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Solid Waste Disposal Site Selection by Using Neutrosophic Combined Compromise Solution Method

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Abstract

Combined Compromise Solution (COCOSO) method is a combination of different aggregation strategies that aim to find final scores of the alternatives with respect to determined criteria based on decision makers judgements. This method is extension of simple additive weighting and exponentially weighted product model In this paper, we extended COCOSO method to its interval-valued neutrosophic version to increase its applicability to the real world problems by using the advantages of neutrosophic sets. The proposed method has been applied to an illustrative example which has multi-criteria and multi-expert decision making problem. The results and the simulations that is applied for the different cases of the problem indicated that the proposed model is a useful decision making tool for the researchers, experts and decision makers who are working at uncertain and indeterminate systems.

Keywords: Combined compromise solution method, Neutrosophic sets, Interval valued sets, Decision making, Waste disposal site.

1 Introduction

In ordinary set theory, an element can belong to a set or not; in optimization, a solution can be feasible or in-feasible; in Boolean logic, a statement can be true or false but nothing in between [14]. If we consider the real life situations, humankind has uncertainty and indeterminacy almost their every decision. When we try to define this system component of it is generally not precise and cannot define as a single value. Thus, to increase the applicability of mathematical models, many researchers introduced solution spaces. In order to represent the uncertainty, fuzzy sets were introduced by Zadeh by using the degree of membership of an element to its set [11]. This representation is extended in many forms to increase of its applicability to different cases. Type-n fuzzy set was developed by Zadeh for handling the uncertainty of the membership function in the fuzzy set theory [12]. After that, interval-valued fuzzy sets (IVFSs) were introduced independently by [12, 2, 3]. In 1986, intuitionistic fuzzy sets (IFSs) introduced by Atanassov to represent not only membership degree of an element but also its non-membership degree [1]. This provides decision makers to represent their judgments with a new perspective which has a larger domain than ordinary fuzzy sets. In 2010, Hesitant fuzzy sets (HFSs) is introduced by Torra which are the extensions of regular fuzzy sets where a set of values are possible for the membership of a single element [8]. In 2013, Yager introduced Pythagorean fuzzy sets (PFSs) which are extension of IFSs to increase the applicability of IFSs [9]. All of this extensions postulated that decision maker has no indeterminacy and his/her decision is absolute. Smarandache introduced neutrosophic sets in 1995 to represent not only uncertainty of the data but also indeterminacy of the decision makers [7]. Neutrosophic sets are defined as the sets where each element of the universe has a degree of truth, indeterminacy and falsity which are between 0 and 1 and these degrees are subsets of neutrosophic sets which are independent from each other [6]. In the neutrosophic sets, uncertainty is represented as truth and falsity values where degrees of belongingness, non-belongingness and indeterminacy value where the factor incorporated as the percent of hesitancy. By using this notation, neutrosophic sets provide to present both uncertainty and indeterminacy All of these properties of neutrosophic sets are the answers to why we use neutrosophic sets in this study. Combined compromise solution (COCOSO) method was introduced to calculate the scores of the alternatives by combining the grey relational generation approach [10]. The method uses a comparability sequence and then the weights are aggregated through two manners. One of them is obtained by the usual multiplication rule and the second one is calculated by the weighted power of the distance from comparability sequence. To validate the ranking index, three aggregation strategy was applied for each alternative. At ultimate, a cumulative equation reports a ranking. This method is introduced for the crisp values. In this paper, we extend the COCOSO method with interval valued neutrosophic numbers and then applied to an illustrative example to validate its applicability for the real case problems. Rest of the paper is organized as follows: Section 2 introduced the preliminaries for neutrosophic sets. In Section 3, extended method is presented. In Section 4, an illustrative example is given with its calculations step by step. The paper end with conclusions and further suggestions.

