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Experton approach to vague information in portfolio selection problem with many views

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Abstract

This paper presents many expert fuzzy extensions of the Black-Litterman portfolio selection model. Black and Litterman identified two sources of information regarding the expected returns, and they combined these two sources of information in one expected return formula. The first source of information is the expected returns from the Capital Asset Pricing Model and thus should hold if the market is in equilibrium. The second source of information is the views held by the investors. The presented extension, owing to the use of a fuzzy random variable and experton, includes two elements that are important from a practical perspective: linguistic information and the multiple experts' views. This paper introduces the model extension sequentially and then illustrates it by empirical example.

Keywords: Black-Litterman model, Fuzzy random variable, Experton

1 Introduction

The Black-Litterman model (further BL model), which was first published by Fischer Black and Robert Litterman [3], provides a framework, which enables investors to combine their unique views regarding the performance of various assets with the market equilibrium by mixing different types of estimates. The BL model was expanded in [4] and [5]. Bevan and Winkelmann [2] provided details on how they use the BL model. The model was discussed pointedly as well in [16] and [25]. Now, there are a variety of models being labelled as Black-Litterman, although they may be very different from the original model created by Black and Litterman. Comprehensive taxonomy and a literature survey were provided in Walters [32] and Meucci [27]. Investors are trapped between the expectations and concerns regarding investing in the markets being persistently volatile. Furthermore, as is well known, the feelings of investors on returns can be different. What for someone is an attractive profit, for others is acceptable; for others still, it does not meet the minimum criteria. Another problem is the precise expression of the investors' feelings. Since the investor's view regarding future asset return is always subjective and imprecise, the fuzzy approach appears to be a natural extension of the BL model. Lawrence et al. [23] proposed fuzzy goal programming and assuming mean portfolio return and beta, the sensitivity of the expected excess asset returns to the expected excess market returns as fuzzy numbers. The researcher used a fuzzy trapezoidal number to represent investor views and omit the consistency aspect in combining prior probabilistic distribution and fuzzy views. Gharakhani and Sadjadi [13] assumed views as fuzzy numbers and mean asset return as well as covariance as fixed estimated parameters. The researchers focused on fuzzy compromise programming to find a solution of fuzzy return maximization and fuzzy beta minimization. Bartkowiak and Rutkowska [1] were the first to propose an approach combining the imprecision of predictions with the Black-Litterman model by using fuzzy random variable. The researchers also began a discussion on the aggregation of opinions of various experts by using the expected value of a fuzzy random variable. Fang et al. [11] also use a fuzzy random approach for investor views and redefine the covariance of the views using variance of a fuzzy random variable. However, both papers use a scalar variance; such a crisp evaluation of the dispersion does not appear to match with an epistemic interpretation of fuzzy data in these cases. In the literature, we were unable to find examples of the model that consider more than one opinion on views.

In this paper, we introduce extensions of the BL model with linguistic expressed views from different experts/many sources. In practice, we are often confronted with random experiments whose outcomes are not numbers but are expressed in inexact linguistic terms, in particular, predictions of events in the stock market. As an example, consider an expert who is questioned about how a planned tax on minerals will affect the valuation of energy companies'. Possible answers would be *slight decrease in values*, *not be* *affected* or *strongly reduce the value* and so on. A natural question is, how shall we consider these opinions when choosing a portfolio? As a tool for handling linguistic information, the fuzzy random variable (further f.r.v.) is used. Operationally, a f.r.v. is a random variable taking fuzzy values.

The concept of f.r.v. was introduced by Feron [12]. Later, different approaches to this concept have been developed: [22], [28], [21] and a unified approach [19]. The overview of different variants can be found in [8] and [15]. Contrary to data-based computations on the market, representing expert knowledge is incomplete; that is, one cannot establish the truth or falsity of proposition. Therefore, in our study, we focus on an epistemic approach to modelling. In this approach, a set represents incomplete information about a specific point value and contains all possible values under a given state of knowledge; therefore, a set can potentially be reduced into a more precise piece of information by collecting more knowledge. Thus, an epistemic model is a mathematical representation both of reality and the knowledge of reality that explicitly explains the limited precision of our measurement capabilities [9]. In the situation here, the probability space (Ω, P) is available; however, each realization of the random variable (an imprecise view) can be represented as a set. Therefore, for modelling, we use ill-known random variables (cf. [9]).

