Territorial System as a Multilevel Complex Structure

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Abstract – The research objective of this article is conducting competitive-optimal forecasting of enterprises that are part of a territorial system, viewed as a multilevel complex structure that takes the factors of inconsistency and uncertainty into account, in conditions of static oligopoly as a multifactorial comparison of competitiveness in the commodity market on the basis of stable and effective compromises (STEC). The following methods were used in the article to achieve this goal: an examination of the contribution of each subsystem of the territorial system in the optimization problem: variational approaches, the maximum principle, dynamic programming methods, and nonlinear programming procedures, complemented by game approaches to ensure stable interactions in the multi-objective multiple-criteria system. The optimization subsystem of the territorial system involves the interaction of objects based on Nash, Pareto, and Shapley’s optimization methods, using the "threats and counter-threats" method. Statistical methods are used in the planning and managing process for business activities, based on STEC. The main results of the research include the presented model of the territorial system as a multi-level complex framework in the form of a system of generalized dynamic balance equations and its structure presented as a system of dynamically-algebraic connections, with the allocation of uncertainty and risk factors in the control system. Three key methods for representing the combinations of control forces and their parameters are described, and five principles of conflict interaction in the territorial system management framework are distinguished. The article presents an analysis of the results of the competitive optimal forecast in conditions of a static oligopoly of two conditional enterprises (one of which is the average) in the form of a multifactorial comparison of enterprises’ competitiveness on the basis of STET. The results of the study are presented using indicator value regions, Shapley points, “threats and counter-threats” method regions, Pareto solutions, and a STEC point [1]. The scientific relevance of the results obtained is that the administrative-territorial structure reform in Ukraine affects the development process of the territory management system, and this process is connected not only with decentralization of the authority, but also with the evolution of mechanisms, technologies, communications, and processes of the management system informatization. There is an urgent need to form a general automated system of territory management based on modeling elements of the territorial system as a multilevel complex structure. As the activity of enterprises has the most significant impact on the development of the territorial system, modeling methods for planning production and commercial activities processes for enterprises to ensure the stability of development of territories in competitive environments are actualized. The practical importance of the obtained results is that territorial authorities and management should solve a wide range of tasks for ensuring the functioning and development of the territory that should provide its population with the appropriate level of life quality. All these tasks are interconnected and cannot be solved separately. Their realization is impossible without an adequate automated control system for these processes at the territorial level.

Keywords – territorial system, multilevel structure, stable and effective compromise, the model of dynamic balance.

1. INTRODUCTION

Decentralization due to the reform of the administrative-territorial system and local self-government is among the most urgent problems of Ukraine. The current development process of the territory management system is connected not only with decentralization of authority but also with the evolution of mechanisms, technologies, communications, and processes of management system informatization [2].

In early 2018, the Cabinet of Ministers of Ukraine approved the Concept of Digital Economy and Society Development of Ukraine for 2018-2020, which identified the following priority tasks and initiatives: development and stimulation of digital transformations into the territorial management system, education, medicine, ecology, non-cash economy, infrastructure, transport, public safety etc. Within the framework of this concept, the peculiarities of the management system within the digital territorial economy are actualized – it is a system of relationships between people that is enabled by a harmonious synchronization of mechanisms and tools, including electronic, innovative, educational, project, and other technologies, due to progress in microelectronics, robotics, information technology and telecommunications.

The informatization of the territorial economy has both advantages and disadvantages. Information, transformed into different forms of knowledge and innovative creativity, becomes an essential factor in the development of industrial and management systems at the state and territorial levels, opening up possibilities of qualitative growth of human life quality, and is the basis for the development of creative and social economics. In the territorial management system,
information relations benefit from the expansion of global information channels, access to information, entering global markets, and cheaper communications. All this creates the topical problems of managing and developing territorial systems with the use of information technologies: significant data arrays and neural networks, machine interaction “Internet of Things”; global and territorial information platforms; trading on global exchanges; information assets and digital as well as cryptocurrency [3].

