Physico-Mathematical Model of the Effect of Ballistic Solid Fuel on the Reservoir of an Oil and Gas Well

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Abstract – The paper studies mathematical modeling of effects by ballistic solid fuel pressure generators on the near-well formation zone. The research is of current relevance since no suitable tools for this type of solid fuel can be found for hydrodynamic simulation of oil and gas wells processing. Which can be used to increase the flow rate of natural hydrocarbons through improvement of the reservoir properties. A simulation algorithm is described; a graph to illustrate relative precision of calculated and actual data is presented.

Key words – pressure generators; ballistic solid fuel; near-well formation zone; mathematical modeling; oil; gas.

I. INTRODUCTION

The global trend of oil production is currently characterized in terms of late stage of well operation, hard-to-recover reserves, effect of various methods and properties on the near-well formation zone. The majority of the applied methods cause enormous harm to the environment due to poisoning of the earth’s interior by chemical reagents (acid treatment, proppant injection, etc.). Non-damaging methods of well treatment to intensify hydrocarbon inflows are well known [1–3].

The study investigates modeling of reservoir treatment by powder pressure generators. To perform the study, the main parameters of the ballistic solid fuel were determined, the characteristics affecting the initiation of cracks for fluid outflow were calculated, an algorithm was developed and a graph was plotted to illustrate the precision of calculations with the data obtained during pilot tests of industrial technology in real conditions.

II. METHODS AND MATERIALS

Methods of mathematical and physical modeling, the theory of shock waves and the theory of state of real gases, experimental studies in wells, and analysis and synthesis of production data were employed to fulfill the task.

The employed methods allowed developing a physico-mathematical model of gas-dynamic effect of the combustion products of solid fuel ballistic charges on the near-well formation zone. The model is a set of interdependent physical processes occurring in the pressure generator, well and reservoir. It provides a complex technology with the developed design, specified size and amount of solid fuel charges, calculated change in pressure in the well and the size of the formed cracks depending on well conditions and characteristics of solid fuel charges prior to well processing. The studied model can be used as the basis for development of software simulator. It determines the interrelation of the working parameters of the device, which generates the powder effect, with the parameters of the well and the well and the formation and contains basic equations to describe the gas-dynamic effect of the combustion products of the solid fuel charge on the reservoir. The model of gas-dynamic effect determines the pressure and temperature in

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the treated well zone not only at any operating time of the gas-generating section of the powder pressure generator, but also for some time after the end of its operation.

In [3], a system of algebraic and differential equations was obtained to describe the combustion of powder charge of a simple geometric shape (cylindrical single-perforate grain) in a well filled with fluid, and a method was developed to calculate the dependence of the pressure generated by the generator as a function of time, taking into account the process of cracks formation in the treated reservoir. It includes the mechanism of solid fuel charge burning represented by relative volume ratios, which do not allow calculation of the parameters of combustion of complex solid fuel charge, i.e. a charge composed of grains of various designs made from solid fuel of different grades, including grains with a more complex channel shape, for example, grains with slotted channels and hexagon channels [1].

To describe the model of gas-dynamic effect on the near-well formation zone, the results were obtained in [3] were used. In this study, basic equations of internal ballistics are derived based on the laws of conservation of mass and energy of a gas bubble. These equations can be used to calculate the pressure and temperature of gases formed by the solid gas-generating device in a well at any treatment time:

a) equation to calculate gas pressure in the gas bubble

\[
dp{t} = \frac{1}{W} [\dot{\lambda}SU \rho_0 (RTv + C(k-1)(T_{init} - 293.15)] - PSU - \frac{kPS_w (U_{up} + U_{bot})}{W}
\]

\[(1)\]

b) equation to calculate gas temperature in the gas bubble

\[
dt{W} = \frac{T}{WP} [SU \rho_s [R(\lambda Tv - T + C(-1)(T_{init} - 293.15)] - (k-1)PS_w(U_{up} + U_{bot})]
\]

\[(2)\]

where \(P\) is current gas pressure in the gas bubble; \(t\) is time; \(S\) is current volume of the gas bubble; \(\lambda\) is heat loss coefficient taking into account heat exchange between the gas bubble and the external environment; \(S\) is current burning surface of the fuel charge; \(U\) is current combustion rate of solid fuel; \(\rho_{0\,gas}\) is solid fuel density; \(R\) is gas constant of the products of combustion of the solid fuel charge; \(T_{init}\) is initial temperature of the solid fuel charge; \(\dot{\lambda}\) is adiabatic index of solid fuel combustion products; \(T\) is temperature of the solid fuel charge; \(C\) is solid fuel heat capacity; \(k\) is an adiabatic index of solid fuel combustion products; \(T_{init}\) is initial temperature of the solid fuel charge; \(S_{well}\) is the area of the flow string in the treated area; \(U_{up}\) is the speed of movement of the gas bubble upper boundary; \(U_{bot}\) is the speed of movement of the bottom boundary of the gas bubble; \(T\) is current gas temperature in the gas bubble.