In a market reality, the quantity of information and recommendations is constantly increasing. Many services, such as Bloomberg or Reuters Thompson, allow access to the predictions of various experts, and investment firms pay their own experts. However, no one is based on a single piece of information, and predictions of individuals and their confidence can greatly vary. Thus, the problem arises considering different (disjointed or partially coherent) opinions of various experts or information from multiple sources. The aggregation method for linguistic expert opinion is one of the current challenges addressed in portfolio optimizing literature.

We use expertons to address and aggregate opinion. Expertons, introduced by Kaufmann [18], are a generalization of a probabilistic set when cumulative probabilities are replaced by intervals, which decrease monotonically. Expertons are used to obtain a better understanding of collected information from clients or decision makers and are particularly popular in economic and social applications. In [7] the mathematical formalization of the concept and its relationship with Φ-fuzzy sets and probabilistic fuzzy sets is analysed. Expertons were used among others in financial diagnosis ([10],[24]) to classify business sectors in the stock market [6], in the sensory analysis of products [33] or during the construction of the semantic scale of questionnaires [29], in group decision making [26] to establish the credit risk level of an investment project [30] and others [31].

In this paper, we propose also a new measure for mean value of experton and discrepancy of opinion. The arithmetic average value of the upper and lower end of experton intervals is usually used as expected/mean value (cf. mathematical expectation of an experton in [7]). Thus, outliers (opinions that stand out from the rest) significantly affect the mean value, which, in our application, is very unwanted. Thus, we use the average value of a function over the whole interval. In addition, it is important to know what degree of uncertainty the aggregated/average opinion has. We propose the intuitive measure define as the difference in area between upper and lower bound of experton. This approach allows one easily to determine the confidence level of aggregated views, which is needed in the Black-Litterman model. The main advantage of the BL model is intuitiveness and ease of application. The extended model presented in this article, despite extensive mathematical tools, retains these two features.

The remainder of this paper is organized as follows. In Section 2, we briefly present BL model. In Section 3, we introduce the fundamental theory of f.r.v. and experton. Section 4 presents the new BL model with linguistic views. Section 5presents an illustrative example based on an opinion/views survey . The last section concludes and summarizes the study.

2 The Black-Litterman model

The BL model enables investors to combine their unique views regarding the performance of various assets with the market equilibrium in a manner that results in intuitive, diversified portfolios. Here, we act in accordance with [16], [32] and [27]. We consider a market of n asset classes whose returns are normally distributed:

$$r \sim N(m, \Sigma). \tag{1}$$

The covariance Σ is estimated by past return realizations. To specify m, BL acknowledge and address the issue estimation risk: since m cannot be known with certainty, it is modelled as a random variable whose dispersion represents the possible estimation error. In particular, BL state that m is normally distributed:

$$m \sim N(\pi, \Sigma_\pi), \tag{2}$$

where π is our estimate of the mean and Σ_π is the variance of the unknown mean m about our estimation. The BL model begins with a neutral equilibrium portfolio for the prior estimate of returns. The model relies on General Equilibrium theory to state that, if the aggregate portfolio is at equilibrium, each sub-portfolio must also be at equilibrium. The model can be used with any utility function, which makes it very flexible. In practice, most practitioners use the Quadratic Utility function and assume a risk-free asset; thus, the equilibrium model simplifies to the Capital Asset Pricing Model (CAPM). The neutral portfolio in this situation is the CAPM market portfolio. CAPM is based on the concept that there is a linear relationship between risk and return. Further, CAPM requires returns to be normally distributed. This model is of the form:

$$E(r) = r_f + \beta r_m, \tag{3}$$

where r_f is the risk-free rate, r_m is the excess return of market portfolio, and β is a regression coefficient. With an equilibrium excess return, we can optimize the utility function. If the utility function is quadratic, we have:

