MODELLING OF THE OPTIMAL INCOME TAX SCHEDULE

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Abstract

This article tackles the issue of choosing the optimal income tax schedule. The author discusses tax schedules' approach based on two mathematical models. The first model is a game-theory model which helps to construct the average tax rates schedule. The second model is an optimization model used to construct the marginal tax rates schedule. Defining the input parameters is the most challenging step in the application of these models.

The restrictions on the choice of input parameters for the first (main) model are defined. Special attention is devoted to the choice of the elasticity parameters in this model.

The obtained results allow to reduce the issue of defining the marginal income tax rates to the definition of the input parameters in the main model. The same remark is valid for the issue of defining the level of tax schedule. Moreover, the quantity of the pointed-out parameters for this model problem turns out to be less than in the case of the direct choice of the marginal tax rates schedule.

Keywords: optimal income taxation, marginal and average tax rates.

JEL code: C61, C72.

Introduction

The urgency of the issue to evaluate the optimal parameters of the progressive income tax schedule is caused by the following reasons. First, the modern theory of the optimal income taxation does not give the exhaustive answer to neither the question of calculating the marginal tax rates nor the question of choosing the boundaries of their application ranges. This theory began to develop rapidly from the early 70s of the twentieth century after the publication of the article by J. Mirrlees (Mirrlees, 1971). Despite this fact, analytical results in this theory were obtained only in a special case with many simplifying suppositions and assumptions (see, e.g., Slemrod, 1983; Saez, 2001; Boadway and Jacquet, 2008; Diamond and Saez, 2011). Moreover, the simplifying suppositions themselves are weakly motivated.

Secondly, the practical significance of solving this issue for the Russian system of income taxation is confirmed by the fact that in the vast majority of developed countries the progressive income tax is charged.

A new approach to the solution of the discussed problem was proposed by S.V. Chistyakov. Initially, this approach was described in relation to the choice of the progressive schedule of average profit tax rates in the form of a variational model (Smirnov and Chistyakov, 1993). Later, in the framework of this approach, a system of two economic-mathematical models for selecting the progressive income tax schedule was proposed (Chistyakov and Ishkhanova, 1998).

The first (main) model of constructing the progressive schedule of average income tax rates is based on the games theory approach. It may be regarded as a modification and development of the original variational model. Within theory of optimal income taxation, the proposed model was the first one to describe explicitly the function, which determines the optimal schedule of average income tax rates. The criteria of maximizing the tax revenues to the state budget is taken into account in this case.

The new technique of selecting the input parameters had been proposed for practical applications of the considered game-theoretic model (Smirnov, 2011).

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However, the application of the model is complicated by the fact that the constructed optimal schedule of average income tax rates can not be represented in the form of a table (schedule of marginal tax rates), which is usually used in practice.

One of the possible ways to overcome this complication is to construct the best approximation of the optimal model schedule of average tax rates by the schedules of average rates which will allow the usual tabular representation. Formulated problem represents the second (optimization) model within the discussed approach.

We will describe only the first of the two above mentioned models, which is the basic one. The second model was studied in (Smirnov, 2016).

Game-theoretic model of constructing the optimal income tax schedule

Here is a brief description of this model. The progressive schedule of average income tax rates is modeled by an absolutely continuous function $y = y(x) \in (0,1)$, $x \in [0, +\infty)$, which almost everywhere on some interval $[x_-, x_+]$ follows differential inequalities

$$0 < \frac{dy}{dx} < \frac{1-y}{x}. \tag{1}$$

In addition, $y = y(x)$ is a constant on each of intervals $(0, x_-]$ and $[x_+, +\infty)$. More precisely

$$y(x) = 0, \ \forall x \in (0, x_-], \tag{2}$$

$$y(x) = y_+, \ \forall x \in [x_+, +\infty), \tag{3}$$

where x is the personal income, x_- is the minimal income subject to the taxation, x_+ is the level of income for which the tax is levied at the maximal average tax rate y_+.

Left side of inequalities (1) implies that function $y = y(x)$ grows on the corresponding interval, i.e. that the schedule is progressive (Musgrave and Thin, 1948). At the same time, from the right side of these inequalities follows that on the same interval function $D(x) = [1 - y(x)]x$ grows as well. It means that the part of the income remaining after the payment of taxes increases while the growth of gained overall income takes place. This statement is valid despite of the average tax rate growth.

Further in the model we are keeping in mind that for $x \in (0, +\infty)$, the set of all absolutely continuous solutions of the system of differential inequalities (1) coincides with the set of solutions of the parametric family of differential equations

$$\frac{dy}{dx} = u\frac{1-y}{x} \tag{4}$$

in the class of all possible Lebesgue-measurable functions satisfying additional condition

$$u = u(x) \in (0,1). \tag{5}$$

Since one of the most important functions of taxes is the fiscal function, we consider the problem of maximizing the functional which describes the total amount of tax revenues to the state budget. Thus, we reduce the choice of the appropriate income tax schedule to the problem of some functional maximization. As it was shown in (Chistyakov and Ishkhanova, 1998, p. 8 – 13), this functional assumes the shape

$$T(y, f) = \int_0^{+\infty} y(x)df(x), \ y \in Y, \quad f \in F, \tag{6}$$

where function $f:(0,+\infty)\to(0,+\infty)$ is so-called income distribution function, whose value $f(x)$ at the point $x\in(0,+\infty)$ represents the total income of all those taxpayers whose personal income does not exceed x; F – the set of admissible functions of distribution of the personal incomes $f=f(x)$; Y – the set of all absolutely continuous functions, satisfying conditions (1) - (3). Functional (6) describes the total amount of taxes which is paid by citizens in case of some given schedule function $y=y(x)$ and the known distribution function of incomes $f=f(x)$.

