

# *Mathematical Modeling of the Differential Dynamics of the Galvanic Process of Restoring the Seats of the Main Supports of Autotractor Engines*

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**Abstract**— Repair of agricultural machinery is one of the most important issues related to maintaining the efficiency of the fleet of vehicles used in the agricultural sector of the country. High equipment with high-performance machines and mechanisms obliges modern repair production to raise its technical level of repair technology, to improve in every way the organization of production, to expand the technical capabilities of production by introducing new technology and advanced technology. Mathematical modeling based on the results of a sample of observations on the current evolution of the galvanic process development for a given technological product (cylinder block bearings) using the third-order spline approximation method, continuous interpolation of the state vector of the galvanic process is constructed. The method of constructing a mathematical model of the electroplating process allows us to raise the question of finding the optimal electrolysis mode in the process of restoring the seats of the engine bearings core blocks. In this case, the optimal electrolysis mode is understood as an electrolytic process, which, on the one hand, provides the specified quality indicators of electroplating, and on the other (simultaneously), minimizes the energy and material costs for its practical implementation. The results of operational tests confirmed the high wear resistance of the electroplated coating, which exceeds the wear resistance of the base material by 20-30%.

**Keywords**—*mathematical model, galvanic process, experiment, repair of agricultural machinery.*

## I INTRODUCTION

One of the main conditions ensuring a high level of organization of repair of agricultural machinery is the correct

and timely provision of repair enterprises with spare parts. Spare parts costs are heavy in the total cost of repairing machines. However, they can be significantly reduced by widespread use in the repair of reconditioned parts. It is known that the complete replacement of worn parts with new ones creates significant difficulties for specialized manufacturers of spare parts. In addition, such a system inevitably leads to a large waste of materials and material resources. Therefore, the search for effective ways to restore parts is of great interest to improve the efficiency of agricultural production.

The modern repair base of the agro-industrial complex has various ways of restoring worn-out machine parts. They in the first place include: welding using electric and gas welding installations vibration arc welding, welding under flux layer, electrolytic plating, metal spraying, electro-spark capacity, method of plastic deformation, recovery with resins, glues, etc. To restore the original dimensions of worn parts, any of the above methods is acceptable, taking into account the technological, physic-chemicals and mechanicals properties of the coating. At the same time, each of these methods has a number of advantages and significant disadvantages that either create technical difficulties in the application of this method, or reduce the economic efficiency of the latter.

In the restoration of parts, especially expensive, it is economically feasible, since the cost of the workpiece in the production is an average of 70 – 75 % of the cost of the parts. Restoration of details is expedient and from the ecological point of view is ecologically destructive, power-consuming metallurgical cycle of production, which gives considerable economy of means at the expense of lower Prime cost in

comparison with cost of a new detail. The main source of economic efficiency of repair is the restoration of worn parts. As a consequence, it is necessary to develop both branded repair and the creation of specialized enterprises for the restoration of parts with a high level of quality of the restored surfaces of the part.

Consequently, the demand for the most economical way to restore parts is of great economic importance [1, 14]. The rejected parts represent the secondary raw materials suitable for the organization of new branch of repair production—the centralized restoration of details which finds application in repair business more and more widely. At the same time, it should be noted that the crucial importance in the production of repair of machines is the rapid and economical restoration of worn seats for bearings of large-sized body parts, as the most responsible and expensive. These include the crankcase of internal combustion engines of cars, the crankcase of the gearbox, the crankcase of the rear axle, hubs, shafts, covers and other.

An important factor affecting the quality of repair of machines is the elimination of defects of body and base parts associated with the loss of accuracy in the mutual arrangement of surfaces by modern methods. Because the wear and aging of the material leads to disruption of the original landing, which is manifested in the increase of the gap in connection with it, or decrease the tightness in the joints mate with an interference fit.

To maintain the operable state of technology, it requires the establishment of technical centers and the development of effective ways to restore parts. Currently, the reliability of the repaired machines is still lower than the new ones. This is due to many factors, the main of which are: violation of the technical conditions of fit of the mating parts and their relative position of the seating and mating surfaces. Since the durability of machines and the service life of their parts are largely determined by the technical condition of the base and body parts, therefore, during repair, it is necessary to monitor and restore the working parameters of these parts.

Each body part has several mounting holes for bearings, bearing cups for bearings, and the wear of these holes causes these expensive parts to be rejected. Depending on their capabilities, various technological processes of repairing holes are used. Recently, work has emerged devoted to the development of new technological processes for the restoration of the mounting holes of body parts [3, 8, 17, 25, 26]. The greatest recognition in the work belongs to electroplating methods, allowing one to receive coatings of any thickness in the range from 0.01 to 1 mm [2, 13]. The problem of more complete use of the residual resource of large-sized basic parts is currently very significant in terms of increasing the durability of machines. It is still possible to explain the fact that such parts are classified as expensive and difficult to manufacture. The following methods of restoring the seats of body parts are most common in the repair practice:

- 1) welding of brass nests [8, 18, 24]; and semicircles in a pre-squandered nest;
- 2) boring nests by displacing their axis to the plane of the connector block (for engines) with a head [3];

- 3) restoration of the original dimensions of the seats by the use of epoxy resins, adhesives and other. [9];
- 4) galvanic method [15, 16, 20].

## II STATEMENT OF THE PROBLEM OF A POSTERIORI MODELING OF THE DYNAMICS OF THE GALVANIC PROCESS

The tasks of a posteriori simulation of the differential dynamics of the galvanic process of restoring parts, on the one hand, are caused by the fact that the observed (current) electrolysis mode may have heterogeneity in its physico-mechanical structure in conjunction with the chemical environment and, therefore, the task of current prediction of the mathematical model of the state equations galvanic process. On the other hand, the fact of the inevitability of a change in the state of the environment of the electrolysis action determines the formulation of its dynamics, adapting to these changes based on the parametric identification of the differential model of the galvanic process of restoring the cylinder block main bearings.