$$U = w^T \pi - \frac{\delta}{2} w^T \Sigma w, \tag{4}$$

where U is the investor utility function, w is vector of weights, π is the vector of equilibrium excess returns, δ is the risk aversion coefficient, and Σ is the covariance matrix of the excess returns. U is a convex function, therefore, it will have a single global maximum. If we maximize the utility with no constraints, there is a closed form solution. If we follow any constraints, we must find the maximum numerically. However, we need to have a value for δ, the risk aversion coefficient. This coefficient characterizes the expected risk-return tradeoff. It is not essential to have an estimate of the investor's risk aversion. The scaling factor can be calculated from the expected return and the variance of the market. One means to find δ is:

$$\delta = \frac{r - r_f}{(\sigma_m)^2}, \tag{5}$$

where r is the total return on the market portfolio and σ_m is the variance of market portfolio. Is it possible to link δ with Sharpe Ratio (SR). Because SR is given by:

$$SR = \frac{r - r_f}{\sigma_m}, \tag{6}$$

thus

$$\delta = \frac{SR}{\sigma_m} \tag{7}$$

We continue to need the variance of our estimate of the mean i.e., Σ_π 2. Black and Litterman made the simplifying assumption that the structure of the covariance matrix of the estimate is proportional to the covariance of the returns Σ (cf. eq. 1). The researchers created a parameter τ, as the constant of proportionality: $\Sigma_\pi = \tau\Sigma$. As noted, the BL model enables the specification of the investors' views on the estimated mean excess returns. It is required that each view is unique and uncorrelated with the other views. Additionally, it is required that views are fully invested, either the sum of weights in a view is 0 (relative view) or is 1 (an absolute view). It is not required a view on any or all assets. We will represent the investors' views on assets using the following matrices:

- P, a $k \times n$ matrix of the asset weights within each view. For a relative view the sum of the weights equals 0, for an absolute view the sum of the weights is 1.
- Q, a $k \times 1$ vector of the returns for each view.
- Ω, a $k \times k$ matrix of the covariance of the views. Ω is diagonal as the views are required to be independent and uncorrelated. Ω^{-1} is known as the confidence in the investor's views. The i-th diagonal element of Ω is represented as ω_i.

Ω, the variance of the views is inversely related to the investors' confidence in the views, however the basic BL model does not provide an intuitive way to quantify this relationship. It is up to the investor to compute the variance of the views Ω. There are several ways to calculate Ω, among others:

- proportional to the variance of the prior,
- use a confidence interval,
- use the variance of residuals in a factor model.

First, in the most common method, it is assumed that the variance of the views is proportional to the variance of the asset returns, as the variance of the prior distribution is. He and Litterman [16] set the variance of the views as follows:

$$\Omega = diag(P\Sigma P'). \tag{8}$$

In the BL model, the prior distribution is based on the equilibrium implied excess returns. One of the major assumptions of the model is that the covariance of the prior estimate is proportional to the covariance of the actual returns, but the two quantities are independent. The parameter τ will serve as the constant of proportionality. The conditional distribution is based on the investor's views. The investors' views are specified as returns to portfolios of assets, and each view has an uncertainty which will impact the overall mixing process. The posterior distribution from Bayes Theorem is the precision weighted average of the prior estimate and the conditional estimate. We can now apply Bayes theory to the problem of blending the prior and conditional distributions to create a new posterior distribution of the asset returns:

$$r \sim N(E(r), M), \tag{9}$$

with mean:

$$E(r) = \left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1} \left[(\tau\Sigma)^{-1}\pi + P'\Omega^{-1}Q\right], \tag{10}$$

and covariance:

$$M = \left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1}. \tag{11}$$

With the posterior distribution it is possible to set and solve a mean-variance optimization, possibly under a set of constrains, such as boundaries on asset classes or a budget constraint. Solving optimization, we use the same utility function as for equilibrium returns 4.

