, 2016), to apply neural networks (Liu, Hou, Liu, 2017; Zhenhua; Zezheng, Chao; 2016), to use the support vector machines and genetic algorithms (Özorhan, Toroslü, Şehitoğlu, 2017), to apply panel data analysis, taking into account macroeconomic indicators and market volatility (Moraes-Arias, Moura, 2013) for forecasting exchange rates. It is worth noting the use of simulation, though on the basis of the training method of support vectors (Yuan, 2013) as well as to improve the quality of the forecast using random processes (Moosa, 2013). Attention is drawn to the account of uncertainty in the forecasting of exchange rates (Kouwenberg, Markiewicz, Verhoeck, 2017; Detken C. 2002). It is proposed to use cointegration methods and random processes models (Moosa, Vaz, 2016) including those that take into account incomplete information (Juselius, 2017). For accounting the influence of external shocks, it is proposed to introduce special noise variables (Gali, Rabanal, 2004; Woodford, 2007) or to use stochastic differential equations describing the dynamics of macroeconomic indicators (Turnovsky, 2000) and their discrete approximations in the simulation mode (Vorontsovskiy, Vyurenko, 2014, 2016).

Exchange rate models are now common, in which the dynamics of the nominal exchange rate of a particular currency is linked to the output and domestic interest rates. Starting with the simplest Keynesian models (Wickens, 2011, pp.367-366), and ending with more meaningful models of Mandell-Fleming (Fleming, 1962; Mundell 1963), Dornbush (Dornbusch, 1976) and Obstfeld-Rogoff (Obstfeld, Rogoff, 1995), etc., only the nominal exchange rate of one currency against another currency is studied in the framework of such models. In the Dornbush model, independent shock variables were introduced for modeling the dynamics of prices and money supply. In the Obstfeld-Rogoff model, a log-linear approximation of its constraints was used.

The literature review shows that the main attention in the forecasting currency exchange rate values is paid to various options of econometric methods. In this article, we will focus on
the medium-term forecasting the Russian ruble REER values calculated and estimated by simulation based on the polynomial residues model.

**Data Description and Problem Statement**

Let us first analyze the dynamics of the real EER index of the ruble over the January 1994 – November 2017 period, using the database compiled by the Bank for International Settlements (BIS Bank), which contains the values of the index of this rate from January 31, 1994 to April 30, 2017. The BIS Bank is a special financial institution, which was settled to support cooperation between Central banks of different countries and facilitate international financial transactions. Real EERs are calculated as geometric weighted averages of bilateral exchange rates adjusted by relative consumer prices. The most recent weights are determined by the 2011–13 period trade volume, 2010 being selected as the base year for calculating these indexes. The EER indices are available as monthly averages, monthly data are released mid-month. An increase in the index indicates an appreciation of the corresponding currency, the reduction tells about the fall of its value.

Looking at the REER of the ruble over the period January 1994 to September 2017, we should note its relative instability associated with the 1998 crises and the problems of 2015–2016 (see Figure 1). The favorable period of exceeding 100% for this index is relatively short. This can be observed at the end of 2008 and from mid-2010 to mid-2014 with small failures. Starting from September 2014, the value of the index of the exchange rate of the ruble becomes less than 100% (Figure 1).

![Figure 1. Dynamics of the ruble REER index over January 1994 to September 2017.](image)

Starting from September 2014, the index under consideration takes values less than 100%. The minimum value of the ruble REER 40.49% was recorded in October 1994. The maximum value of this index 109.27% was achieved in January 2013. From July 2014 to the present, the observed value of the ruble REER was below 100%. During this period, its trend was very contradictory, the decline turned into growth and vice versa. The maximum value of the index over this period was 94.15% in April 2017. The ruble REER trend is quite changeable even over each relatively short time period so it is quite difficult to single out a certain common trend for this index over such period of time. This makes it impossible to construct a reliable forecast for REER values, based on its values over for the entire period under review. Forecasts should be based on the study of actual trends in the REER of the ruble either over separate time periods when the general tendency of the change in the index is preserved, or only over the last data period, in the case in question, it is 2015-2017.

The analysis of the ruble REER over the selected time period shows that the emerging shocks can both significantly change trends in the index dynamics, as it was at the time of 1998 crisis, and make the index fluctuate along with a more or less stable trend of its change, as it...
was, for example, it was from September 1998 to October 2008. The next fall of the index was caused by the 2008 global financial crisis. In subsequent years until mid-2017, the impact of the shocks was changeable. All this confirms the necessity of considering the shocks in the forecasting of the real effective exchange rate of the ruble.