The pressure in the gas bubble \(P\) formed in the well under gas-dynamic effect is determined for any time point \(t\) multiple of the given integration step \(\Delta t\) by numerical integration of equation (1) using the Euler method:

\[
P_{t+\Delta t} = P_t + \frac{\Delta P}{\Delta t}\Delta t
\]

\[(3)\]

where \(P_t\) and \(P_{t+\Delta t}\) are current values of the pressure in the gas bubble, respectively, at time \(t\) and \(t + \Delta t\); \(\frac{\Delta P}{\Delta t}\) is the pressure gradient in the gas bubble in the time interval equal to the specified integration step; \(\Delta t\) is the specified integration step of equations (1) and (2).

At the time of the beginning of the gas-dynamic effect on the near-well formation zone \(t = 0\), the pressure \(P_t\) is calculated to the pressure of detonation products in the gas bubble calculated by the formula

\[
P = \frac{m_{equiv}RT_k}{NW_k + (3-K_c)W_{gb} - m_{equiv}b}
\]

\[(4)\]

where \(P\) is pressure in the internal cavity at the time of the end of work of shaped charges, \(Pa\); \(m_{equiv}\) is mass of the equivalent charge, kg; \(W_p\) is volume of the perforator section, \(m^3\); \(W_{gb}\) is volume of the gas-generating section, \(m^3\); \(R\) is the specific gas constant of detonation products, \(J/(kg\,K)\); \(T\) is temperature of the detonation products in the internal cavity, \(K\); \(K_c\) is thermodynamic temperature of the detonation products calculated on the assumption that all chemical transformations proceed ideally and no heat exchange between detonation products and the external environment, \(K\); \(N\) is the number of cumulative charges placed in the perforator section of the integrated device, pcs; \(W_k\) is the volume of the perforator section punched by the cumulative charge, \(m^3\); \(\chi\) is the coefficient of temperature variation of detonation products in the internal cavity during heat losses due to heat exchange between detonation products and the external environment, and mixing of the detonation products with atmospheric air in the internal cavity; \(b\) is specific cavom, \(m^3/kg\); \(K_s\) is the space factor of the gas-generating section

\[
K_s = \frac{W_{sfc}}{W_{gb}}
\]

\[(5)\]

where \(W_{sfc}\) is the amount of solid fuel charge, \(m^3\).

The value \(t_{stop}\) is a boundary condition that indicates the time interval when the integration of equations (1) and (2) takes place. The time \(t_{stop}\) is set as a multiple of the integration step \(\Delta t\) that is sufficient to calculate the current pressure and temperature values in the gas bubble during combustion of the solid fuel charge of the integrated device, \(t_{add}\), and after combustion of the solid fuel charge, \(t_{comb}\), i.e.

\[
t_{stop} = t_{comb} + t_{add}
\]

\[(6)\]

The volume of the gas bubble formed in the gas-dynamic effect is determined by:

\[
W = W_{init} + W_{burnt} t + W_{comp} t + W_{cr} t
\]

\[(7)\]

where \(W_{init}\) is initial volume of the gas bubble formed at the time point \(t\) at the cumulative charges of the perforating section stop operating; \(W_{burnt} t\) is the volume of the burnt portion of the solid fuel charge at the considered time point \(t\) of the gas-generating section operation; \(W_{comp}\) is the volume formed due to compressibility and movement of the well fluid at the considered time point \(t\); \(W_{cr} t\) is the amount of cracks formed in the reservoir due to its rupture, which are filled with combustion products of the solid fuel charge at the considered time point \(t\).

The rate of solid fuel combustion depends on pressure in the
gas bubble and is determined from empirical equations – the laws of the rate of solid fuel combustion, which have the form:

\[ U(P_t) = a \left( \frac{P_t}{P_{atm}} \right)^\nu \quad \text{or} \quad U(P_t) = a + b \left( \frac{P_t}{P_{atm}} \right), \quad \text{(8)} \]

where \( P_t \) is gas pressure in the gas bubble at the time point \( t \); \( P_{atm} \) is pressure equal to normal (atmospheric) pressure; \( a, b, \nu \) are factors and index, respectively (the law of the rate of solid fuel combustion).

In the general case, the change (deformation) of a unit volume of well fluid is determined by the formula:

\[ \Delta W = W \beta_t (P_u - P_b), \quad \text{(9)} \]

where \( \Delta W \) is change (deformation) of a unit volume of the well fluid; \( W \) is a unit volume of well fluid; \( \beta_t \) is isothermal compressibility factor of well fluid; \( P_u \) and \( P_b \) are pressures affecting a unit volume of the well fluid from below and above, respectively.