The variety of ways in which one can determine the structure of the equations of state of a galvanic process under study (as a formal D-system; [23, p. 10]) raises the question of choosing among them those that are optimal from the point of view of some physically “transparent” criterion characterizing a certain structural quality models of the dynamics of the observed development of the galvanic process. The difficulty here is that there is no definite answer to this methodological question. The choice of one or another point of view depends not only on the nature of the state variables of the D-system, but also on the objectives of the study, on how the restrictions are set, whether the experimental data are complete or not, and on other factors, in particular, related to the context of the subject areas of study of the mathematical model. And it is unlikely that the methodological position of the D-system characterized by “creative” search (searching for an acceptable model structure, searching for a representative model within the adopted structure, etc.) is doubtful. Essentially, this process is interactive, when, before forming an acceptable model, several models have to sort out, discard. And this process cannot be fully automated, since the decisions made by the researcher-designer will always be intertwined with both formal calculations and rigorous numerical calculations, and heuristic assumptions related to the modeling context.

Recent advances in cybernetics and computer science have led to the emergence of new tools for a posteriori mathematical modeling, for which the following typical components of a decision support system in mathematical modeling of the differential equations of state of the D-system under study can be distinguished:

- a) data processing (filtering, interpolation, extrapolation);
- b) parametric identification;
- c) display of model properties (simulation modeling);
- d) confirmation procedures.

In order to formalize the previous reasoning on a fixed time interval  $T=[t_0, t_1] \subset \mathbb{R}$  (physical experiment interval), we link the current state vector  $x(t) \in \mathbb{R}^n$ ,  $t \in T$  modeling to the

differential dynamics of the galvanic process with the characteristics of its physical and mechanical properties available observational (calculated from experimental data):

- $x_1(t)$  - current value of coating thickness, mm;
- $x_2(t)$  - current value of the micro hardness of the coating, kg mm<sup>-2</sup>;
- $x_3(t)$  - current value of Fe content in the coating;
- $x_4(t)$  - current value of coating brittleness.

The fact that the parameters of the physic-mechanical properties of electroplating coatings and their derivatives are absent as variable phase vector of the state of electrolysis is due to the general diffusion concept of the model of the dynamics of the evolution of the electroplating process. The components of the vector  $x(t) \in R^n$  are piecewise monotone functions (by virtue of their physical nature), therefore, almost everywhere in the time interval T has a derivative (Lebesgue's theorem on the differentiation of a monotonic function [12, p. 324]).

Further, to the current variables of the vector of input actions that "purposefully" affect the dynamics of electrolysis (which determine the required quality of electroplating coatings on parts subject to the physicochemical effect of the electroplating process), we assign the following physical factors:

- $u_1(t)$  - current value of the cathode density of the electric current, A·cm<sup>-2</sup>;
- $u_2(t)$  - current value of the flow rate of the electrolyte, m·s<sup>-1</sup>;
- $u_3(t)$  - current value of the anode rotation frequency, about, turns min<sup>-1</sup>;
- $u_4(t)$  - current pH value of the electrolyte, pH;
- $u_5(t)$  - current value of FeSO<sub>4</sub> / ZnSO<sub>4</sub>.

Thus, the dynamics of the electrolysis process (in the implementation of electroplating) is described by some non-linear system of differential equations:

$$dy_i(t)/dt = f(y_1(t), \dots, y_n(t), u_1(t), \dots, u_m(t)), \quad 1 \leq i \leq n, \quad (1)$$

with appropriate initial conditions

$$y_i(t_0) = c_i, \quad 1 \leq i \leq n; \quad (2)$$

$col(c_1, c_2, \dots, c_n) \in R^n$  is the initial ( $t=t_0$ ) state of the dynamic system S.

System (1) - (2) is due to the complex (non-stationary) interconnection of the kinetics of chemical reactions, hydrodynamics and mass transfer in the electrolyte flow, kinematics of electrode plates, as well as the influence of the anode-cathode electric field on all these processes.

To describe the time interval T, a linearized (with respect to the state  $x = 0, u = 0$ ) non-stationary model of the dynamics of the state vector of a physicochemical process, let us consider the class of controlled models of the system of homogeneous linear differential equations (SADDU):

$$\begin{aligned} dx_i(t)/dt &= f(c_1 + x_1(t), \dots, c_n + x_n(t), u_1(t), \dots, u_m(t)) = \\ &= f_i(c_1, c_2, \dots, c_n) + \sum_{1 \leq j \leq n} \alpha_{ij} x_j(t) + \sum_{1 \leq k \leq m} \beta_{ik} u_k(t) + \dots, \\ &1 \leq i, \quad j \leq n, \quad 1 \leq k \leq m; \end{aligned}$$

$$\begin{aligned} \alpha_{ij} &= \partial f_j(y_1, \dots, y_n, u_1, \dots, u_m) / \partial y_i \Big|_{y_1=c_1, \dots, y_n=c_n, u_1=0, \dots, u_m=0}, \\ \beta_{ik} &= \partial f_j(y_1, \dots, y_n, u_1, \dots, u_m) / \partial u_k \Big|_{y_1=c_1, \dots, y_n=c_n, u_1=0, \dots, u_m=0}, \end{aligned}$$

where dots (+ ...) denote, as usual, the members of a series of a higher order of smallness, (for  $n = 4, m = 5$ ), described by the vector-matrix differential equation:

$$dx(t)/dt = Ax(t) + Bu(t), \quad t \in T \quad (3)$$

where  $\dot{=}$  denotes equality almost everywhere on the interval T,  $x(\cdot) \in AC(T, R^n)$  - Carathéodory type solution (K-solution)  $u(\cdot) \in L_2(T, R^m)$ ,  $A(\cdot) \in L_2(T, L(R^n, R^n))$ ,  $B(\cdot) \in L_2(T, L(R^m, R^n))$  and  $L(R^n, R^n)$  - the space of all linear operators (matrices) acting from the space  $R^n$  to  $R^n$  (here  $AC(T, R^n)$  and  $L_2(T, R^m)$  - respectively, the standard function spaces of absolutely continuous vector functions and equivalence classes of all Lebesgue integrable [12] vector functions with the standard  $L_2$ -norm [12]). As usual [6], the pair  $(A(\cdot), B(\cdot)) \in L_2(T, L(R^n, R^n)) \times L_2(T, L(R^m, R^n))$  is called the (A,B)-model differential system (3).