II. LITERATURE REVIEW


III. RESEARCH OBJECTIVE

The aim of this article is to investigate the framework of the territorial system as a multilevel complex structure using a project-targeted approach to management systems, taking into account factors of varying inconsistency and uncertainty. A four-component mathematical model of a conflict situation in the management framework of the territorial system is described. A combination of approximate flexible computational schemes and classical optimization control structures is presented.

IV. RESEARCH MATERIAL PRESENTATION

The United Territorial Community (UTC) as a territorial system (TS) is a multilevel complex structure (multi-objective multiple-criteria system (MMS)) that consists of the following subsystems: a subsystem-object, a horizontal subsystem of equivalent objects, and a hierarchical subsystem (HS). Each subsystem forms its "contribution" to the optimization task, which includes approaches to ensure the effectiveness of the object, namely: variational approaches, the maximum principle, dynamic programming and nonlinear programming methods, which substantially supplement game approaches with their optimization principles to ensure stable interactions in BBS, providing efficiency to the object and the system as a whole in terms of inconsistency. In general, the TS model as a complex territorial MMS can be described by a system of generalized dynamic equilibrium equations (1) – (4):

\[ Mv = Au + Bu + A^{(c)}z + B^{(c)}w + p - v' + v' \]  
\[ \frac{dX}{dt} = u, \quad \frac{dX^{(c)}}{dt} = w; \]  
\[ \frac{dR}{dt} = Q(R - R') - (Cv + Du + Fp + D^{(c)}w + F^{(c)}L) + Jz + r' - r'; \]  
\[ 0 \leq v \leq V(t, X, R), 0 \leq z \leq Z(t, X^{(c)}, R). \]

where \( v, p \) are vectors of outputs and final non-productive consumption of goods and services; \( u, v \) are intensities of capital investments on the development of main and renewable funds; \( z \) is the intensity of resources recovery; \( X, X^{(c)} \) are volumes of main and recovery funds; \( Z \) is the vector of indicators that characterize resource potential; \( r', r' \) are resource flows; \( v', v' \) are the export and import of goods and services; \( A, A^{(c)} \) are matrices of specific direct costs; \( B, B^{(c)} \) are matrices of foundation-making costs; \( Q \) is a matrix of self-healing coefficients and mutual influence of natural resources; \( C \) is a matrix of specific resource costs; \( D, D^{(c)} \) are matrices of specific fund-raising costs of resources; \( F \) is a matrix of resource costs for non-productive consumption of products; \( J \) is a diagonal matrix with elements \( J_{ii} = 1 \) if recovery of the resource \( i \) leads to an increase in the index \( R \) and \( J_{ii} = -1 \) otherwise; \( F^{(c)} \) is the vector of coefficients characterizing non-industrial load on resources; \( L \) is the population size; \( R' \) is the unrestricted state of natural resources, \( V(t, X, R) \), \( Z(t, X^{(c)}, R) \) are production functions of output power depending on time, fixed assets and potential of resources; \( M \) is a matrix that takes into account the difference in technology.

The main factor of the development of TS is human capital (labor resources), which can be presented in the form of a restriction:

\[ \lambda \nu + \mu \lambda^{(c)}u + \lambda^{(c)}v + \mu^{(c)}w \leq \beta N, \]  

where \( \lambda, \mu, \lambda^{(c)}, \mu^{(c)} \) are the vectors of specific human resource costs of production, the growth of fixed assets, restoration of resources, the growth of recovery funds respectively; \( \beta \) is the proportion of human resources in the population [4].

In the TS functioning framework, different factors of inconsistency (conflict) and uncertainty are actualized with the application of the project-target approach to the management system of the territory (MST). The modeling methods take into account the multifaceted structure, multi-criteria nature of tasks, and properties of conflict interaction of objects when designing and managing MMS of antagonistic, non-coalition, coalition, and cooperative as well as combined character. A sufficiently complete set of methods for optimizing the MMS is created to solve the problem. An important task of the MMS management theory is to develop MMS management methods that have properties of stability and effectiveness in the conflict.
providing compromises on tactical and informational bases [5].

