Digital model of optimal investment budget formation

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***Abstract* – The study discusses the problems associated with substantiating the optimal size of the investment budget. At present, the generally accepted approach is the one that involves a joint analysis of two schedules: of investment opportunities and of marginal cost of capital. This approach assumes that the optimal size of the investment budget lies at the intersection of these schedules. In this paper, it is shown that the classical approach does not take into account the nonlinear nature of the dependence of the return on investment portfolio on the cost of capital sources. The digital model constructed in the framework of this research of the dependence of project profitability on the cost of capital sources shows that with an increase in the cost of capital, the decrease in the internal rate of return is increasing. The classical approach does not take this factor into account, which may lead to the fact that projects with a factually negative or rather low internal rate of return are included in the investment portfolio. The study proposes for the first time a digital model for the formation of an optimal investment budget, offering instead of a graphical model of investment opportunities to use a graph of the internal rate of return of the investment portfolio as a whole, depending on the size of the budget. Joint consideration in the model of data on the internal rate of return on the investment portfolio and the marginal cost of capital allows to avoid the inclusion of inefficient projects in the portfolio. In addition, to form the investment budget, it is proposed to additionally consider information on changes in the net present value of the investment portfolio depending on the size of the budget. Using in the model data on changes in the internal rate of return and net present value of the investment portfolio as a whole on the size of the investment budget allows us to increase the validity of management decisions related to the formation of the investment portfolio.**

Keywords — digitalization, model, budget, investment

I. Introduction

The task of forming the optimal investment budget arises in those cases when in the conditions of a limited investment budget it is necessary to form a certain investment portfolio that ensures investment efficiency acceptable for the investor [1]. When forming the investment budget, it is essential to consider many factors related to the selection of possible sources of financing, taking into account the fact that with the growth of the investment budget, the cost of capital usually changes upwards [2].

II. Research Methodology

Traditionally, when forming the investment budget, a joint analysis of the graphical model of investment opportunities or IOS (Investment Opportunity Schedule) and the graphical model of the marginal cost of capital or MCC (Marginal Cost of Capital) is used [1-7].

When constructing a graphical model of investment opportunities (Fig. 1), the abscissa axis depicts the total amount of investment associated with the investment portfolio, and the ordinate axis represents the internal rates of return IRR of the investment options considered, sorted in descending order.

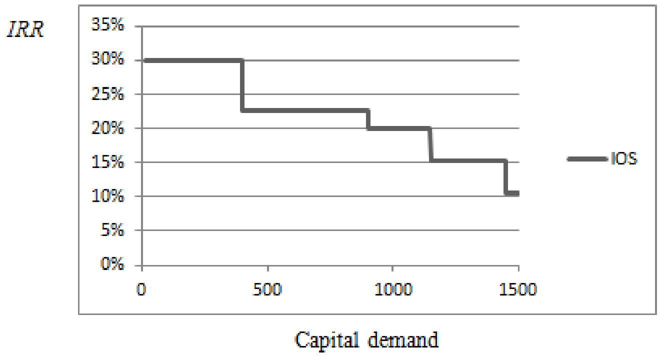


Fig. 1. Investment Opportunity Schedule

Since when creating a graphical model of investment opportunities, it is assumed that investment projects should be placed in descending order of IRR, therefore, the IOS is a non-increasing function of the investment budget size.

The MCC Schedule (Fig. 2) reflects the value of the last unit of investment. Obviously, when forming the budget for investments, the sources of investment with the lowest cost of capital are selected first, therefore, the marginal cost of capital is a non-decreasing function of the size of the investment budget size [3-4].

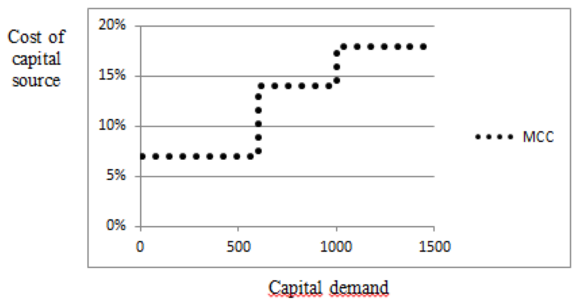


Fig. 2. Marginal Cost of Capital Schedule

Joint plotting of IOS and MCC schedules allows us to find their intersection point (Fig. 3). In Fig. 3 proposed for inclusion in the portfolio projects are to the left of the point of intersection of the schedules, and projects lying to the right of this point should be turned away.