In such a mathematical formulation, the problem of a posteriori simulation of a certain (formed experimentally on the time interval T, that is, a posteriori) dynamic process  $(x(\cdot), u(\cdot)) \in AC(T, R^n) \times L_2(T, R^m)$  in the class of differential systems with control from the family of non-stationary models SOLDU formalizes

**Definition 1 [6].** *If for a given (a posteriori) dynamic process  $(x(\cdot), u(\cdot))$  there is some linear differential system (3), which includes the dynamic process  $(x(\cdot), u(\cdot))$  in the class of its admissible K-solutions, then the (A, B) model of this system is called a strong irrefutable (A, B) model over the process  $(x(\cdot), u(\cdot))$ .*

The immediate goal of the research is the experimentally obtained process  $(x(t), u(t))$ ,  $t \in T$ , formed according to current observations on the time interval T for the state vectors and control actions of the galvanic process of restoring the cylinder blocks' main supports, the conditions for the existence of a strong irrefutable (A, B) - model over the process  $(x(t), u(t))$  and construction of an algorithm for the parametric identification of its matrices A (t) and B (t).

### III COMPUTATIONAL SCHEME FOR IDENTIFYING THE STATE OF THE GALVANIC PROCESS

As a rule, the physico-chemical nature of the development of electrolysis is stationary-isotropic in nature, which advances an important class of models of the differential dynamics of the galvanic process in the SOLDU model class, namely, - class of autonomous differential (A, B) models [4], i.e. when in the structure of equations (3) the matrices A(t) and B(t) on the time interval T have constant elements. In such a mathematical formulation (simulation) of the equation of the state vector of the electroplating process (according to formula (3)) will be:

$$dx(t)/dt = Ax(t) + Bu(t), \quad t \in T := [t_0, t_1] \in R, \quad (4)$$

where  $x(t_0) =: x_0 \in R^n$  is the initial conditions of the FHP,  $u(\cdot) \in L_2(T, R^m)$ , the vector of input actions, A is the constant ( $n \times n$ ) - matrix, B is the constant ( $n \times m$ ) - matrix; The matrices

A and B are mediated by the FHP environment (their elements are subject to parametric identification, since they are initially unknown, because they depend in a complex way on the interrelationship of numerous factors: the temperature and pressure of the electrolyte, the geometry and microrelief of the product surface, etc.).

Consider the differential equation (4), and ask the following question (it concerns a preliminary structural-parametric analysis of equations (4) characteristic of the overall dynamics of electrolysis). What are the necessary and sufficient conditions imposed on the elements of matrix A, so that all components of the state vector  $x(t)$  were non-negative when  $t \geq 0$  provided that the components of  $x_0$  are non-negative and the components of the vector function  $f(t) := Bu(t)$  are non-negative when  $t \geq 0$ . Referring to the general solution (Cauchy formula [4, p. 109]), we see that a sufficient condition reduces to the nonnegativity of the elements of the matrix exponent  $e^{At}$  for each  $t \geq 0$  (here A is the matrix of the differential system (4)). It is easy to show that this condition is also necessary.

Theorem 1 [5, p. 201]. For nonnegativity of all elements of the matrix exponent  $e^{At}$  at  $t \geq 0$ , it is necessary and sufficient that:

$$\alpha_{ij} \geq 0, \quad i \neq j, \quad i, j = 1, 2, \dots, n.$$

where  $\alpha_{ij}$  - elements of the matrix A.

The task of mathematical modeling of an autonomous (A, B)-model describing the developmental dynamics of the physicochemical process under study is set as follows. To determine the conditions from the current electrolytic process data  $(x(t), u(t)) \in R^{n+m}, t \in T$  characterizing the fact of the existence of a mathematical model (4) for the dynamics of electrolysis and to identify the elements of its matrices A and B.

Let us consider auxiliary constructions (results of processing experimental data):  
 $\omega(t) = \text{col}(x_1(t), \dots, x_n(t), u_1(t), \dots, u_m(t)), \quad \omega_d(t) = \text{col}(dx_1(t)/dt, \dots, dx_n(t)/dt),$   
 $\Delta = \text{det}[\int_T \text{col}(\omega_d(t), \omega(t)) [\text{col}(\omega_d(t), \omega(t))]^* dt],$   
 where  $\text{col}$  is a column vector,  $\text{det}$  is the matrix determinant,  $*$  - column vector transposition.

The characteristic of the situation, when the system (4) serves as the differential model of its dynamics for the observed FHP process, is the following statement [22].

Statement 1. For the electrolytic process  $(x(t), u(t)) \in R^{n+m}, t \in T$  there is a differential system (4) for which this process is its solution, if and only if  $\Delta = 0$ .

Comment. This statement allows one, along with other things, to analyze the separation of experimental data into state variables of the process under study and its control actions.