Thus, the model of choosing the average tax rates schedule can be described as the problem of defining the optimal strategy for the first (maximizing) player in the antagonistic game $\Gamma=\langle Y,F,T\rangle$ for which the payoff function T implies form (6). In this problem the set of strategies for the second player F is represented by the set of admissible functions describing the personal incomes distribution $f=f(x)$. Moreover, the set of strategies for the first player Y coincides with the set of all absolutely continuous functions $y=y(x)$, satisfying conditions:

$$\frac{dy}{dx}=u\frac{1-y}{x}, \tag{7}$$

$$u=u(x)\in[\delta,\sigma], \tag{8}$$

$$0<\delta\le\sigma<1, \tag{9}$$

$$y(x_-)=0, \tag{10}$$

$$y(x_+)=y_+, \tag{11}$$

$$(0<x_-<x_+,\ 0<y_+<1).$$

Note that in this case we realize transition to a closed set of values for the control function $u=u(x)$. The transition from condition (5) to conditions (8), (9) provides the existence of exact solution of the problem. This transition makes it possible to choose two extra exogenous parameters δ and σ of the model. These parameters determine the minimal and maximal (on the segment $[x_-,x_+]$) values of elasticity for the tax schedule function $y=y(x)$ corresponding to income x.

We are paying attention to the fact that income distribution, generally speaking, depends on conditions of the taxation and, in particular, on the chosen schedule. Because of this the application of game theory model seems more preferable in comparison with the usual optimization approach to maximize functional (6) under restrictions (7) – (11). More precisely, the above described game is considered as a game against the nature. It is remarkable, that in this game the maximizing player has a dominant strategy which is obviously his optimum strategy. Thus, it turns out that this strategy does not depend on the set of admissible distributions of incomes F (admissible set of strategies of the second player). As the government's interests with respect to the taxation field are basically of fiscal character, the model of antagonistic game in contrast to the model of non-antagonistic one seems more relevant for this problem. Evidently the model of non-antagonistic game gives the opportunity to take under consideration some other interests of taxpayer except the unwillingness to pay taxes. One of such interests, for example, is the consumption level of public goods.

In the explicit form the optimal modeling schedule of the average tax rates which is determined on the basis of solution to the described game takes the following form (Chistyakov and Ishkhanova, 1998, p. 14 – 31):

$$y_{opt}(x)=\begin{cases} 0, & 0 \le x < x_- \\ 1-\left(\dfrac{x_-}{x}\right)^{\sigma}, & x_- \le x < x_0, \\ 1-(1-y_+)\left(\dfrac{x_+}{x}\right)^{\delta}, & x_0 \le x \le x_+, \\ y_+, & x > x_+, \end{cases} \tag{12}$$

where

$$x_0=\left((1-y_+)\frac{x_+^{\delta}}{x_-^{\sigma}}\right)^{\frac{1}{\delta-\sigma}}. \tag{13}$$

At the same time the inequality

$$\left(\frac{x_-}{x_+}\right)^{\sigma} \le 1-y_+ \le \left(\frac{x_-}{x_+}\right)^{\delta},$$

providing compatibility of conditions (7) – (11) must be valid.

For practical use of the above described model (6) – (11) it is necessary to give recommendations for the choice of five input parameters: x_-, x_+, y_+, δ and σ.

In spite of the well-known concept of elasticity in economic theory, the choice of input parameters δ and σ brings to certain difficulties. In other words, valid recommendation for the choice of these indicators is hardly possible. Because of this, instead of parameters δ and σ was suggested to choose parameters x_0 and y_0 which are related to the original parameters by relationships (12) and (13). More precisely x_0 is defined by ratio (13) and y_0 is defined by the value of function (12) at the point x_0 (see Smirnov, 2011).

The choice of parameter x_0 depends on the objectives of the tax policy and it can be given different economic interpretations. For instance, this parameter may be interpreted as the bottom boundary of the middle-class incomes. It is calculated similar to parameter x_+ while in this case parameter y_0 is regarded as the average tax rate for income x_0.

Thus, keeping in mind, the necessity to use the optimal schedule (12) simultaneously with the procedure of replacing exogenously given parameters δ and σ by parameters x_0 and y_0 we arrive to the necessity of solving the following system of equations:

$$x_0=\left((1-y_+)\frac{x_+^{\delta}}{x_-^{\sigma}}\right)^{\frac{1}{\delta-\sigma}},$$

$$y_0=1-(1-y_+)\left(\frac{x_+}{x_0}\right)^{\delta},$$

under constraints

$$0<\delta\le\sigma<1,$$

$$0 < x_- < x_0 < x_+ ,$$
$$0 < y_0 < y_+ < 1 .$$

Parameter y_0 corresponding to a certain parameter $x_0 \in (x_-, x_+)$ must be selected taking into account the following restrictions (see Smirnov, 2011):

$$1-(1-y_+)^{\frac{\ln x_- - \ln x_0}{\ln x_- - \ln x_+}} \le y_0 < \min\left\{ y_+, 1-\frac{x_-}{x_0} \right\}.$$

Conclusion

Thus, the obtained results make it possible to reduce the problem of defining the marginal rates of income tax and corresponding grades to the choice of five input parameters in the considered model. These parameters can be determined through statistical data and on the basis of the tax policy objectives taken into account. The choice of the above-mentioned parameters must satisfy the imposed restrictions. Moreover, it turns out that the quantity of such parameters in the model is less comparing with the approach when direct solution of the marginal tax rates schedule is considered.

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