2 Preliminaries

Definition 1. [7] Let E be a universe. A neutrosophic set A in E is characterized by a truth-membership function T_A, an indeterminacy-membership function I_A and a falsity-membership function F_A. $T_A(x)$, $I_A(x)$, and $F_A(x)$ are real standard elements of [0,1]. It can be written as;

$$A = \langle x, (T_A(x), I_A(x), F_A(x))\rangle : x \in E, \quad \text{where } (T_A(x), I_A(x), F_A(x)) \in [0,1] \tag{1}$$

The sum of $T_A(x)$, $I_A(x)$, and $F_A(x)$ can be represented as $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

Definition 2. [6] X be a universe of discourse. An interval-valued neutrosophic set N in X is independently defined by a truth-membership function $T_N(x)$, an indeterminacy-membership function $I_N(x)$, and a falsity-membership function $F_N(x)$ for each $x \in X$, where $T_N(x) = [T_N^L(x), T_N^U(x)] \subseteq [0,1]$, $I_N(x) = [I_N^L(x), I_N^U(x)] \subseteq [0,1]$, and $F_N(x) = [F_N^L(x), F_N^U(x)] \subseteq [0,1]$. Also, they meet the condition $0 \le T_N^U(x) + I_N^U(x) + F_N^U(x) \le 3$. So, the interval-valued neutrosophic set N can be shown as:

$$N = \{ \langle x, [T_N^L(x), T_N^U(x)], [I_N^L(x), I_N^U(x)], [F_N^L(x), F_N^U(x)] \rangle \mid x \in X \} \tag{2}$$

where L and U represent the lower bound and upper bound of the neutrosophic values, respectively.

Definition 3. Deneutrosophication formula is given in Eq. 3 [4];

$$\mathfrak{H}(x) = \frac{T^L(x) + T^U(x)}{2} + \left(1 - \frac{I^L(x) + I^U(x)}{2}\right) I^U(x) - \frac{F^L(x) + F^U(x)}{2}\left(1 - F^U(x)\right) \tag{3}$$

Definition 4. [6] Let $a = \langle [T_a^L, T_a^U], [I_a^L, I_a^U], [F_a^L, F_a^U]\rangle$ and $b = \langle [T_b^L, T_b^U], [I_b^L, I_b^U], [F_b^L, F_b^U]\rangle$ be two interval-valued neutrosophic numbers and the relations of them are given below:

$$a \oplus b = \langle\ [T_a^L + T_b^L - T_a^L T_b^L, T_a^U + T_b^U - T_a^U T_b^U], [I_a^L I_b^L, I_a^U I_b^U], [F_a^L F_b^L, F_a^U F_b^U]\ \rangle \tag{4}$$

$$a \otimes b = \langle\ [T_a^L T_b^L, T_a^U T_b^U], [I_a^L + I_b^L - I_a^L I_b^L, I_a^U + I_b^U - I_a^U I_b^U], [F_a^L + F_b^L - F_a^L F_b^L, F_a^U + F_b^U - F_a^U F_b^U]\ \rangle \tag{5}$$

$$a^k = \langle\ [(T_a^L)^k, (T_a^u)^k], [1-(1-I_a^L)^k, 1-(1-I_a^U)^k], [1-(1-F_a^L)^k, 1-(1-F_a^U)^k]\ \rangle \tag{6}$$

$$a \times k = \langle\ [1-(1-T_a^L)^k, 1-(1-T_a^U)^k], [(I_a^L)^k, (I_a^u)^k], [(F_a^L)^k, (F_a^u)^k]\ \rangle \tag{7}$$

Definition 5. The weighted aggregation operation for interval-valued neutrosophic numbers (INNWAO) is given in Eq. 8 [13]:

$$\text{INNWAO}_w(A_1, A_2, \cdots, A_n) = \langle\ [1 - \prod_{i=1}^{n}(1 - \text{inf}T_{A_i})^{w_i}, 1 - \prod_{i=1}^{n}(1 - \text{sup}T_{A_i})^{w_i}], [\prod_{i=1}^{n}(\text{inf}I_{A_i})^{w_i}, \prod_{i=1}^{n}(\text{sup}I_{A_i})^{w_i}], [\prod_{i=1}^{n}(\text{inf}F_{A_i})^{w_i}, \prod_{i=1}^{n}(\text{sup}F_{A_i})^{w_i}]\ \rangle \tag{8}$$

where $W = (w_1, w_2, \cdots, w_i$ is the weight vector of $A_i (i = 1, 2, \cdots, n)$ and $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$.