We introduce auxiliary notation. Let  $X_j := \text{Mat}\{x(t_0 + i\Delta T)\}_{j \leq i \leq j+n+m-1}, X_j^\# := \text{Mat}\{\int_{T_i}(x(t), u(t)) dt\}_{j \leq i \leq j+n+m-1}, D := [A, B], T_i := [t_0 + i\Delta T, t_0 + (i+1)\Delta T], j = 0, 1, 2, \dots$  Turning to an integral system equivalent to differential equation (3), it is easy to establish that matrix equality takes place:

$$X_{j+1} - X_j = DX_j^\#.$$

Now, conditionally assuming that (on the current measurements of our vector function  $(x(t), u(t)) \in R^{n+m}, t \in T$ ) the initial conditions [4] are fulfilled, we can propose the following direct computational identification algorithm (PVAI) (let's call it PVAI-1) of the matrix D:

$$D = (X_{j+1} - X_j) X_j^{\#-1}. \quad (5)$$

The proposed parametric identification of the autonomous model of the differential dynamics of the FHP state vector, based on the PVAI-1 numerical procedure, for all its visual appeal (measurements of the physico-mechanical parameters of the FHP and input actions are carried out discretely - over a fixed time interval  $\Delta T$ ) contains one internal (when  $\text{rank}[X_j^{\#-1}] < n+m$ ) flaw. The methodological nature of this deficiency for passive ( $u(t)=0$ ) dynamic objects (4) was revealed in [7]. Its essence is:

Theorem 2. For parametric identification of system (4) by measurements through the step  $\Delta T$  of the state vector at one particular solution with  $u(t)=0$ , it is necessary and sufficient that:

- a) the minimal polynomial of the initial vector  $x_0$  with respect to the matrix A coincides with the minimal polynomial of the matrix A;
- b) the elementary divisors of the matrix A were mutually simple;
- c)  $\Delta T \neq 2\pi k / \text{Im}\lambda_i - \text{Im}\lambda_j^{-1}$ , where  $\lambda_i, \lambda_j$  - own values of the matrix A with  $\text{Re}\lambda_i = \text{Re}\lambda_j, k = 1, 2, \dots, \text{Im}, \text{Re}$  - respectively, an imaginary and real part of a complex number.

Condition a) essentially means that the initial state vector  $x_0$ , according to the terminology used in [11], should excite all harmonics of matrix A, while condition b) is equivalent to the condition of similarity of matrix A to a single Frobenius cell or, equivalently, to the possibility of differential matrix type system:

$$dx(t)/dt = Ax(t)$$

to the scalar differential equation of order n.

In general, the analytical conditions a) and b) are similar to the well-known Kalman conditions for the controllability of linear systems with one input [11], while condition c) essentially clarifies the question of the possibility of constructing a PVAI-1 computational scheme. This condition is not constructive and only characterizes the discrete structure of the set of "unsuccessful" values of the period of "retrieving information" about the galvanic process through the interval  $\Delta T$ .

In order to methodologically overcome the technical deficiencies of the PVAI-1 computational circuit identified above (when  $\text{rank}[X_j^{\#-1}] < n+m$ ), let us consider another parametric identification algorithm for the differential model of linear, autonomous dynamics (4) of the controlled physicochemical process - its PVAI-2. We note that the ideology of the PVAI-2 approach is based on the exclusion of the discretization procedure for monitoring the current development of electrolysis.

The calculation scheme of the proposed PVAI-2 algorithm is as follows: we multiply (matrix) the differential equation (4) on the right by the row vector  $[(x(t), u(t))]$ , after which we integrate both parts of the new (already matrix) equation into the whole time interval  $T$ . As a result of the calculations, we obtain the following integral equality:

$$\int_0^T (dx(t)/dt)[(x(t), u(t))]dt = D \int_0^T f_{col}(x(t), u(t))[(x(t), u(t))]dt$$

and if condition b) [6] holds, then obviously the numerical matrix:

$$f_{col}(x(t), u(t))[(x(t), u(t))]dt$$

is not special, that is, the rank of the  $[\int_0^T f_{col}(x(t), u(t))[(x(t), u(t))]dt]$  is equal to  $n + m$  and, therefore, is algebraic (matrix) ratio:

$D = \int_0^T f_{col}(dx(t)/dt)[(x(t), u(t))]dt \times [\int_0^T f_{col}(x(t), u(t))[(x(t), u(t))]dt]^{-1}$ , which completes the construction of the computational scheme of the PVAI-2 algorithm.

Although we considered only the problem of the existence of a numerical procedure for identifying the parameters of a linear, autonomous model of the differential dynamics of the galvanic process, overcoming the "shortcomings" of the calculation scheme of the PVAI-1 algorithm. We note that the issue of the noise immunity of the PVAI-2 algorithm is essential for its feasibility positively on the basis of the above Theorem 1.2 and the stability factor of linearly independent systems known in the linear analysis. At the same time, the autonomous character of the model of the differential dynamics of the galvanic process suggests that the time of the direct computational method of parametric identification of a linear differential model of the electrolysis state vector is, in principle, limited from below only to the technical capabilities of the measuring and computing equipment "serving" the physical experiment.

#### IV CALCULATION OF THE OPTIMAL ELECTROLYSIS MODE FOR THE RESTORATION OF THE MAIN SUPPORTS OF THE CYLINDER BLOCKS

The method of constructing a mathematical model of the electroplating process, developed in the previous section, allows us to raise the question of finding the optimal mode electrolysis (OME) in the process of restoring the seats of the engine block engine bearings. In this case, OME is understood as an electrolytic process, which, on the one hand, sets the quality indicators of electroplating, and on the other (at the same time), minimizes the energy and material costs of its practical implementation.

Formally, the mathematical formulation of the OME problem is formulated as follows. Let as the equations (4) of the dynamics of the galvanic process be given (by identification) a linear stationary differential system:

$$dx(t)/dt = Ax(t) + Bu(t), \quad t \in T = [0, \tau]$$

with a fixed initial condition  $x(0) = x_0$  and a fixed duration  $\tau$  electrolysis. It is required to find the law of control actions  $u(t)$  in the form of a vector function of  $t \in T$ , which in the electrolytic process minimizes the functional "energy losses" of the form:

$$J(u) = \int_0^T (u_1^2(t) + \dots + u_m^2(t))dt$$

and at the same time translates the process state vector at a

given time interval  $T = [0, \tau]$  from the initial state  $x_0$  to the final  $x(\tau) = x_T$ , where  $x_T$  is the required parameters of the electroplating process (indicators of its quality).