As noted above, the management and planning system of TS is created under conditions of uncertainty and risk. The classification of uncertain factors includes:

1) natural (environmental) uncertainty; insufficient information about object-subsystem functioning processes (initial conditions, external influence factors, perturbations, current state and positions, function parameters, namely distribution laws and random function moments);

2) factor uncertainty, or inconsistency in mutual information related to the description, actions of objects-subsystems in complex MS, uncertainty associated with conflicts of interacting objects-subsystems;

3) uncertainties that reflect incorrect formulations of the goal and its indicators in a complex system [6].

The mathematical model of the conflict situation includes four components: the conceptual mathematical model of the MMS, the managerial forces, the vector target indicator, the principle of conflict interaction based on stability and efficiency.

In general, the structure of the TS can be represented as a system of dynamical-algebraic relations (6):

\[
\begin{align*}
x^d &= f(t, x, q, u_1, ..., u_N), \quad x(t_0) = x_0; \quad a \\
x^a &= \varphi(t, x, q, u_1, ..., u_N), \quad x \in X; \quad b \\
y &= y(x, q, t), \quad q \in Q; \quad c \\
u &= u(t, x, y, q), \quad u \in U; \quad d
\end{align*}
\]

where N is the number of objects; \( x = (x^d, x^a) \) is a state vector with \( x^d \) – dynamic and \( x^a \) – algebraic states; \( X \) is a set of states; \( y \) – an exit vector; \( u \in U \) is a management vector; \( q \in Q \) is a vector of parameters characterizing the parametric uncertainty in (6a – c), the possible parametrization in (6d). Expressions (6) characterize the dynamic bonds (a), algebraic bonds (b), the exit vector (c), the decision-making and control function (d). Management \( u \in U = U_1 \times \cdots \times U_N \), \( u \in U \), is a subvector of control of the i-th TS object. Properties of the right parts (6a), (6b) – a continuity and differentiation, for (6a) – a fulfillment of Lipschitz conditions.

Let’s note three key ways of representing controlling forces: the parameter vector \( q \in Q \); the program management \( u = u(t) \); the law of management (or positional management) \( u = u(t, x) \), \( u \in U \), while the properties of vectors and sets of controls can vary. The most desirable U properties are bulge and compactness (or weak compactness).

Taking into account the complexity of boundary problems in the MMS, it is advisable to focus on a combination of approximate flexible computational schemes and classical optimization structures of management: mathematical programming and operational management, with significant parametrization of control forces in temporary intervals of their application.

The control forces can be represented by the following combinations:

1) program-corrected law of management (PCLM) (strategy) at a given segment split \([t_i, T] \) on \( \Delta t = t_j - t_{j-1} \), herewith \( u_j(t) = \{u_j(t)\} : \)

\[
u_j(t) = \sum_{j=1}^{N} u_j(x(t_{j-1}, t) t_j - t_{j-1}), \quad t_i = T
\]

where \( u_j(x(t_{j-1}, t)) = u_j(t) \) – the permissible program management \( u_j \in U_j \) on the segment \([t_{j-1}, t_j]\) under the known initial condition \( x(t_{j-1}) \), that is implemented on \( t \in [t_{j-1}, t_j] \);

2) parametrized PCLM of the form (7), where

\[
u_j(x(t_{j-1}, t)) = \sum_{k=1}^{N} q_{j-k}^t \Delta t
\]

with a breakdown \([t_{j-1}, t_1, t_2, ..., t_{k-1}, t_k, ..., T]\) on the segment \([t_{j-1}, T]\) with the fixed \( x(t_{j-1}) \) [7].

Control force parametrization procedures enable the use of parametric networks to reduce the global optimization issues in multicriteria tasks, to estimate approximately the availability and optimality of the solution, and to assign the initial approximation for the local search of the exact solution. The methods and algorithms are structured in two or more stages:

1) The solution set is evaluated using network approaches, and the initial approximation is chosen in the “beneficial” local area;

2) Based on initial approximation, the exact task of determining parameterized optimal management is solved in forms (7) - (8) [8].