3 Proposed Method

Step 1. Construct the neutrosophic decision-making matrix X_l of each decision maker as in Eq. 9:

$$X_l[x_{ijl}]_{n\times m} = \begin{bmatrix} x_{11l} & \cdots & x_{1ml} \\ \vdots & \ddots & \vdots \\ x_{n1l} & \cdots & x_{nml} \end{bmatrix} \tag{9}$$

where x_{ijl} denotes the interval valued neutrosophic evaluation score of $i^{th}(i \in \{1, 2, \cdots, n\}$ alternative with respect to $j^{th}(j \in \{1, 2, \cdots, m\}$, and $l^{th}(l \in \{1, 2, \cdots, q\}$ decision maker.

In here, we recommend to use the scale which is constructed by using interval valued neutrosophic sets given in Table 1 as follows:

Linguistic Terms		<T, I, F>
CL	Certainly Low	<[0.05, 0.2], [0.1, 0.3], [0.85, 0.95]>
VL	Very Low	<[0.15, 0.3], [0.2, 0.4], [0.75, 0.9]>
L	Low	<[0.25, 0.4], [0.3, 0.5], [0.65, 0.8]>
BA	Below Average	<[0.35, 0.5], [0.4, 0.6], [0.55, 0.7]>
A	Average	<[0.45, 0.6], [0.5, 0.5], [0.45, 0.6]>
AA	Above Average	<[0.55, 0.7], [0.4, 0.6], [0.35, 0.5]>
H	High	<[0.65, 0.8], [0.3, 0.5], [0.25, 0.4]>
VH	Very High	<[0.75, 0.9], [0.2, 0.4], [0.15, 0.3]>
CH	Certainly High	<[0.85, 0.95], [0.1, 0.3], [0.05, 0.2]>

Table 1: Scale for decision matrix

Step 2. Compute the aggregated neutrosophic decision matrix (X) by using Definition 5 as in Eq. 10:

$$X[x_{ij}]_{n\times m} = \begin{bmatrix} x_{11} & \cdots & x_{1m} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nm} \end{bmatrix} \tag{10}$$

where x_{ij} represents the aggregated neutrosophic score of i^{th} alternative with respect to j^{th} criterion.

Before the weighted comparability sequence, we recommend to use the scale which is constructed by using interval valued neutrosophic sets given as in Table 2.

Linguistic Terms		<(T, I, F)>
CLI	Certainly Low Importance	<[0.05, 0.25], [0.1, 0.3], [0.75, 0.95]>
VLI	Very Low Importance	<[0.15, 0.35], [0.2, 0.4], [0.65, 0.85]>
LI	Low Importance	<[0.25, 0.45], [0.3, 0.5], [0.55, 0.75]>
BAI	Below Average Importance	<[0.35, 0.55], [0.4, 0.6], [0.45, 0.65]>
AI	Average Importance	<[0.40, 0.60], [0.5, 0.5], [0.40, 0.60]>
AAI	Above Average Importance	<[0.45, 0.65], [0.4, 0.6], [0.35, 0.55]>
HI	High Importance	<[0.55, 0.75], [0.3, 0.5], [0.25, 0.45]>
VHI	Very High Importance	<[0.65, 0.85], [0.2, 0.4], [0.15, 0.35]>
CHI	Certainly High Importance	<[0.75, 0.95], [0.1, 0.3], [0.05, 0.25]>