To solve the aforementioned optimization-boundary value problem

$$\min J(u), \quad x(0) = x_0, \quad x(\tau) = x_T \quad (6)$$

We introduce two auxiliary mathematical constructions:

$$U := [B, AB, \dots, A^{n-1}B],$$

$$W(\tau) := \int_0^T e^{A\tau} B B^* e^{A^*t} dt$$

The first is called the controllability matrix, the second is the manageability grammar [19] (note that a pair of matrices (A, B) is called a controlled pair if the rank of the matrix  $U$  is equal to  $n$ ); here  $*$ - matrix transposition operation.

Statement 2 [19, p. 50]. If the differential model of dynamics (4) for FHP is such that its controllability matrix  $U$  has a rank equal to  $n$ , then the optimal control law  $u_{opt}(t)$ ,  $t \in T$ , which solves the OME (6), exists and has the following analytical form:

$$u_{opt}(t) = B^* e^{A^*(\tau-t)} W^{-1}(\tau) (x_T - e^{A\tau} x_0). \quad (7)$$

Statement 2 indicates a clear rule for constructing an optimal program (the  $u_{opt}(t)$  law) of forming on the time interval  $T$  values - electric current strength, electrolyte flow rate, anode (cathode) rotation frequency and solution acidity, while ensuring that ultimately the parameters of electroplating The process will acquire a given  $x_T$  quality for - galvanic coating thickness, adhesion strength, and concentrations of electrolyte components. To do this (analysis of the formula (7)):

- set the duration of the electrolytic process (select  $\tau$ ), as well as the required quality indicators of the coatings (select  $x_T$ );

- construct the inverse matrix for the grammar of controllability (calculate the matrix  $W^{-1}(\tau)$ );

- integrate over the interval  $T$  the homogeneous system (4)

- with the initial condition  $x_0$  (calculate the vector  $e^{A\tau} x_0$ );

- construct the matrix function  $B^* e^{A^*(\tau-t)}$

(the last three phases of the OME calculation procedure can be performed using the accompanying functions of the MATLAB and SCILAB software environments [7]).

#### V THE RESULTS OF A POSTERIORI SIMULATION OF THE DYNAMICS OF THE GALVANIC PROCESS

To solve the above-mentioned range of problems of a posteriori differential modeling of the electroplating process, the "REDIM" software package (implementation of a dynamic model) developed at the Institute of systems dynamics and control theory of the Russian academy of sciences, including several specialized problem modules, was used. The design of this software environment was conducted under the assumption that it will be entrusted with two main methodological goals of computer modeling:

- software that reproduces weakly structured dynamic properties is modeled on a computer to generate operational scenarios with different assumptions about the internal and external physical environment of the simulated process;

- discovery of useful physical and mathematical properties describing the dynamics of the object under study, as well as empirical testing of hypotheses and assumptions put forward regarding this dynamics.

The system (applied software package) "REDIM" is implemented in a modular type and consists of: a package monitor (MP), an input language translator (TVL), a package resident (RP) and a problem module library (PM). Dialogue with the user (decision maker (decision maker)) is built according to the "menu principle", i.e., after entering a general directive, the system starts requesting the necessary information from the decision maker, analyzes this information (checking data integrity), points to some errors and whenever possible, prompts the necessary actions.

In the "REDIM" software environment, the problem of parametric identification of differential equations of the state of a physicochemical process in the class of linear autonomous models (4) and determining (based on this model) the optimal mode of the galvanic process was carried out in the course of full-scale numerical experiments to restore the cylinder block bearings automotive engines (see figures 1-9). At the same time, the PVAI-2 parametric identification algorithm showed sufficient performance and efficiency in terms of a posteriori restoration of the parameters of the differential model (4) of the galvanic surface treatment process of the restored part and, on its basis, finding (building on the basis of numerical simulation) the optimal "energy-saving" electrolytic process.

The sequence of mathematical calculations (processing of experimental data and technological planning of the optimal galvanic process of restoring the cylinder block core bearings) in the software environment of the problem modules of the "REDIM" package can be divided into three stages.

At the first stage of mathematical modeling, based on the results of a sample of observations on the current evolution of the galvanic process development for a given technological product (core supports of cylinder blocks) by the spline approximation method of the third order, continuous interpolation of the state vector of the galvanic process was built.

At the second stage of the simulation, a numerical calculation of the parameters of the dynamic model of the galvanic process was carried out, i.e., the results of the spline approximation of the current phase coordinates generated during the first stage were used in the numerical procedure corresponding to the PVAI-2 algorithm for parametric identification of the differential equations of the state vector of the galvanic process - matrices  $A$  and  $B$  of system (6) with  $n = 4$ ,  $m = 5$ .

Identified matrix  $A$ :

$$\begin{bmatrix} -0,0207 & 0,0011 & -0,0033 & -0,0017 \\ -0,2605 & -0,0259 & 0,1028 & -0,0204 \\ 0,0289 & -0,0033 & 0,0107 & 0,0011 \\ -0,1013 & 0,0010 & 0,0127 & -0,0230 \end{bmatrix}$$

Identified matrix  $B$ :

$$\begin{bmatrix} 0,0955 & 0,0453 & -0,0039 & -0,0124 & 0,0192 \\ -7,6580 & -15,2468 & 0,9402 & 1,3282 & -0,9613 \\ -0,8368 & -2,0469 & 0,1210 & 0,1055 & -0,2113 \\ 1,9518 & 1,4397 & -0,1080 & 0,2087 & 0,0301 \end{bmatrix}$$