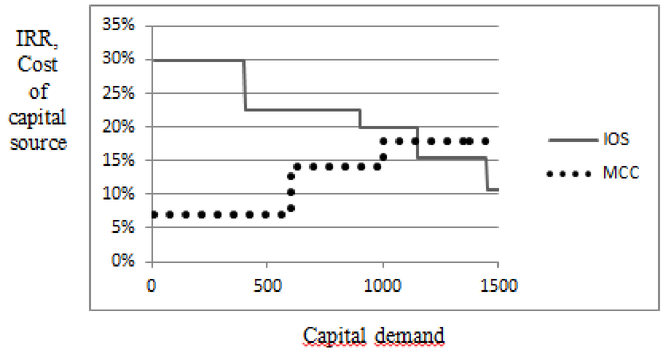


Fig. 3. Combined model of investment opportunities and marginal cost of capital

It is assumed that this approach will enable all profitable projects to be included in the investment portfolio and to refuse the implementation of all inefficient projects with negative returns. This approach, at first glance, is simple and clear, which results in its popularity.

III. Problem Statement

Despite the advantages of the above approach to the formation of an investment portfolio, it should be noted that under certain conditions it can lead to the situations when projects with virtually negative returns or with rather low returns will be included in the investment portfolio. This is due to the fact that when calculating the IRR values of individual investment projects, cash outflows associated with servicing the initial investments are not fully taken into account (for example, interest payments on loans, dividends to shareholders, etc.).

It is believed that the intersection point of the IOS and MCC schedules allows us to determine the maximum allowable investment budget. In this case, depending on the conditions for the implementation of the investment projects under consideration (whether they are subject or not subject to splitting), the capital investment budget may be adjusted downwards [5].

This approach to the formation of the investment budget is well-known and repeatedly described in the literature on investment analysis and corporate governance [5]. The obvious advantage of this approach is its simplicity and clarity. Unfortunately, this approach in many respects does not take into account the influence of the cost of capital on the internal rate of return of the formed investment portfolio. In fact, this approach comes down to reviewing a graphical model formed as the difference between the IOS and MCC schedules. Such a schedule (denoted by IOS*) is a chart of the investment opportunities, adjusted for the MCC (Fig. 4).

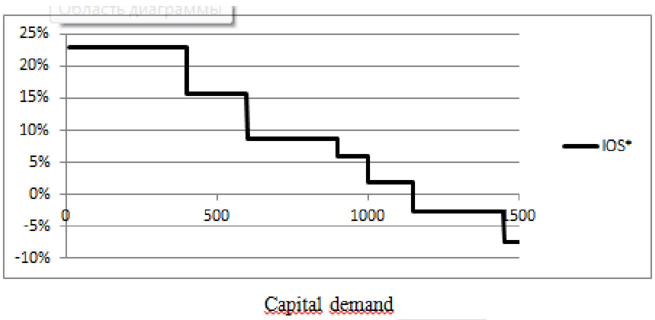


Fig. 4. IOS adjusted for the MCC

In this case, to determine the optimal investment budget, it is proposed to focus on the intersection point of this schedule with the abscissa axis [6].

Unfortunately, the IRR of the project, taking into account the cost of attracted capital, non-linearly depends on the cost of capital, which does not allow the use of IOS and MCC schedules (similarly, the IOS* schedule) as a correct tool for investment budget formation.

IV. Research Results

We will research the nature of this dependence on the example of an isolated project. Let us review a more general case when a project can be partially implemented, i.e. the project can be splitted. Suppose a project with a duration of T years has the following net cash flow: –IC, CF1, ..., CFT, where IC are the project investments, CFt are the net cash flow elements corresponding to the tth year of the project.

Investments with a cost of I percent per annum are attracted to finance the project. In this case, the possible volume of attracted investments is d·IC, where d is the share of the project that received financing (0 <d≤1) with the cost of capital I. Then the internal rate of return IRR0 excluding the cost of invested capital (which corresponds to IOS) is found from the equation

$$-d \cdot IC + \sum_{t=1}^{T} \frac{d \cdot CF_t}{(1+IRR_0)^t} = 0. \tag{1}$$

In case if we intend to take into account the outflows associated with servicing the invested capital, then the net cash flow of the project will be: d·IC, d·CF1 – d·IC·I, ..., d·CFT – d·IC·I , where d·IC·I – outflows associated with servicing investments. From here we find the IRR from the equation

$$-d \cdot IC + \sum_{t=1}^{T} \frac{d \cdot CF_t - d \cdot IC \cdot I}{(1+IRR_1)^t} = 0. \tag{2}$$

Obviously, IRR1<IRR0. Moreover, due to the nonlinear nature of the dependence of the IRR on the cost of capital (I), in this case, the following inequality will be accomplished

$$IRR1<IRR0–I. \tag{3}$$

It should be noted that when forming the investment budget based on the IOS and MCC schedules, the following ratio is used

$$IRR1=IRR0–I, \tag{4}$$

which seems incorrect.