Table 2: Scale for weighting the criteria

Step 3. Compute the total of the weighted comparability sequence (S_i) and the whole of the power weight of comparability sequence (P_i) for each alternative by using Eqs. 11 and 12 as follows:

$$S_i = \sum_{j=1}^{n} w_j x_{ij} \tag{11}$$

$$P_i = \sum_{j=1}^{n} x_{ij}^{w_j} \tag{12}$$

Step 4. Obtain the relative weights through the three appraisal score strategies by using Eqs. 13, 14 and 15. In here, we deneutrosophicated the S_i and P_i values for the applicability of the method.

$$k_{ia} = \frac{P_i + S_i}{\sum_{i=1}^{m}(P_i + S_i)} \tag{13}$$

$$k_{ib} = \frac{S_i}{\underset{i}{min} S_i} + \frac{P_i}{\underset{i}{min} P_i} \tag{14}$$

$$k_{ic} = \frac{\lambda S_i + (1-\lambda)P_i}{\lambda \underset{i}{max} S_i + (1-\lambda)\underset{i}{max} P_i} \tag{15}$$

where $0 \leq \lambda \leq 1$.

Step 5. Calculate the final ranking of the alternatives based on k_i values by using Eq. 16 -the greatest is the best one- as follows:

$$k_i = (k_{ia}k_{ib}k_{ic})^{\frac{1}{3}} + \frac{1}{3}(k_{ia} + k_{ib} + k_{ic}) \qquad (16)$$

4 Application

We applied our proposed model for the selection of the most appropriate waste disposal site in the city of Istanbul. Figure 1 presents the locations of the alternatives as follows:

Figure 1: Location of the alternatives

The following criteria are used for the assessment process [5]:

- C1 - Environmental effect
- C2 - Social effect
- C3 - Economics
- C4 - Operational activities

The process is evaluated by based on 3 decision makers judgments as in Tables 3, 4, and 5 by using the given scales, respectively.

		DM1 Weight = 0.35				
		AL1	**AL2**	**AL3**	**AL4**	**AL5**
C1	**Benefit**	AA	H	VH	AA	CH
C2	**Benefit**	H	H	H	H	CH
C3	**Cost**	AA	L	CL	L	L
C4	**Cost**	L	L	VL	L	L

Table 3: Evaluations of Decision Maker 1

		DM2 Weight = 0.4				
		AL1	**AL2**	**AL3**	**AL4**	**AL5**
C1	**Benefit**	A	BA	AA	H	H
C2	**Benefit**	BA	A	A	H	AA
C3	**Cost**	L	H	L	BA	L
C4	**Cost**	H	H	L	BA	VL

Table 4: Evaluations of Decision Maker 2

		DM3 Weight = 0.25				
		AL1	**AL2**	**AL3**	**AL4**	**AL5**
C1	**Benefit**	BA	BA	CL	CH	VH
C2	**Benefit**	A	A	CL	CH	H
C3	**Cost**	CL	H	BA	L	VL
C4	**Cost**	BA	BA	H	H	L

Table 5: Evaluations of Decision Maker 3

The evaluation of the criteria weights is given in Table 6 as follows:

	DM1	DM2	DM3
C1	HI	HI	AAI
C2	CHI	AI	AI
C3	BAI	LI	VLI
C4	AI	BAI	HI

Table 6: Evaluations of decision makers

The aggregated decision matrix is given in Table 7 as follows:

	AL1
C1	<[0.47, 0.62], [0.44, 0.56], [0.43, 0.59]>
C2	<[0.5, 0.66], [0.38, 0.54], [0.4, 0.55]>
C3	<[0.33, 0.49], [0.25, 0.47], [0.56, 0.71]>
C4	<[0.47, 0.63], [0.32, 0.52], [0.43, 0.59]>
	AL2
C1	<[0.48, 0.64], [0.36, 0.56], [0.42, 0.58]>
C2	<[0.53, 0.69], [0.42, 0.5], [0.37, 0.52]>
C3	<[0.54, 0.71], [0.3, 0.5], [0.35, 0.51]>
C4	<[0.47, 0.63], [0.32, 0.52], [0.43, 0.59]>
	AL3
C1	<[0.56, 0.74], [0.22, 0.44], [0.32, 0.49]>
C2	<[0.46, 0.63], [0.28, 0.44], [0.43, 0.58]>
C3	<[0.21, 0.37], [0.22, 0.44], [0.68, 0.82]>
C4	<[0.35, 0.52], [0.26, 0.46], [0.54, 0.7]>
	AL4
C1	<[0.69, 0.84], [0.25, 0.47], [0.19, 0.36]>
C2	<[0.72, 0.86], [0.23, 0.44], [0.17, 0.34]>
C3	<[0.29, 0.44], [0.34, 0.54], [0.61, 0.76]>
C4	<[0.41, 0.58], [0.34, 0.54], [0.48, 0.64]>
	AL5
C1	<[0.76, 0.9], [0.18, 0.4], [0.13, 0.29]>
C2	<[0.71, 0.86], [0.23, 0.45], [0.16, 0.34]>
C3	<[0.23, 0.38], [0.27, 0.47], [0.67, 0.82]>
C4	<[0.21, 0.36], [0.26, 0.46], [0.69, 0.84]>

Table 7: Aggregated decision matrix

The aggregated weights of the criteria are given in Table 8 as follows:

C1	<[0.53, 0.73], [0.32, 0.52], [0.27, 0.47]>
C2	<[0.56, 0.81], [0.28, 0.42], [0.19, 0.44]>
C3	<[0.26, 0.47], [0.3, 0.5], [0.53, 0.74]>
C4	<[0.42, 0.63], [0.4, 0.54], [0.37, 0.58]>

Table 8: Aggregated criteria weights

S_i and P_i values are calculated as in Table 9.

	S_i
AL1	<[0.6, 0.88], [0.1, 0.33], [0.15, 0.45]>
AL2	<[0.64, 0.9], [0.1, 0.33], [0.13, 0.41]>
AL3	<[0.58, 0.87], [0.06, 0.27], [0.17, 0.47]>
AL4	<[0.71, 0.94], [0.07, 0.31], [0.07, 0.33]>
AL5	<[0.69, 0.93], [0.05, 0.27], [0.08, 0.35]>
	P_i
AL1	<[0.993, 0.994], [0, 0.009], [0.001, 0.024]>
AL2	<[0.996, 0.997], [0, 0.009], [0, 0.015]>
AL3	<[0.989, 0.992], [0, 0.004], [0.001, 0.032]>
AL4	<[0.997, 0.999], [0, 0.007], [0, 0.009]>
AL5	<[0.996, 0.998], [0, 0.004], [0, 0.012]>

Table 9: S_i and P_i values

The appraisal score strategies which are introduced in Step 4 are calculated by using Eqs. 13, 14 and 15 as in Table 10.

	AL1	AL2	AL3	AL4	AL5
k_a	1	1	0.99	1	1
k_b	2.08	2.13	2	2.22	2.16
k_c	0.94	0.96	0.91	1	0.98

Table 10: Appraisal score strategies of the alternatives

The final ranks of the alternatives and their scores are calculated by using Eq. 16 as in Table 11.

	AL1	AL2	AL3	AL4	AL5
k_i	2.588	2.637	2.516	2.711	2.661
Rank	4	3	5	1	2

Table 11: Final ranks of the alternatives and their scores

Through our application AL4 is determined as the most appropriate location for the waste disposal site. A comparative analysis is also conducted to check the validity of our proposed method. Neutrosophic CODAS method is applied the same decision matrices [4]. Table 12 presents the distances to negative solution of the neutrosophic CODAS as follows:

		AL1	AL2	AL3	AL4	AL5
ED	C1	0.000	0.096	0.217	0.385	0.473
	C2	0.079	0.117	0.000	0.469	0.472
	C3	0.128	0.000	0.247	0.189	0.233
	C4	0.000	0.000	0.099	0.008	0.239
	Total	0.207	0.213	0.563	1.052	1.417
HD	C1	0.000	0.194	0.500	0.852	1.116
	C2	0.175	0.260	0.000	1.055	1.055
	C3	0.258	0.000	0.501	0.397	0.471
	C4	0.000	0.000	0.237	0.019	0.536
	Total	0.433	0.454	1.239	2.323	3.179

Table 12: Hamming and Euclidean distances

As in Table 12, the results of the CODAS method are obtained by combining two distances, Euclidean distance and Hamming distance of alternative to the negative ideal solution. In here, we calculated each alternative distance to the negative ideal solution with respect to each criterion.