3 Preliminaries

3.1 Fuzzy random variable

In the case of predictions about the future rate of return we model real-valued random attributes from which the available information is imprecise. Thus, it is natural to use fuzzy random variable. There are two main approaches to random mechanisms producing fuzzy values. According to Puri and Ralescu [28] the observations of certain random experiments do not consist of numerical outputs but only of vague linguistic terms. According to Kwakernaak's concept [22] the random mechanism behind fuzzy random variables produce real-valued data that cannot be exactly perceived. An explanation of formal differences and their interpretations in different types of fuzzy random variables can be found in [14]. We act in accordance with Kwakernaak's idea, which was later extended by Kruse [20] and Kruse and Meyer [21].
Let (Ω, A, P) be a probability space modelling a random experiment and $F_c(\mathbb{R})$ be the space of all fuzzy numbers.
A mapping $\chi : \Omega \to F_c(\mathbb{R})$ is said to be a fuzzy random variable associated with (Ω, A, P) if it satisfies for each $\alpha \in (0;1]$ that both:

$$inf\chi_\alpha : \Omega \to \mathbb{R} \text{ and } sup\chi_\alpha : \Omega \to \mathbb{R} \tag{12}$$

are real-valued random variables, where:

- χ_α is the interval-valued α-level mapping,
- $\chi_\alpha(\omega) = \{x \in \mathbb{R} : \chi(\omega)(x) \geq \alpha\}$,
- $\inf \chi_\alpha(\omega), \sup \chi_\alpha(\omega) \in \chi_\alpha(\omega), forall \omega \in \Omega$.

3.2 Fuzzy views

As Black and Litterman did, we will not specify the method for utilizing an expert or experts' views. Consider a group of experts who evaluate one view. Each expert expresses his/her evaluation by interval. The goal is to combine all experts' evaluations into a single one. Moreover, we also need information on how reliable/consistent this aggregate opinion is.

3.3 Experton theory

Let D be a set of opinion (views). The group of b experts is requested to express their subjective opinions on each element from D in the form of a confidence interval

$$\forall d \in D : \left[a_*^j(d), a_j^*(d)\right] \subset [0,1], \tag{13}$$

where $\subset$ denotes an interval inclusion and j is the number of an expert.
For example, we ask 15 experts their opinion regarding how they understand the small increasing return rate. We received 15 interval values, as shown in Figure 1.Using

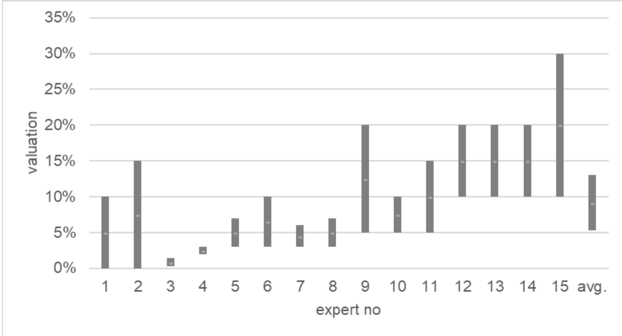

Figure 1: Interval valuation of term "small increase".

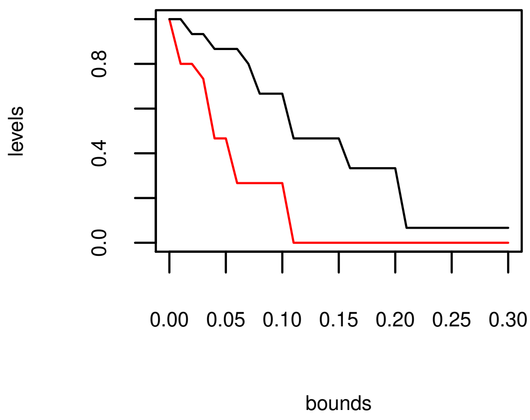

Figure 2: Experton for term "small increase".

the standard arithmetic average, we can say that, on average, experts have considered *a small* return rate of *approximately 9%*, with the values *from 5% to 13%*. Suppose we have a set of monotone compatibility levels: $0 < \alpha_1 < \alpha_2 < \ldots < \alpha_l < 1$ We consider the statistics for which each possible alternative (opinion) $d \in D$ involves the values of both the lower and the upper bound of confidence intervals. The cumulative distribution $F_*(\alpha, d)$ (the lower cumulanta) is then given by $a_*^j(d)$, and $F^*(\alpha, d)$ (the upper cumulanta) is then given by $a_j^*(d)$ as follows:

$$F_*(\alpha, d) = \frac{\sum_{j: a_*^j(d) \geq \alpha_i} 1}{r}, j = 1, 2, \ldots, r, i = 1, 2, \ldots, l, \tag{14}$$

$$F^*(\alpha, d) = \frac{\sum_{j: a_j^*(d) \geq \alpha_i} 1}{r}, j = 1, 2, \ldots, r, i = 1, 2, \ldots, l, \tag{15}$$

where $\sum_{j: a_*^j(d) \geq \alpha_i} 1$ $(\sum_{j: a_j^*(d) \geq \alpha_i} 1)$ counts the number of lower (upper) bounds of confidence intervals for which