Modern macroeconomics assumes that an economy is withdrawn from an equilibrium state by shocks of different size and nature with a random periodicity, the impact of these shocks being spread. A certain possibility of modeling the influence of external shocks can be associated with empirical analysis in the mode of simulating the dynamics of macroeconomic indicators on the basis of the use of functional models that are formed taking into account special "noise variables" (see Gali, Rabanal, 2004; Woodford, 2007) or stochastic processes (see Vorontsovsky, Vyvumenko, 2014; Vorontsovsky, Vyvumenko, 2016; Turnovsky, 2000).

Currently, the widely used are exchange rate models which link the dynamics of the nominal exchange rate of a specific currency to the production and domestic rates and rates of interest. Starting with the simplest Keynesian models (Wickens, 2011), to more valuable models of the Mundell-Fleming (Fleming, 1962; Mundell 1963), Dornbusch (Dornbusch, 1976), Obstfeld-Rogoff (Obstfeld, Rogoff, 1995), etc. Only the nominal exchange rate of one currency relative to another currency is investigated in the framework of these models. In the Dornbusch model, independent shock variables were used for modeling the price dynamics and the money supply. In the Obstfeld-Rogoff model, a log-linear approximation of its constraints was applied.

In terms of such models it is rather difficult to introduce a real effective exchange rate, since this rate depends on a significantly greater number of internal and external factors leading to its shock oscillations. In this paper, we propose to use shock variables reflecting the rather complex and ambiguous influence of many factors on the exchange rate in question. With respect to the real effective exchange rate, we are talking about unanticipated shocks, which impact most of all on the exchange rates of not freely convertible currencies. Such shocks are often treated as not predictable using only prior information. They are modeled in the form of stochastic variables. It requires rigorous additional assumptions concerning the behavior of the model parameters or applying computer simulation methods based on some discrete approximation of model constraints containing stochastic variables.

From a practical point of view, the assumption of macroeconomic indicators and their ratios instability can be considered as a definite hypothesis, which requires empirical confirmation. The development of macroeconomics, and hence potential changes and fluctuations of considered parameters can be investigated by means of computer simulation the initial conditions of the considered macroeconomic indicators.

In this article, we propose an approach to forecasting REER of the ruble, using simulation based on the polynomial residues model. The main task of the research is to construct a medium-term forecast for the ruble REER index in the simulation mode, taking into account the impact of the shock variables.

The hypothesis is that the proposed approach allows building a relatively accurate forecast for the ruble REER using data for a prior period of a certain length.

the second is that the choice of the time period length significantly affects the quality of the forecast.

The quality of the generated forecast is to be estimated with the help of MAPE and RMSE, with respect to the mean calculated trajectory, as well as using the boundaries of the confidence interval.

**Polynomial Residuals Model and Simulation Algorithm**

The problem of forecasting REER values is essentially a time series forecasting problem that is a very challenging job. There are many research methods used in this field, such as different kinds of exponential smoothing method (Lawton, 1998), ARIMA (Chevillon, Hendry,
and others mentioned in the Introduction. Besides, there are some combining forecasts, mixing, or pooling quantitative forecasts obtained by different time series techniques (Hibon, M., Evgeniou, T., 2005).

The main feature and difference of the proposed forecasting approach, capable of building a medium-term forecast, from classical methods is as follows. Calibration of the model and formation of the mean calculated REER trajectory in the simulation mode involves accounting the dynamics of the indicator and changes in its trend during the selected period, while classic methods are fundamentally based on the construction of local linear regression equations. In addition, the proposed method takes into account the random factor in the simulation process, while traditional forecasting methods take into account the random factor as a characteristic or the remainder of the equation, and give it only an indirect estimate.

The idea of the polynomial residuals model is based on the method of successive (variable) differences, designed for smoothing time series characterized of two components: trend component and irregular component. The random variations in the time series are caused by short-term, unanticipated and nonrecurring factors that affect the time series.

If a time series \( r_t \) contains a random component \( \varepsilon_t \) (an independent and identically distributed process with mean 0 and variance \( s^2 \)) and a deterministic (systematic), which is described by a polynomial \( Q^p \) of a degree \( p \), then one can construct the corresponding successive differences series of orders 1, 2, \ldots, \( p+1 \), which are determined as

\[
\Delta^{p+1} r_t = \Delta^p r_t - \Delta^p r_{t-1},
\]

\( \Delta^p r_t \) stands for the \( p\)-order difference calculated for the time series \( r_t \). According to this method, the difference (1) is assumed to contain only the random component.