Results of the neutrosophic CODAS method are given in Table 13 as follows:

	AL1	AL2	AL3	AL4	AL5
Score	-7.9	-7.7	-2.1	5.79	11.9
Rank	5	4	3	2	1

Table 13: Results of the neutrosophic CODAS

When we examined the results, first and second orders are swapped. The main reason of this result is the deneutrosophication of the values that is applied in Step 4. Therefore, the neutrosophic division operation for the Step 4 can be developed for more liable results for further researches.

5 Conclusions

Neutrosophic logic presents an excellent tool to capture no only the vagueness of the data but also the indeterminacy of the decision makers in the assessment processes. In this paper, we have extended COCOSO method with interval-valued neutrosophic fuzzy numbers in order to select the most appropriate location for the waste disposal site under fuzziness. Neutrosophic COCOSO method produces meaningful results and can be used as an alternative MCDM method for the applications that have uncertainty.

We believe the applied approach is an appraisal framework which can be used as a decision-making tool by the managers or researchers to make useful inferences, judgments, and decisions. Since our considers both quantitative and qualitative data, it is very practical to use for the areas that have uncertainty and vagueness.

For further research, the data can be extended by using the experts judgments and opinions that are from the environmental sciences. Also, an integrated decision making process consists of fuzzy MCDM method and fuzzy inference system can be used and the obtained results can be compared.

References

[1] K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1) (1986) 87 – 96.

[2] I. Grattan-Guinness, Fuzzy membership mapped onto intervals and many-valued quantities, Mathematical Logic Quarterly 22 (1) (1976) 149–160.

[3] K. U. Jahn, Intervall-wertige mengen, Mathematische Nachrichten 68 (1) (1975) 115–132.

[4] A. Karasan, E. Boltürk, C. Kahraman, A novel neutrosophic codas method: Selection among wind energy plant locations, Journal of Intelligent & Fuzzy Systems 36 (2) (2019) 1491–1504.

[5] A. Karasan, E. Ilbahar, C. Kahraman, A novel pythagorean fuzzy ahp and its application to landfill site selection problem, Soft Computing (2018) 1–16.

[6] U. Rivieccio, Neutrosophic logics: Prospects and problems, Fuzzy sets and systems 159 (14) (2008) 1860–1868.

[7] F. Smarandache, A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability: Neutrosophic Logic: Neutrosophy, Neutrosophic Set, Neutrosophic Probability, Infinite Study, 2003.

[8] V. Torra, Hesitant fuzzy sets, International Journal of Intelligent Systems 25 (6) (2010) 529–539.

[9] R. R. Yager, Pythagorean fuzzy subsets, in: 2013 Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS), IEEE, 2013, pp. 57–61.

[10] M. Yazdani, P. Zarate, E. Kazimieras Zavadskas, Z. Turskis, A combined compromise solution (cocoso) method for multi-criteria decision-making problems, Management Decision.

[11] L. A. Zadeh, Fuzzy sets, Information and control 8 (3) (1965) 338–353.

[12] L. A. Zadeh, The concept of a linguistic variable and its application to approximate reasoningi, Information sciences 8 (3) (1975) 199–249.

[13] H. Y. Zhang, J. Q. Wang, X. H. Chen, Interval neutrosophic sets and their application in multi-criteria decision making problems, The Scientific World Journal 2014.

[14] H. J. Zimmermann, Fuzzy set theory and its applications, Springer Science & Business Media, 2011.