For a more detailed study of the dependence of the internal rate of return IRR1 on the cost of capital I, we will construct a linear digital model [8–9] of this dependence using the Maclaurin's series. For a linear digital model of the analytic function y(x), it is sufficient to construct a Maclaurin's series of the form

$$y(x) = y(0) + y'(0)\cdot x, \tag{5}$$

where y(0) is the value of the function at the point x=0; y'(0) – value of the first derivative function at the point x=0.

For convenience, we will denote the internal rate of return IRR1 as y, and the cost of capital I as x. Then, in accordance with the notations introduced, we must construct the Maclaurin's series for the function

$$-d\cdot IC + \sum_{t=1}^{T} \frac{d\cdot CF_t - d\cdot IC\cdot x}{(1+y)^t} = 0. \tag{6}$$

According to formula (1), the value of this function at the point x=0 will be equal to IRR0. Since the function under study is given implicitly, therefore, to find the first derivative, it is necessary to find the partial derivatives with respect to the variables x and y at the point x=0. The partial derivative with respect to x has the form [10]

$$\frac{\partial F(x,y)}{\partial x} = -d\cdot IC \sum_{t=1}^{T} \frac{1}{(1+y)^t}, \tag{7}$$

where F(x, y) is an implicit function of the form (6). The partial derivative with respect to y has the form

$$\frac{\partial F(x,y)}{\partial y} = -d \sum_{t=1}^{T} \frac{(CF_t - IC\cdot x)\cdot t}{(1+y)^{t+1}}. \tag{8}$$

Then at the point x=0 the first derivative is found the following way

$$y'(0) = -\left.\frac{\frac{\partial F(x,y)}{\partial x}}{\frac{\partial F(x,y)}{\partial y}}\right|_{x=0} = -\left.\frac{-d\cdot IC\sum_{t=1}^{T}\frac{1}{(1+y)^t}}{-d\sum_{t=1}^{T}\frac{(CF_t - IC\cdot x)\cdot t}{(1+y)^{t+1}}}\right|_{x=0} = -IC\frac{\sum_{t=1}^{T}\frac{1}{(1+IRR_0)^t}}{\sum_{t=1}^{T}\frac{CF_t\cdot t}{(1+IRR_0)^{t+1}}}. \tag{9}$$

Since for the investment project under consideration the conditions IC>0 and CFt>0 are satisfied, therefore, the first derivative is y'(0)<0, which confirms the validity of the ratio IRR1<IRR0. As a result, we obtain the following digital model of the dependence of the internal rate of return of a project on the cost of attracted capital.

$$IRR_1 = IRR_0 - I\cdot IC\frac{\sum_{t=1}^{T}\frac{1}{(1+IRR_0)^t}}{\sum_{t=1}^{T}\frac{CF_t\cdot t}{(1+IRR_0)^{t+1}}}. \tag{10}$$

V. Results Discussion

Let us consider the dependence of the internal rate of return on the cost of investment in the following example.

Example 1. Let the initial investment in the project be IC=100. The project lasts for five years. Net cash flow CFt is constant and equal to 40 each year. In this case, the internal rate of return excluding the cost of capital I (IRR0) will be 28.65%.

Taking (10) into consideration, the following equation was obtained for a linear digital model

$$IRR_1 = 28,65\% - 1,2816\cdot I. \tag{11}$$

Table 1 shows the calculated values of the internal rate of return taking into account the cost of capital I, as well as the results of the linear digital model.