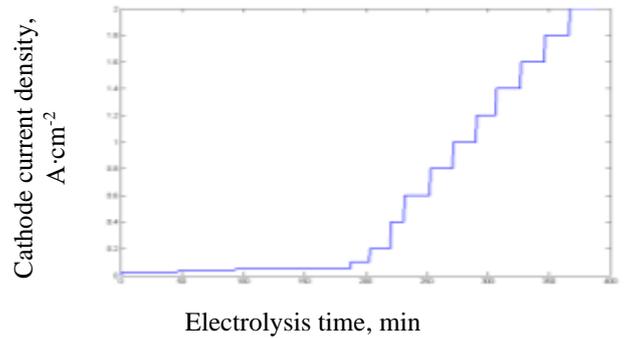


Fig. 1. Experimental data for cathode current density –  $u_1(t)$

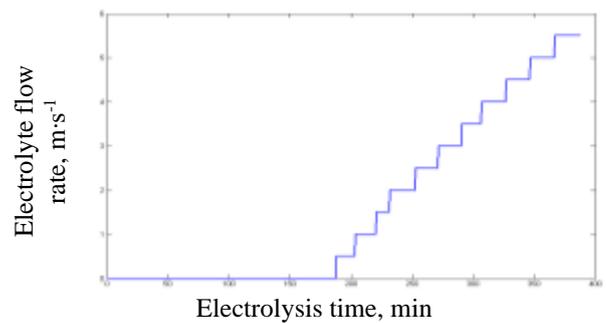


Fig. 2. Experimental data on the rate of flow of electrolyte –  $u_2(t)$

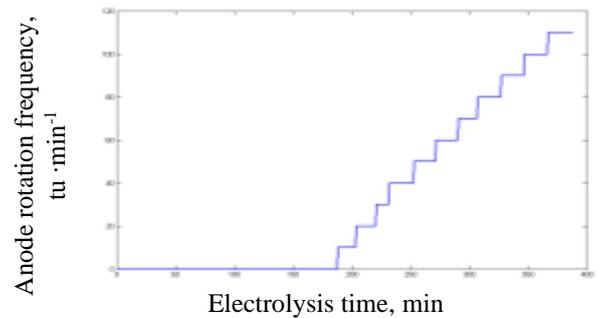


Fig. 3. Experimental data of the frequency of rotation of the anode –  $u_3(t)$

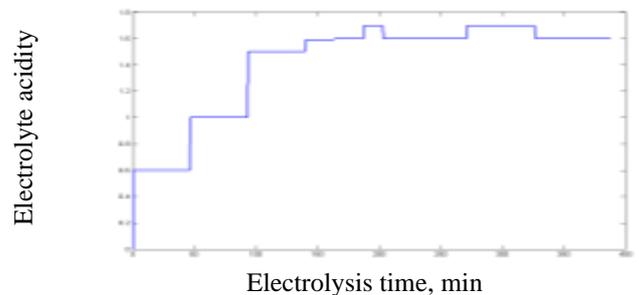


Fig. 4. Experimental data of electrolyte acidity –  $u_4(t)$

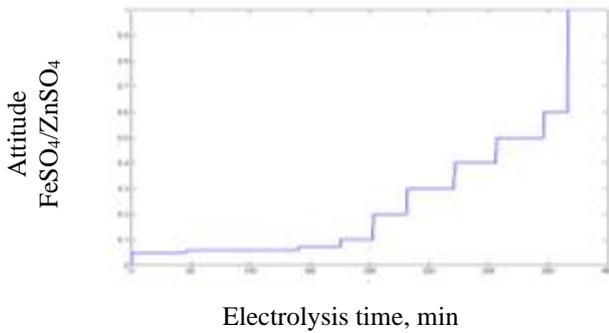


Fig. 5. Experimental data  $\text{FeSO}_4/\text{ZnSO}_4 - u_5(t)$

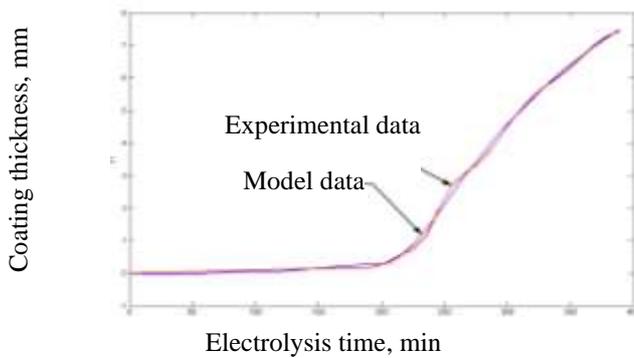


Fig. 6. Modeling the process of changing the coating thickness  $- x_1(t)$ .

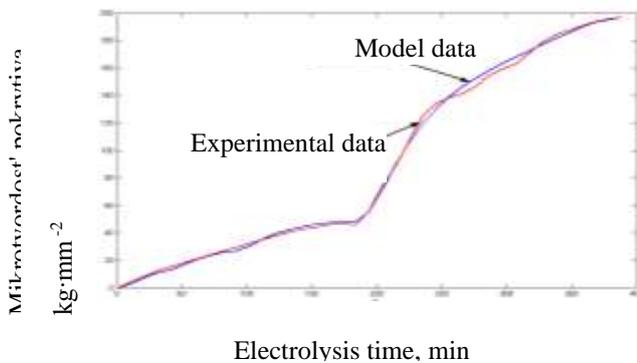


Fig. 7. Modeling the process of changing the microhardness of the coating  $- x_2(t)$ .

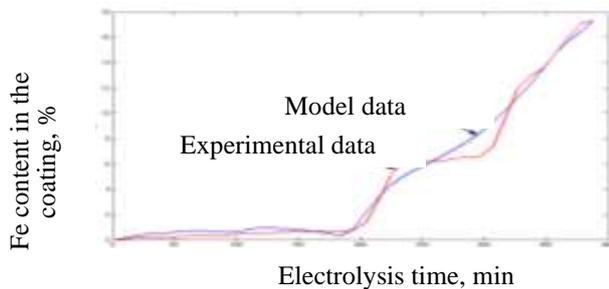


Fig. 8. Modeling the process of changing the content of Fe in the coating  $- x_3(t)$ .