The target properties of TS are characterized by a vector (9):

\[
J = J[x_0, t_0, T, q, x(\cdot), u(\cdot), y(\cdot)] = (J_i, ..., J_m),
\]

which reflects a complex functional relation with the mentioned values. The typical view of the i-th winning (loss) function is a functional on \( t_0 \leq t \leq T \) (10):

\[
J_i(u_1, ..., u_N) = \Phi_i(T, x(T)) + \int_{t_i}^{T} F_i(t, x, u_1, ..., u_N) dt, \quad i = 1, ..., m
\]

In addition to the continuity (10) by \( (x, u) \) and differentiation by management, the optimal properties are
concavity-quasi-concavity (convexity-quasi-convexity) of the functional (10) on a set of controls. The discrepancy of the dimension $J$ with the number of objects means that some objects have a vector structure. The dimension of the indicator will coincide with the number of objects in the TS if the rate of each object is scalarized. Coalition structure of actions and interests of the TS (11): 

$$P = \left\{ K_1, \ldots, K_{m_0} : K_i \cap K_j = \emptyset, \ \sum_{i=1}^{m_0} K_i = [R, M] \right\}$$  

(11)

where $R$ is the set of indices, for example, managers, $M$ is the set of indices of the indicator vector. The indicator of each coalition usually takes one of two types (12a) - (12b):

$$J_K = \{ J_1, \ldots, J_{m_0} \};$$

$$J_K = \sum_{i=K} \alpha_i J_i, \quad 0 \leq \alpha_i \leq 1, \quad \sum_{i=K} \alpha_i = 1$$  

(12)

where the sum of the indices $i_1$ is $m$.

The coalition management without parametrization takes the form (13):

$$u_k = (u_{i_1}, \ldots, u_{i_m}), \quad u_K \in U_K = \prod_{i=K} U_i,$$  

(13)

Expressions (12a) turn into the form (14):

$$x = f \left[ I, x, u_{i_1}, \ldots, u_{i_m}, \ldots, u_{i_{m_0}} \right], \quad l \in M_K.$$  

(14)

Indicator in option (12b) turns into (15):

$$J_{K_k} = \Phi_{k_k}(x, t) + \sum_{i=K_k} \lambda_i J_{i_k}, \quad \sum_{i=K_k} \lambda_i = J_{K_k}, \quad \sum_{i=K_k} \lambda_i = 1$$  

(15)

where $\Phi_{k_k} = \sum_{i=K_k} \lambda_i \Phi_{i_k}$; $F_{k_k} = \sum_{i=K_k} \lambda_i F_{i_k}$.

There are five principles of conflict interaction: antagonism $\{ M_k = \{1,2\}, J^k = -J^2 \}$; cooperative interaction; non-cooperative interaction; coalition interaction; hierarchical interaction (with the right of the first move). The properties of conflict interactions are robust because they enable the making of healthy efficiency assessments in uncertain environments, “active partner” purpose, taking into account the nature of uncertainty and conflict. In these principles of conflict interaction, three basic concepts of game theory are put into place: stability, efficiency, and STEC. Stability is the provision of inter-object-stable (balanced by goals) processes for operation and design of the MMS in conditions of conflict (inconsistency) and/or uncertainty. Efficiency is the achievement of maximum target quality of objects or coalitions based on sustainable and rational coalition creation.

Stability-effective compromise is the combination of stability and efficiency within a set of solutions. It includes from the complete coincidence of these properties at one point of the space $J$ (or $U$) to provide the possible degree of approximation in terms of information-tactical extensions of agreements [9]. The optimization subsystem of the TS assumes the existence of modules that separately and together allow to find the optimal control or control law. It applies to non-coalition, coalition and cooperative interaction of objects based on the optimization methods of Nash, Pareto, and Shapley, using the method of “threats and counter-threats” (TCT). The range of problems that have to be solved by territorial authorities and administration is much wider: the provision of functioning and development of farms (enterprises/firms); formation of financial resources (budget revenues), necessary for fulfillment of TS functions; development of the social sphere, which is necessary for the vital activity of the population of this territory, its reproduction; protection of the environment as the only source of living. All these problems are interconnected and cannot be solved separately. Their realization is impossible without an adequate management system of these processes at the territorial level [10]. It follows that the TS should manage the economy, the social sphere and the environmental protection, as it has the necessary powers, methods and levers of management and appropriate management structures for the task.