TABLE I. Exact and approximated values of the internal rate of return depending on the cost of capital

Cost of capital (I), %	Exact value IRR1, %	Linear approximation IRR1, %	IRR0 – I, %
0%	28.65%	28.65%	28.65%
1%	27.36%	27.37%	27.65%
2%	26.07%	26.09%	26.65%
3%	24.76%	24.80%	25.65%
4%	23.44%	23.52%	24.65%
5%	22.11%	22.24%	23.65%
6%	20.76%	20.96%	22.65%
7%	19.40%	19.68%	21.65%
8%	18.03%	18.40%	20.65%
9%	16.64%	17.11%	19.65%
10%	15.24%	15.83%	18.65%
11%	13.82%	14.55%	17.65%
12%	12.38%	13.27%	16.65%
13%	10.92%	11.99%	15.65%
14%	9.43%	10.71%	14.65%
15%	7.93%	9.43%	13.65%
16%	6.40%	8.14%	12.65%
17%	4.85%	6.86%	11.65%
18%	3.26%	5.58%	10.65%
19%	1.65%	4.30%	9.65%
20%	0.00%	3.02%	8.65%

Fig. 5 shows the corresponding graphs.

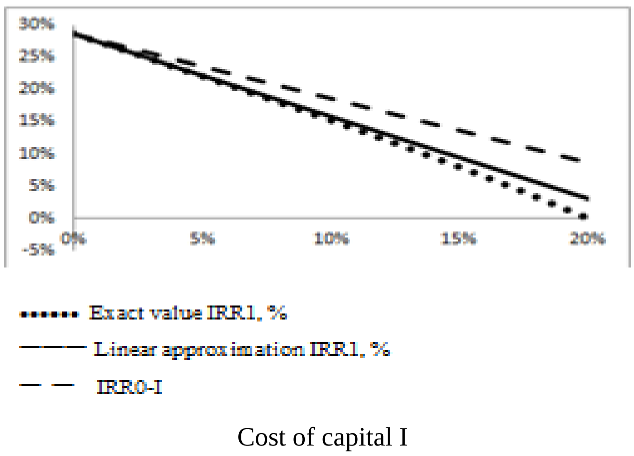


Fig. 5. A graphical model of the dependence of the internal rate of return on the cost of capital

It can be noticed that with an increase in the cost of capital I, the 'exact' internal rate of return of a project decreases nonlinearly. At the same time, a simplified assessment of the profitability of a project of the type IRR0 - I gives the largest deviations from the exact values of the returns. It should be noted that with an increase in the cost of capital I, the margin of error in estimating the profitability of a project of the type IRR0 - I significantly increases, which results in a significant overstatement of the profitability estimates of the latest projects considered for inclusion in the investment portfolio. Nevertheless, traditionally, when substantiating the investment budget based on the IOS and MCC schedules, it is this option for evaluating the profitability that is used.

Thus, the use of IOS and MCC schedules (or equivalent to the IOS* graphical model) as a tool to substantiate the investment budget leads to an overestimation of the profitability of investment projects (especially for projects involving capital raising with a fairly high cost). As a result of such an incorrect implementation of the procedure of budget forming for investment, the formed investment portfolio can include either projects with a negative internal rate of return or with a rather low real rate of return.

For forming the investment budget, it is proposed to consider, instead of profitability limit values (IOS), indicators calculated by the total net cash flow of the investment portfolio formed for a specific budget. In particular, it is proposed to consider the internal rate of return and the net present value of the investment portfolio as a function of the investment budget. Thus, for each possible investment budget, a net cash flow is formed; it takes into account the cash flows associated with servicing various sources of capital, and for it the internal rate of return and the net present value are calculated.

Let us consider the following example 2.

Example 2. Table 2 shows the projected net cash flows associated with the implementation of various investment projects. In Table 2 projects are already ordered in descending order of internal rate of return. We will assume that projects can be splitted, i.e. can be partially implemented. Table 3 shows data on possible sources of capital, which are put in order of increasing cost of capital.

TABLE II. POSSIBLE NET CASH FLOWS OF INVESTMENT PROJECTS, C.U.

Project	Year							IRR, %
	0	1	2	3	4	5	6	
A	–400	120	150	160	170	180	180	29.9%
B	–500	160	160	160	160	160	160	22.6%
C	–250	75	75	75	75	75	75	19.9%
D	–300	80	80	80	80	80	80	15.3%
E	–150	35	35	35	35	35	35	10.6%

TABLE III. DATA ON CAPITAL SOURCES

Source of capital	Maximum possible size, c.u.	Capital cost, %
1	600	7%
2	400	14%
3	500	18%