It is known that in this case (Anderson, 1976) \( (p+1)\)-order difference is written in the form

\[
\Delta^{p+1} r_t = \varepsilon_t - C_{p+1}^1 \varepsilon_{t-1} + C_{p+1}^2 \varepsilon_{t-2} - \ldots + (-1)^{p+1} \varepsilon_{t-p-1},
\]

where \( C_k^j = \frac{k!}{j!(k-j)!} \).

The expectation and the variance for time series (2) are defined as follows:

\[
E^{p+1} \Delta r_t = 0, \quad D^{p+1} \Delta r_t = s^2 C_{2(p+1)}^{p+1}.
\]

The estimate for the sample variance of the original series random component is

\[
s^2 = \frac{1}{(n-p-1)C_{2(p+1)}^{p+1}} \sum_{t=1}^{n-p-1} (\Delta^{p+1} r_t)^2
\]

Thus, the simulation process for such time series can be presented as a five-step algorithm:

Step 1. To determine the degree \( p \) of the polynomial \( Q^p \) that describes the deterministic component.

Step 2. To obtain estimates for the polynomial \( Q^p \) coefficients.

Step 3. To estimate the variance \( s^2 \) of the random component \( \varepsilon_t \).

Step 4. To calculate the values \( r_t \) for a given value \( r_1 \) by the recurrence formula.
where $\xi_t$ is a random variable distributed normally with mean of 0 and variance 1. It simulates the complex influence of internal and external shocks on the index in question.

Step 5. To process the simulation results and to obtain probabilistic estimates for the forecast value.

Our analysis of the data on the monthly averaged REERs, $\Delta$REERs, and $\Delta^2$REERs showed that the polynomial residuals model corresponding to $p = 1$

$$r_{t+1} = r_t + q_t^p + s\xi_t,$$  \hspace{1cm} (4)

and $p = 2$

$$r_{t+1} = r_t + \beta_{0t} + \beta_{1t} + s\xi_t.$$  \hspace{1cm} (5)

and

$$r_{t+1} = r_t + \beta_{0t} + \beta_{1t} + s\xi_t.$$  \hspace{1cm} (6)

can be applied with acceptable accuracy over 18–30 month time intervals.

Figure 2 presents the dynamics of the REER, $\Delta$REER, and $\Delta^2$REER for the ruble from October 2009 to September 2017 and the distribution histogram for the values of indices under consideration.

![Figure 2](image_url)

**Figure 2.** Dynamics (a, b, c) and distribution histogram (d, e, f) of the ruble REER, $\Delta$REER, $\Delta^2$REER from January 1994 to September 2017.
We used the test of Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) to assess the hypothesis that univariate ΔREER, Δ²REER are trend stationary. It turned out that the result depends on which part of the time series the test is applied to so the degree of the polynomial depends on the length of the time period, hereinafter called the “estimation depth”.

The parameters of models (5), (6) can be found in different ways. The most widely used is the method of least squares. To estimate the model coefficients the deviations $\xi_t$ around the polynomial line are assumed to be normally and independently distributed with a mean of 0 and a standard deviation $s$ which does not depend on $t$. The variance estimate $s^2$ of the stochastic component is calculated in accordance with (3). Another way to find the parameters is to formulate a criterion for matching the calculated and observed values and apply the method of search of the parameters minimizing the criterion.

Provided the parameters of models (5), (6) are found we are enabled to construct the REER trajectories and obtain stochastic estimates of the forecast value.

**Simulation Results**

The parameters in models (5), (6) were calculated using the Nelder–Mead algorithm (Bunday, 1984; Nelder, Mead, 1965), known as one of the best algorithms for multidimensional unconstrained optimization. The criterion for estimating the required parameters $(\beta_0, \beta_1, s)$ and $(\beta_0, \beta_1, \beta_2, s)$ was a minimum for standard deviation of the REER index calculated values from the observed ones over the time period from the chosen starting month to September 2017. The calculated values were understood as a result of the REER simulation based on relations (5), (6) and subsequently averaged over 1000 trajectories of the REER index.

The REER simulation was performed for different values of the estimation depth. Parameters for model (5) (polynomial degree $p = 1$) at three values of the estimation depth 18, 23 and 28 months, are given in Table 1.

<table>
<thead>
<tr>
<th>Estimation depth, mo.</th>
<th>$\beta_2$</th>
<th>$\beta_1$</th>
<th>$\beta_0$</th>
<th>$s_k$</th>
<th>Width of the 50% confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>-0.002</td>
<td>-0.082</td>
<td>8.972</td>
<td>2.586</td>
<td>7.36</td>
</tr>
<tr>
<td>25</td>
<td>-0.022</td>
<td>1.899</td>
<td>39.751</td>
<td>2.972</td>
<td>10.27</td>
</tr>
<tr>
<td>30</td>
<td>-0.021</td>
<td>1.875</td>
<td>39.838</td>
<td>3.618</td>
<td>14.92</td>
</tr>
</tbody>
</table>

Parameters for model (6) (polynomial degree $p = 2$) at three values of the estimation depth 20, 25 and 30 months, are given in Table 2.