Generally, in the process of planning the industrial and commercial activities of enterprises, in the process of the current management of production and product flows it is necessary to provide elements of the enterprise stability in a competitive environment. For example, approaches based on STEC are relevant in static and dynamic models of oligopoly in the commodity market, where an oligopoly is a group of enterprises that owns production and market of one or several types of goods [11].

Scalar Nash equilibrium. A set of solutions $q^* = \{ q_1^*, \ldots, q_{m_0}^* \}$ is a Nash equilibrium in relation to the scalar indicator $\Phi_{i_k}(q_1^*, \ldots, q_{m_0}^*)$, that is an effectiveness function of the coalition $K_i$, if for any $q^* \in Q, i, j \in M_k = \{1,2, \ldots, m_k\}$, $\Phi_{i_k}(q^* || q^*) \leq \Phi_{i_k}(q_1^*)$, where $\Phi_{i_k}(q^* || q^*) = \{ q_1^*, \ldots, q_{j-1}^*, q_j^*, \ldots, q_{m_k}^* \}$. If $M_k = \{1,2\}$ and these goals are antagonistic, i.e. $\Phi_{i_k} + \Phi_{i_k}^* = 0$, then the Nash equilibrium turns into a saddle point: $\min_{q^*} \max_{q^*} \Phi_{i_k}^* = \min_{q^*} \max_{q^*} \Phi_{i_k}^*$. The concept of choosing the most effective Nash solution is based on the fact that the need for Nash-SETEC occurs when a scalar Nash equilibrium under a fixed MMS structure is not the only one – a selection of non-dominant Nash solutions. Nash-solution of the game (16):

$$u_i^* = (u_{i_1}^*, \ldots, u_{i_m}^*), \quad i = 1, \ldots, l; \quad u \in U$$  

(16)

dominates the solution $\bar{u}$, if

$$J_{K_i}(u_i^*) \geq J_{K_i}(u^*), \quad i = 1, \ldots, l.$$  

(17)
Within the framework of the Nash-STEC, a non-dominant solution \( u^- \) is the only one that is most effective for the entire coalition breakdown of the MMS and is therefore accepted by players as a conditional agreement. An algorithmic circuit of the STEC may be formed using one of the methods of Pareto optimization in the final set of points where technologically convenient procedure becomes Pareto optimization based on cones of dominance. The condition of dominating the solution \( J^* \) over \( J' \) regarding the cone \( \Omega \) with matrix \( B \) has a simple look:

\[
BAJ \geq 0, \quad (18)
\]

where \( \Delta J = J^- - J^* \), \( J'' = J(u^*) \), \( J' = J(u') \).

The inequality sign changes if the efficiency in minimization of losses. For \( B = E \), the multifold cone \( \Omega \) becomes rectangular, and the optimization procedure based on the cone \( \Omega \) is reduced to Pareto-optimization. When these considerations are taken into account, the final set of values of the vector \( J \) sets the test table, which establishes a pairwise comparison of points of the table and allocation of non-dominant ones. Herewith, on each iteration, the points \( J' \) that provide the return sign of the ratio are excluded [12].

The iteration of the algorithm for finding the Nash-STEC consists of the following steps.

Stage 1. Obtaining a solution, a Nash equilibrium.

Stage 2. The comparison of this solution based on (18) with the one received previously.

Stage 3. The exclusion of dominant decisions on a given subset.