Fig. 5 shows the IOS and MCC schedules. Considering these graphs, we can conclude that projects A, B and C are acceptable. At the same time, the necessary investment will be 1150 c.u. At the point of intersection of the schedules IOS=19.9% (corresponds to project C) and MCC=18% (corresponds to source of capital No. 3). Thus, it seems that project C can be included in the investment portfolio, since the ratio IOS>MCC is fulfilled. However, if to form the net cash flow of project C, taking into account the cost of servicing the capital, the overall situation does not look so bright. Let us calculate the cost of servicing capital for project C. The required investment for the project is 250 c.u. We can obtain 100 c.u. from a source of capital No. 2 at 14% and 150 c.u. from a source of capital No. 3 at 18%. Then the weighted average capital cost for project C is

$$WACC=(100/250)\cdot 14\%+(150/250)\cdot 18\%=16.4\%.$$

Then the annual cost of servicing the capital will be 250·16.4%/100%=41 c.u. and the annual cash flow for the project will decrease from 75 c.u. down to 34 c.u. As a result, the net cash flow for project C will be

–250; 34; 34; 34; 34; 34; 34.

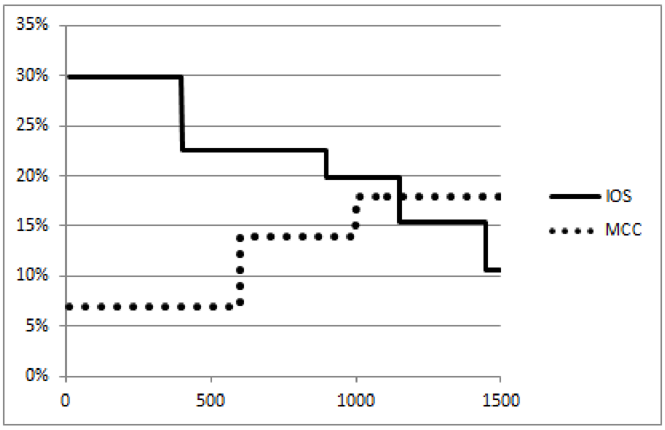


Fig. 6. Graphical model combining IOS and MCC examples

As a result, we will get that the internal rate of return of project C with the selected financing option will in fact be negative and equal to –5.52%.

To increase the substantiation of decisions made on the investment budget formation, it is recommended that instead of constructing and considering isolated IOS and MCC schedules, we introduce a schedule reflecting a change in the internal rate of return of the investment portfolio depending on the size of the investment budget (Portfolio's Internal Rate of Return, PIRR).

In Fig. 6, IOS, MCC, and PIRR schedules are plotted together. This schedule clearly demonstrates that project C is somewhat "self-critical". Although the inclusion of project C in the portfolio does not fundamentally affect the overall return on the portfolio, the cost of capital required to implement this project is not offset by the income from the portfolio of projects (the PIRR schedule crosses the MCC schedule and goes down).

Thus, the intersection point of the PIRR and MCC schedules can serve as a guideline in the formation of the investment budget. An increase in the budget beyond the point of intersection of these schedules leads to the situation in which the cost of attracted capital begins to exceed the return on the investment portfolio. However, it can be noted that if the investor is guided by a certain lower limit of the rate of return when forming the portfolio, he can continue to include projects with low or even negative returns in the portfolio. Returning to our example, an investor forming a portfolio with a return of at least 10% could include project C in his portfolio. If the investor focused on a return of 5% or higher, then he could include project D in the portfolio as well. It is clear that such an expansion of the investment portfolio is not economically feasible, since it significantly reduces portfolio profitability, but the implementation of such projects that seem to be ineffective at first glance often allows us to solve problems associated with conducting scientific research, the implementation of social, environmental, infrastructural, etc. projects aimed at ensuring sustainable development in the long run.