$a_*^j \geq \alpha_i$ $(j : a_j^*(d) \geq \alpha_i)$; Thus, we obtain

$$\forall_d \in D, \forall_\alpha \in [0,1] : \tilde{A}(d) = [F_*(\alpha,d), F^*(\alpha,d)] \tag{16}$$

Experton for *a small increase*, based on the valuation presented in Figure 1, is shown in Figure 2. We define the mean value of experton as an interval $\mu = [\varepsilon_*, \varepsilon^*]$, where

$$\begin{aligned} F_*(\varepsilon_*) &= \frac{1}{max(a_j^*(d)) - min(a_*^j(d))} \int_{min(a_*^j(d))}^{max(a_j^*(d))} F_* dx \\ F^*(\varepsilon^*) &= \frac{1}{max(a_j^*(d)) - min(a_*^j(d))} \int_{min(a_*^j(d))}^{max(a_j^*(d))} F^* dx \end{aligned} \tag{17}$$

The area between upper and lower bounds can be interpreted as the dispersion of opinions. Thus, the ω_i is calculated as:

$$\omega_i = \frac{\int_{min(a_*^j(d))}^{max(a_j^*(d))} F^* dx - \int_{min(a_*^j(d))}^{max(a_j^*(d))} F_* dx}{max(a_j^*(d)) - min(a_*^j(d))} \tag{18}$$

The example valuation has mean value $[0.1038, 0.1515]$ with dispersion 0.2767 (see Figure 3) The most important feature of the above approach, next to the intuitive calculation, is robustness for extreme values and no loss of information in case of dispersed (volatile) data sets. Let us once more examine an example *small increase* from our survey. In A let the respondents be no. 12, 13, 14; in group B, respondents are no. 11, 12, 15. As we observe (Figure 1) in group A, respondents are unanimous, and everyone noted the same interval [10%; 20%]. Kaufmann's mean is 15%; in the proposed approach, the mean is also 15% with dispersion 5% resulting from the interval's width. In group B, respondents pointed to respective intervals: [5%; 15%], [10%; 20%], [10%; 30%]. The Kaufmann's mean remains 15%; however, our mean this time is the interval, [13.25%; 17.67%], with dispersion of 12.5%. Expertons for group A and B with their means are shown in Figure 4.

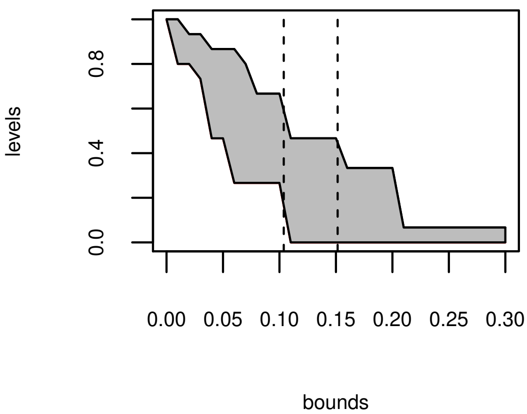

Figure 3: Mean value (interval between dashed lines) and dispersion (grey area) of experton "small increase".

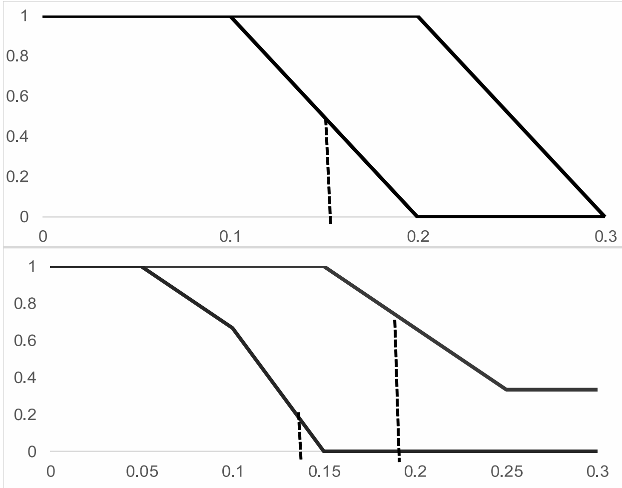

Figure 4: Mean value and dispersion of "small increase" expertons for group A (upper panel) and B (bottom panel).