To validate this method, it is expedient to use the following output data:

- 1) the annual characteristics of demand \( D(Q) \) for goods, determined by the budget constraints of consumers, their advantages and elasticity \( D(Q) = a \cdot 1g Q - b \) \( (E = 2) \) are proposed to be calculated on the basis of \( \text{Const}_1 = 1m \text{ c.u., Const}_2 = 2m \text{ c.u.} \)
- 2) the average wage of people working in this field \( w = 10k \text{ c. u./year}; \)
- 3) share of capital paid for equipment lease \( r = 120\% \text{year}; \)
- 4) indicators of the technological process of enterprises \( A_1 = A_2 = 6; \ g_1 = 0.3; \ g_2 = 0.3; \)
- 5) planned production costs of enterprises \( TC_1 = TC_2 = 16m \text{ c. u.}, \) allowable values of labor and capital resources: \( 100 \leq L \leq 400 \) people; \( 1 \leq K \leq 4m \text{ c. u.} \)
- 6) penalty lags, which regulate accounting excess of real costs over the planned, for each enterprise takes the value: \( \alpha_1 = \alpha_2 = 2.5 \).

In the article, the analysis of competitive-optimal forecast of management in a static oligopoly in the form of multivariate competitiveness comparison of the enterprises in the commodity market on the basis of STEC is conducted. For the convenience of graphing the results of the analysis \( N = 2 \), one of the enterprises is average. The results are presented in the form of figures with areas of indicators values, points of Shapley, areas of TCT, Pareto solutions, a point of STEC.

Draw multivariate analysis of the impact of changing demand function of the base value \( \text{Const} \) with recalculation of \( D(Q) \), wage level \( w \), production level \( A \), the share of leasing capital \( r \), planned costs \( TC^* \) on the results of the static oligopoly of the enterprise with changed data and the average enterprise.

The degree of imperfection of the territory market depends on the kind of imperfect competition: in monopolistic conditions, the degree of influence of each enterprise on the market is small; with oligopoly it increases due to a limited number of enterprises; in the degeneration of oligopoly into monopoly on the market there is a domination of only one enterprise with full corresponding degeneration of market self-regulation and lack of competition.

![Fig. 1. Optimization of static oligopoly with the average enterprise at \( A_1 = 6, A_2 = 8 \)](image)

The degree of competitiveness depends on enterprise’s impact on the development of a territorial system, the degree of differences in properties of an enterprise from some average enterprise for this branch, the homogeneity of market goods and other factors. When aligning properties of an enterprise, the quantitative characteristic of competitiveness tends to a constant value \( K_{\text{const}} = 1/N \); where \( N \) is a number of enterprises in market territory. In conditions of monopoly, \( K_{\text{const}} = K = 1 \). In a market with increasing number of participants, \( K_{\text{const}} \to 0 \). In the case of "refusal" of stable state conditions, the coefficient of \( r \)-th enterprise competitiveness will vary within \( 0 \leq K_{\text{const}} \leq 1 \).

Today, an existing market mechanism can be characterized as imperfect competition, the prerequisites for the emergence of which were the ownership of certain market shares by individual producers, entry barriers into the field, heterogeneity of production, and imperfection (limitation) of market information [13].
A characteristic of the components of the territorial functioning system as a multilevel complex structure, which consists of the subsystem-object, the horizontal subsystem of equivalent objects, the hierarchical subsystem, and functions based on inconsistency and uncertainty, is presented in this article. The model of the TS as a complex territorial MMS in the form of systems of generalized dynamic balance and the structure of the TS as a system of dynamical-algebraic connections is described. The classification of uncertainty and risk factors in the management and planning system of the TS is highlighted. The article describes three key ways of representing the combinations of control forces, describes the procedures of their parametrization, and selects five principles of conflict interaction in the TS management system. An analysis of the results of the TS management competitive-optimal forecast in conditions of the static oligopoly of 2 enterprises (one of which is the average) was carried out in the form of a multifactorial comparison of the enterprises’ competitiveness based on STEC. The results of the research are presented with indicator value areas, Shapley points, “threats and counter-threats” method areas, Pareto solutions, and the STEC point. It is expedient to continue the research of competitive-optimal forecasts of TS management in conditions of duopoly and open competition, taking into account the principles of conflict interaction: antagonism; cooperation, non-coalition, coalition, and hierarchical interaction.

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