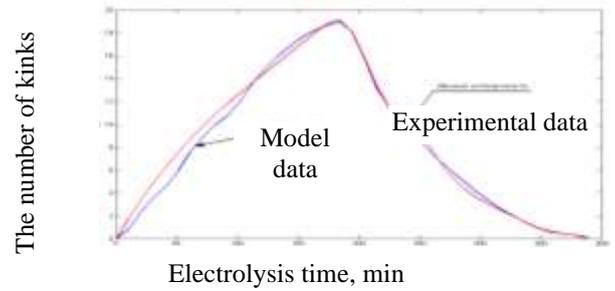


Fig. 9. Modeling the process of changing the fragility of the coating  $- x_4(t)$ .

Thus, at the second stage, the identification algorithm of the PVAI-2 “automatically” restored 36 ( $n \times n + n \times m$ ,  $n=4$ ,  $m=5$ ) parameters in the system of differential equations (4) galvanically according to field tests (experiment). -process. The good performance of the PVAI-2 is apparently explained by the fact that, as shown in [21], this algorithm solves the problem of differential approximation [27]:

$$\min_{f_T} \|dx(t)/dt - Ax(t) - Bu(t)\|_{R^4} dt,$$
  
 $\|\cdot\|_{R^4}$  – Euclidean norm in  $R^4$ , where min is considered by the coefficients of the matrices  $A$  and  $B$  (in [27, p. 392], it is noted that the differential approximation method can be considered as a direct generalization of the correlation algorithms).

At the third stage, numerical modeling of various simulation scenarios for the development of the electroplating process was carried out in order to build the optimal technological map of the physico-chemical processing of a given mechanical machine part.

#### VI CONSTRUCTION OF THE OPTIMAL DEPOSITION OF GALVANIC ZINC-IRON COATING BASED ON MATHEMATICAL MODELING OF THE ELECTROLYTIC PROCESS

The MDEP-equations proposed above can be used when calculating the optimal mode of the electrolytic process (OMEP) for resource-saving technology of deposition of zinc-iron electroplating coatings in a non-stationary electrolysis environment. At the same time, under the OMEP, we will understand the process of electroplating zinc-iron coatings during the restoration of seats, providing, on the one hand, given indicators of their quality, and on the other (simultaneously), minimizing the energy and material costs of its implementation.

Statement of the OMEP problem: for the differential system (6), we construct the vector function of control actions  $u_{opt}(t)$ ,  $t \in T = [0, \tau]$ , satisfying the criterion of minimum energy and material costs:

$$\min J(u) = \int_T (u_1^2(t) + u_2^2(t) + u_3^2(t) + u_4^2(t) + u_5^2(t)) dt, \quad (8)$$

providing initial-boundary conditions of the electrolytic process:

$$x(0) = x_0, \quad x(\tau) = x_T,$$

where  $x_0$  and  $x_T$  - respectively, the initial (initial) and required (final) parameters of electrolysis (indicators of its quality), with restrictions:

$$x_i(t) \geq 0, \quad (i=1, \dots, n), \quad u_j(t) \geq 0, \quad (j=2, \dots, m);$$

these limitations reflect the physicochemical nature of the OMEP.

The considered OMEP-Technology belongs to the class of linear optimal control problems with a quadratic (with respect to control) functional with complex and diverse constraints of spatial-phase and terminal type, as well as constraints on control actions. For its numerical solution, an optimization software package developed at IDSTU SB RAS, including various gradient-type methods and improvement methods based on sufficient Krotov optimality conditions, was used. These methods and software system are described in more detail in [28]. Given there is a complex structure of the OMEP problem, it was solved in several stages.

First, the task of minimizing the penalty functional was posed:

$$I(x) = \sum_{i=1}^n s_i^t \int_T (\max(0, -x_i(t)))^2 dt + \sum_{i=1}^n s_i^f (x_i(\tau) - x_T)^2,$$

what allowed one to receive the conditional decision of the OMEP for which trajectory and phase restrictions were fulfilled; here  $s_i^t, s_i^f$  – penalty factors. Then, by increasing the values of the penalty coefficients for the terminal constraints, we managed to achieve an approximate constraint at the final time instant. At the final stage, the total functional was minimized,  $s_0 J(u) + (1 - s_0) I(x)$ ,  $s_0 \in (0, 1]$ , which essentially made it possible to refine the approximate optimal control. Summarizing the modeling of the OMEP in general, we note that in its first stages, gradient-type methods were used, and in the latter, second-order methods; the calculation results are presented in figures 10-14.

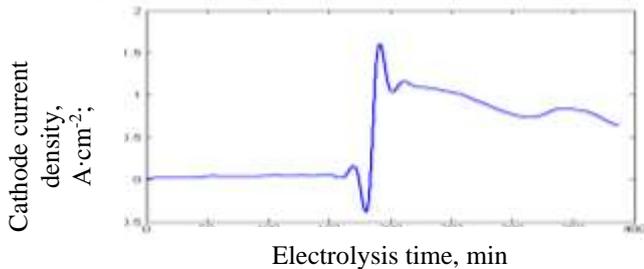


Fig. 10. The optimal program for the selection of cathode current density –  $u_{1opt}(t)$ .

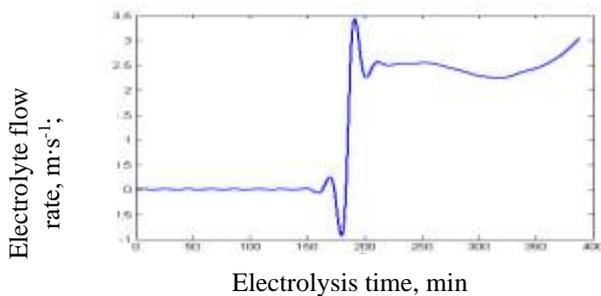


Fig. 11. Optimal electrolyte flow rate selection program –  $u_{2opt}(t)$ .