4 Black-Litterman model with linguistic views

This section introduces the BL model with linguistic views. The Black-Litterman model begins with a neutral equilibrium portfolio for the prior estimate of returns, because the model relies on General Equilibrium theory. This portion of the BL model is in the new approach with no changes.

4.1 Investors' view

Investors have specific views regarding the expected return of certain assets in a portfolio, which differ from the implied equilibrium return. The Black-Litterman model allows such views to be expressed in either absolute or relative terms. The new BL model also allows the views to be expressed in linguistic form and the occurrence of many predictions/views in relation to a single asset.

Let us consider the investment planning for 2011; therefore, we use the period with the Eurozone crisis. The standard BL format view will appear as *Banks will have an absolute excess return of -5%*. Fuzzy BL format views: *Banks will lose slightly/significant*.

After fuzzy modification in BL model we can consider multiple views in the following forms:

- expert 1: *Banks will lose slightly*,
- expert 2 *The situation will not affect the bank's valuation*,

- expert 3 *The Eurozone banking system may lose approximately 3% of its value.*

We assume that there are b experts. Every expert formulates views about each asset. We aggregate the views using experton; as a fuzzy view, we use the proposed experton's mean and confidence: $1-\omega_i$, where ω_i is dispersion level, calculated according to eq. 18.

4.2 The new BL formula

Having specified the prior estimate of returns (π, Σ), the scalar τ, fuzzy view $\tilde{Q}$ and the covariance matrix of the error Ω, all of the inputs are then entered into the Black-Litterman formula, and the new combined return vector $\tilde{E[r]}$ is derived as follows:

$$\tilde{E[r]} = \left[(\tau\Sigma)^{-1} + P^{'}\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\pi + P^{'}\Omega^{-1}\tilde{Q}\right] \tag{19}$$

The covariance matrix of join distribution is:

$$\tilde{M} = \left[(\tau\Sigma)^{-1} + P^{'}\Omega^{-1}\tilde{Q}\right]^{-1} \tag{20}$$

Optimal portfolio weights are computed by solving the optimization problem. This problem can be a traditional mean-variance approach beginning with equilibrium expected returns as well as the maximization of utility function. Using the same methodology as computing the equilibrium returns (4), we will use the quadratic utility function, as follows:

$$\tilde{U} = w^T\tilde{E} - \frac{\delta}{2}w^T\tilde{M}w \tag{21}$$

where w vector of weights invested in each asset, $\tilde{E}$ - the new combined return vector, $\tilde{M}$ - new covariance matrix. As expected returns are fuzzy vector, it is a fuzzy optimization problem. A different method for fuzzy optimization can be found in [17]. Arriving at the optimal portfolio is somewhat more complex in the presence of constraints. We consider only budget constraints that force the sum of the total portfolio weights to be one.

5 Empirical example

The goal of this section is to illustrate the new BL model and compare the results with those of the standard BL model.

5.1 Data

To create an empirical example, we choose data: gold (XAUUSD), stocks (S&P500), bonds (S&P U.S. Aggregate Bond Index). The chosen instruments are easily recognizable. Additionally, these instruments represents different class of assets, which investors treat interchangeable. We consider only 3 assets because the BL model is generally used to choice from different types of assets. In April 2017, we asked 15 members of Financial Engineering Students Interest Group to provide a linguistic assessment of future rates of return in the May to September 2017 period for stocks, gold and bonds. The same group was surveyed on the meaning of linguistic terms; therefore, we can calculate expertons and what it means. The determined values are presented in Table 1.

assets type	crisp mean	experton mean
shares	-0.0053	[-0.0120, -0.0077]
bonds	0.0080	[0.0005, 0.0045]
gold	0.0021	[0.0089, 0.0169]

Table 1: Mean value of views

5.2 Algorithm

The algorithm can be determined by the following steps.