<table>
<thead>
<tr>
<th>Estimation depth, mo.</th>
<th>$\beta_1$</th>
<th>$\beta_0$</th>
<th>$s_k$</th>
<th>Width of the 50% confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>-0.188</td>
<td>9.695</td>
<td>2.360</td>
<td>5.909</td>
</tr>
<tr>
<td>23</td>
<td>-0.004</td>
<td>0.628</td>
<td>3.007</td>
<td>9.470</td>
</tr>
<tr>
<td>28</td>
<td>0.082</td>
<td>-3.497</td>
<td>3.501</td>
<td>13.561</td>
</tr>
</tbody>
</table>

Examples of typical simulation results are presented in Figure 3 and Figure 4. Figure 3 shows the simulated REER trajectories (thin dotted line), the mean trajectory calculated over 1000 trajectories (solid line) at the selected period of time, the borders of the 50% confidence...
domain (bold dotted line), and statistic data (bold points). Graph (a) in Figure 3 corresponds to the first order polynomial residuals model, graph (b) to the second order one.

Figure 3. Results of the ruble real effective exchange rate simulation: simulated REER index trajectories (‘-’), the mean calculated trajectory (–), the borders of the 50% confidence domain (–·–), statistic data (–·–), real values of the REER (–––) in August–October 2017. Subgraph a) corresponds to the first order model at the "estimate depth" 18 months; graph b) corresponds to the second order model at the "estimate depth" 20 months.

Figure 4 shows the mean calculated trajectory (solid line), the borders of the 50% confidence domain (dotted line), statistic data (bold points), real REER values (empty circles) in September-December 2017 which were forecasted.

Figure 4. Results of the ruble real effective exchange rate simulation: simulated REER index trajectories (‘-’), the mean calculated trajectory (–), the borders of the 50% confidence domain (–·–), statistic data (–·–), real REER values (–––) in September-December 2017. Subgraph a) corresponds to the first order model at the "estimate depth" 18 months; graph b) corresponds to the second order model at the "estimate depth" 24 months.

To estimate the quality of the generated forecast for every simulation were used the most common metrics – Mean Absolute Percentage Error (MAPE) and Root Mean Square Error (RMSE), calculated for errors $e_k$ and percentage errors $P_k$ as follows:

$$MAPE = \frac{1}{n} \sum_{k=1}^{n} |P_k| , \quad RMSE = \sqrt{\frac{1}{n} \sum_{k=1}^{n} e_k^2}.$$
Typical values of MAPE and RMSE are presented in Table 3.

<table>
<thead>
<tr>
<th>Estimate depth, mo</th>
<th>p=1</th>
<th>p=2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MAPE</td>
<td>RMSE</td>
</tr>
<tr>
<td>12</td>
<td>0.0125</td>
<td>2.9879</td>
</tr>
<tr>
<td>18</td>
<td>0.0088</td>
<td>1.9506</td>
</tr>
<tr>
<td>24</td>
<td>0.0101</td>
<td>1.9326</td>
</tr>
<tr>
<td>30</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

**Discussion and Conclusions**

Real effective exchange rates reflect the stability, strengthening or weakening of the national currency, taking into account foreign trade turnover and the prospects for the competitive ability development of the country in foreign markets. To analyze the dynamics of exchange rate data, indices are used, whose value comparability for different countries is achieved on the basis of a common reference period. In the BIS Bank database, 2010 is taken into account as the reference period.

The dynamics of the real effective exchange rate of the ruble may be differently variable for different time intervals. This assumes accounting actual trends in the ruble REER over different time intervals for constructing forecasts for this index values.

The polynomial residues model assumes a formal representing the index residues under study in the form of a polynomial over the selected time period and taking into account a noise variable. This allows us to consider the features of the actual REER dynamics of the ruble in the simulation mode and ensures a quite reliable medium-term forecast. The quality of the forecast is confirmed by the significantly low values of MAPE and RMSE. At the same time, the predicted values of the ruble REER are located within the constructed confidence interval. This confirms the hypothesis that proposed approach allows building a relatively accurate medium-term forecast for the ruble REER using data for a prior period of a certain length. The problem of optimal "estimate depth" period for the forecast requires a separate study, as well as the possibility of applying the proposed approach to the REER of other currencies.

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