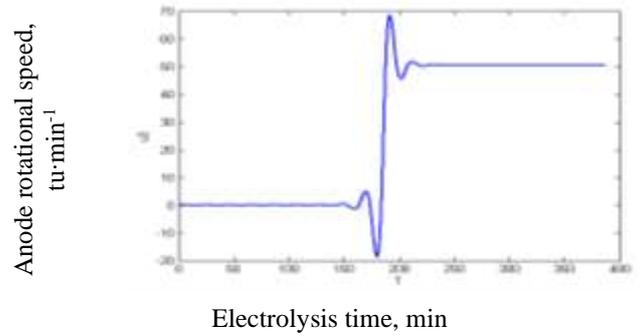


Fig. 12. Optimal anode speed selection program –  $u_{3opt}(t)$ .

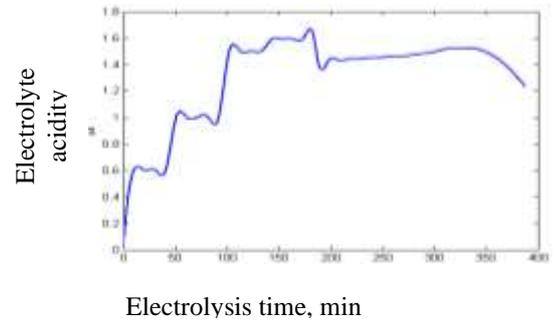


Fig. 13. Optimal electrolyte acidity selection program –  $u_{4opt}(t)$ .

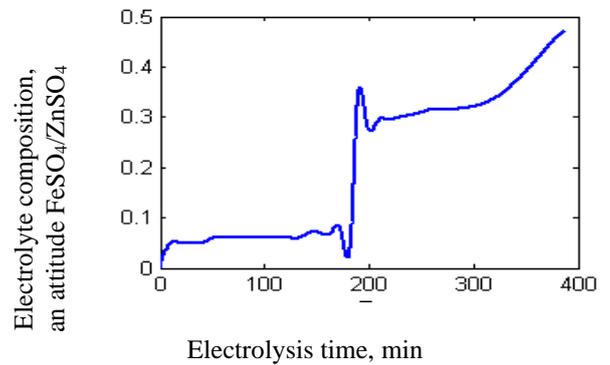


Fig. 14. Optimal electrolyte composition selection program  $FeSO_4/ZnSO_4$  –  $u_{5opt}(t)$ .

The result of numerical simulation showed that the OMEP-Technology for non-stationary deposition of galvanic zinc-iron coating when restoring the cylinder block core supports the required values (Fig. 10-14) allows for savings (resource savings expressed through the cost functional (7)) relative to MDEP- Technology by 50%.

### VII RESULTS PRODUCTION TEST

Restoration of worn parts of autotractor machines for operational tests using a zinc-iron galvanic alloy contact-flow method was conducted by us in the workshops of Irkutsk State Agrarian University, at Kasyanovsky Automotive Repair Plant OJSC, Svirsky ASO OJSC, where the semi-industrial electroplating plant was first manufactured on the industrial universal galvanic installation.

For testing during operation, the basic parts were restored, (table 1). The reason for the wear of the seats under the liners

of the main bearings of the engine block is the complex nature of the deformations and movements under the action of a number of forces arising during the operation of the connecting rod-crank mechanism. Although the liner cover in the nest is fixed A1 / A. Nevertheless, in the process of work, the surface is bent due to bending of the liner, as well as alternating movements of its elements and then all surfaces of the liner in the nest. As a result, there is wear on the surface of both the liner and the block nest, which reaches significant values, of the order of  $70 \div 160$  microns. The wear is uneven, the hole is oval, the major axis of which is located in the vertical plane of the block.

In addition, as a result of the aging of the metal of the block, its deformation occurs, causing a loss of coaxiality of the nest hole [10]. The bore holes for the outer race of the primary, intermediate and secondary shaft bearings, like the shafts themselves, wear out mainly as a result of plastic deformation of both the block and the shafts themselves. The cause of wear is the misalignment of the holes for the outer race of the front bearings (at the end of the crankshaft and the rear in the casing of the gearbox [14]. Allowable wear values lie within  $0.02 \div 0.03$  mm, although in practice wears up to 0.1 mm. Up to 30% of gearbox shafts and 50% of gearboxes of cars arriving for repairs have wear.

TABLE I. Wear base parts

№	The name of detail	Q-ty	Prev/ fur. processing	Average wear, mm
1	2	3	4	5
1	2	3	4	5
1.	Engine block ZIL -130	2	-	0,08-0,1
2.	Engine block YMZ -238	2	-	0,07
3.	Carter change box gear GAZ -53A	2	1	0,05
4.	Cardan shaft Intermediate GAZ -53A	12	6	0,03
5.	Cardan shaft intermediate ZIL -130	8	-	0,04
6.	Shaft primary gearbox GAZ -53A	6	6	0,03
7.	Shaft secondary gearbox GAZ -53A	4	4	0,05
8.	Dynamo cover	8	-	0,02
9.	Starter cover	9	-	0,02
10.	Tube axis ZIL -130	8	8	0,02

As can be seen from table 1, all parts had wear of bearing seats in the range from 0.02 mm to 0.11 mm and were restored to the nominal size of zinc-iron alloy according to the technology developed by us.

All of the details listed in the table were divided into two groups:

1. The part rotated, the anode head with the anode inside it remained stationary.
2. Body parts remained fixed, and the anode rotated.

All parts restored to nominal sizes were installed on vehicles operating in car fleets and agricultural enterprises of the Irkutsk Region.

The results of operational tests presented in TABLE 1 confirmed the high wear resistance of the electroplated coating, which exceeds the wear resistance of the base material by 20-30%.

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