1. Vector of equilibrium calculation. At the beginning of each half-year, we calculate the vector of equilibrium excess return and the covariance matrix of the excess returns. For these calculations, we use data from the previous year. We calculate the risk aversion parameter for benchmark portfolio to be 4.46, and we use $\delta = 4$ for our model. However, our results are not materially affected by this choice of parameter. Then, we find the maximum of function (4) in three cases for each model:
   - with no constraints,
   - subject to the sum of weights is equal 1,
   - subject to the sum of weights is equal 1 and no short selling.

   Thus, we find equilibrium portfolios (marked with CAPM) that are used as a benchmark. In addition, the benchmark (futher: berk) will also be a portfolio built in equal measure from all of the considered assets.

2. Views aggregation and following matrices creation:
   - $P - 3\times 3$,
   - $Q - 3\times 1$,
   - $\tilde{Q} - 3\times 1$,
   - $\Omega - 3\times 3$.

3. Combined return vector calculation. With matrices P, Q and Ω, we calculate the new combined return vector $E(r)$ (10) and covariance matrix of the join distribution M (11). Similarly, in the fuzzy case, with matrices P, $\tilde{Q}$ and corresponding Ω we calculate the new combined return vector $\tilde{E(r)}$ (19) and the covariance matrix of the join distribution $\tilde{M}$ (20). We use parameter $\tau = 1$.

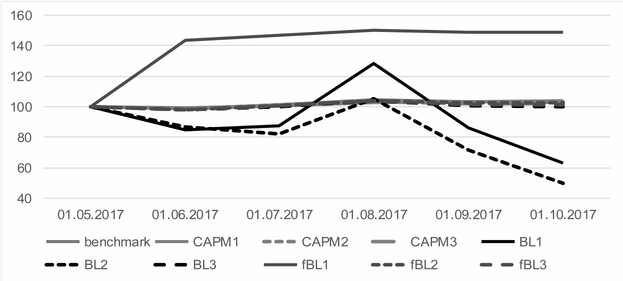

Figure 5: Results of portfolios with an initial value of 100

4. Utility function optimization. Thereafter, we again optimize utility function (4) and fuzzy utility function (21) with constraints.

5.3 Results

We check performance of all portfolios every month during the next six months, start value for each portfolio equals 100; the results are presented in Table 2 and Figure 5. As we can observe, the views in study period were not achieved. However, the use of expertons and fuzzy views helps maintain the value of portfolios at the benchmark level, and, in the case without constraints, attains higher profits from investment.

	30.06	31.07	31.08	30.09	31.10
Berk	99.34	100.92	102.59	103.63	102.54
CAPM1	98.55	101.41	104.67	103.33	104.00
CAPM2	98.55	101.11	103.98	102.66	103.18
CAPM3	98.65	100.78	103.53	102.01	102.19
BL1	84.86	87.55	128.25	85.97	62.89
BL2	86.87	82.47	105.11	71.82	50.11
BL3	97.86	100.06	104.10	100.95	100.22
fBL1	143.84	146.80	150.53	148.68	149.10
fBL2	98.28	100.92	104.22	102.40	102.66
fBL3	98.39	100.54	103.71	101.66	101.55

Table 2: Results of portfolios

6 Conclusion

The mean-variance optimization is an essential tool for portfolio managers. It does have certain weaknesses, such as the model's sensitivity to small changes in initial values and the estimation errors. The BL model, in particular, offers a useful framework that can increase the performance and robustness of the portfolio. Additionally, the BL model has proven very successful in reducing the estimation errors. The fuzzy extension of BL model, presented in the paper, allows to include linguistic views for many view sources. To model the linguistic view, f.r.v is used. This approach lets one formulate intuitive views, as well as set the opinions of a group of experts. The use of linguistic views of investors requires their acquisition. There is a natural question about the subjectivity and imprecision of information. To examine differences in the opinions of investors, the survey has been conducted. In our study, respondents' opinions were significantly overstated in comparison to historical rates of return. This finding suggests inclusion of views of expectations and desires. In our extension, investors are independent. In future works, this assumption will be repealed.

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