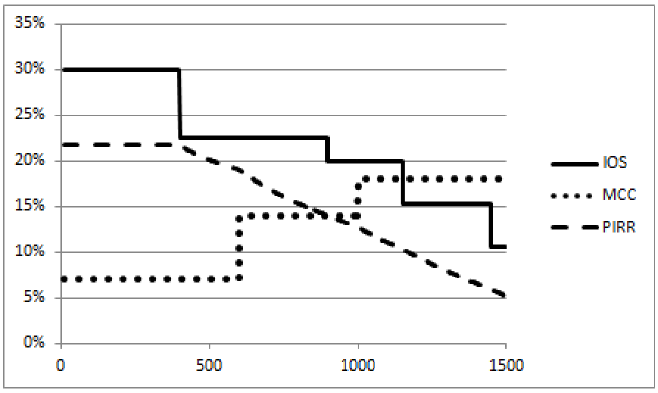
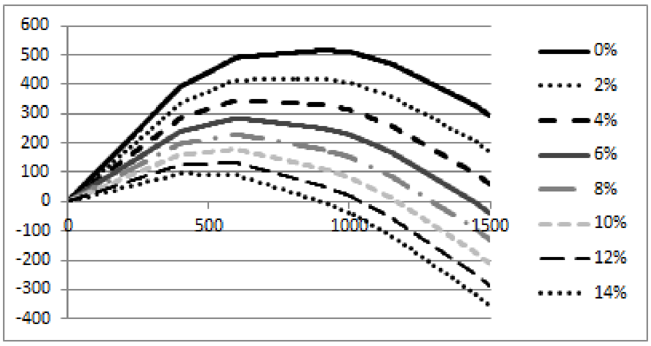


Fig. 7. Graphical model combining IOS, MCC and IPIRR examples

To calculate various indicators related to the analysis of the investment budget, software was developed for Microsoft Excel spreadsheets using the programming language Visual Basic for Applications and a certificate of state registration of the computer program ‘Investment budget forming’ was received [9]. In particular, apart from IOS, MCC and PIRR, the program calculates the values of the net present value of the investment portfolio (Portfolio’s Net Present Value, PNPV) for various investment budget sizes and discounting rates. Fig. 7 shows the schedules of changes in the NPV portfolio for our example. It can be noted that for all the schedules presented in Fig. 7 the existence of a maximum value is characteristic. This optimum point can be used in the formation of the investment budget. The advantages of this criterion include the fact that the value of the discounting rate set by the investor is taken into account here. With an increase in the discounting rate, the capital budget that maximizes NPV decreases, as insufficiently effective projects are removed from the portfolio. In our example, at a discounting rate of 14%, it is advisable for the investors to restrict themselves to implementation of only project A (the maximum value of NPV equal to 96 c.u. is achieved with a budget of 400 c.u., i.e. only if one project A is implemented).

On the contrary, with a reduction in the discounting rate, a budget that maximizes the portfolio’s NPV may include less efficient projects (for example, project B). In our example, at a discounting rate of 2%, the maximum NPV is 418 c.u. achieved with a budget of 900 c.u., i.e. during the implementation of projects A and B. However, since the vertices of the schedules under consideration are quite flat and the maximum is slightly pronounced, therefore, under certain conditions, for example, project C could be included in the portfolio, although its inclusion would lead to a slight decrease in the NPV of the investment portfolio.



Capital demand

Fig. 8. Schedules of changes in the net present value of the investment portfolio depending on the investment budget of the example

VI. Conclusion

The traditional model of forming the investment budget, involving the joint use of a graphical model of investment opportunities and a graphical model of the marginal cost of capital, proceeds from the fact that to assess the profitability of an individual project and the investment portfolio as a whole, it is enough to consider the internal rate of return of the project and the cost of investments associated with the implementation of this project (project portfolio). The paper shows that such an approach to assessing the profitability of projects taking into account the cost of investment capital leads to an overestimation of the profitability of projects in comparison with real profitability. It should be noted that with an increase in the cost of capital, the mentioned above margin of error in estimating the profitability of projects increases significantly. As a result, there may be risks of errors [12] when making managerial decisions related to the formation of an investment portfolio, especially regarding the inclusion of projects with a relatively low profitability and a relatively high cost of capital in the investment program.

Instead of a simplified assessment of the profitability of projects based on the joint use of a schedule of investment opportunities and a schedule of the marginal cost of capital, it is proposed to use a schedule of the internal rate of return for the investment portfolio, built depending on the investment budget size. Such a schedule clearly represents a change in the real profitability of the investment portfolio, taking into account the cost of capital, which allows you to make more realistic decisions when forming a portfolio, identifying and cutting off inefficient projects.

The paper also proposes to use the schedules of changing the net present value of the investment portfolio depending on the size of the investment budget at various discounting rates to formulate the investment budget. These models allow you to visualize the impact of investor preferences, expressed in discounting rate, on the composition of the investment portfolio.

Since the implementation of the proposed approach to forming the investment budget involves a fairly large amount of computation, a digital technology was developed for Microsoft Excel spreadsheets using the Visual Basic for Applications programming language, which automates both the execution of the necessary computational tasks and the construction of relevant schedules [11].

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