The change in the gas bubble volume for any time interval \( \Delta t \) due to compressed upper column of the well fluid in the gasdynamic effect on the the near-well formation zone, for example, by the ZGRP-01-1 pressure generator [3], is determined by the formula:

\[ W_{c.a.t} = W_{c.a.t} + \sum_{i=1}^{n} \Delta W_i \Delta t, \quad \text{(10)} \]

where \( W_{comp.a.t} \) is the volume that will change the volume of the gas bubble within the time interval \( \Delta t \) due to compressed upper column of the well fluid; \( W_{comp.t} \) is the volume that changed the volume of the gas bubble at the time point \( t \) due to compressed upper column of the well fluid, at the beginning of the gasdynamic effect \( t = 0 \); \( \Delta W_i \) is the change in \( n \)-th unit volume of the upper column of well fluid affected by pressure change within the time interval \( \Delta t \) that is equal to the integration step of equations (1) and (2); \( n \) is the number of unit volumes of the upper column of the well fluid affected by pressure change at the time point \( t \).

The volume of gases \( W_{c.t} \) that fill cracks during their opening is determined by numerical integration of the equation:

\[ \frac{dW_{c.t}}{dt} = n h S_{well} \left( \frac{P_{cr.c}}{p} \right)^{\frac{k}{2}} \sqrt{2k \frac{P_{cr.c}}{k-1} \frac{P_{cr.c}}{p}} \left[ 1 - \left( \frac{P_{cr.c}}{p} \right)^{k-1/k} \right], \quad \text{(11)} \]

where \( n \) is the number of opening cracks; \( h \) is height of the opening cracks; \( P_{cr.c} \) is gas pressure (combustion products) at the crack entrance; \( k \) is adiabatic index of combustion products for the solid fuel charge; \( \rho_{cr.c} \) is density of combustion products of the solid fuel charge.

The length and width of the crack of height \( h \) opening during gas-dynamic effect on the the near-well formation zone are determined by the formulas:

\[ l = \sqrt{\frac{W_{cr.b}}{(2h(1-v)^2)h(P_{cr.c}-P_{res}-q_m)}}, \quad \text{(12)} \]

\[ \omega = \sqrt{\frac{4(1-v^2)xl}{E}} \times (P_{cr.c} - P_{res} - q_m). \quad \text{(13)} \]

where \( l \) and \( \omega \) are length and width of the formed crack, respectively; \( E \) is dynamic Young’s modulus of the reservoir; \( \nu \) is Poisson’s ratio of the reservoir; \( P_{res} \) is reservoir rock pressure; \( q_m \) is lateral rock pressure.

For the time interval \( \Delta t \) equal to the integration step, the burning surface of the solid fuel charge is determined by the formula:

\[ S_{\Delta t} = \frac{S_{t} + S_{t+\Delta t}}{2}. \quad \text{(14)} \]

The rate of movement of the upper boundary of the gas bubble \( U_{bot} \) is determined for each time interval \( \Delta t \) that is equal to the integration step of equations (1) and (2). Since the volume of the gas bubble \( W_{comp.t} \) formed due to compressibility and movement of the upper column of well fluid is known for the time interval \( \Delta t \) of the gasdynamic effect on the the near-well formation zone, the rate of movement of the upper boundary of the gas bubble in the time interval \( \Delta t \) is calculated by the equation:

\[ U_{bot} = \frac{W_{comp.t}}{S_{well}\Delta t}. \quad \text{(15)} \]

The rate of movement of the bottom boundary of the gas bubble \( U_{bot} \) is taken to be equal to zero, since the compressibility of the lower column of the well fluid is not taken into account.

The temperature of gases in the gas bubble \( T \) is determined for any time point \( t \), which is multiple of the given integration step \( \Delta t \), by numerical integration of equation (2), for example, by the Euler method:

\[ T_{t+\Delta t} = T_{t} + \frac{dT}{dt} \Delta t, \quad \text{(16)} \]

where \( T_{t} \) and \( T_{t+\Delta t} \) are temperature of gases in the gas bubble at the time points \( t \) and \( t + \Delta t \), respectively; \( \frac{dT}{dt} \) is temperature gradient of gases in the gas bubble in the time interval equal to a given integration step; \( \Delta t \) is a given integration step for equations (1) and (2).

III. RESULTS

The above equations of internal ballistics and the methods to determine parameters in these equations that show physical processes form a physico-mathematical model of the gasdynamic effect on the near-well formation zone by ballistic solid fuel. Figure 1 presents the flowchart of the model.

Figure 2 shows the calculated parameters of the gasdynamic effect on the reservoir versus the actual well treatment data using the ZGRP-1-01 powder pressure generator. The pressure in the well was measured by an independent pressure recorder (IPR) with a recording step of a fast process of \( 10^3 \) located 200 m from the device. The thickness of the treated reservoir is 3 m and perforation density is 30 hole/m. As can be seen in Figure 2, the agreement between the actual and design pressures in the well is satisfactory.
Fig. 1. Flowchart of a physico-mathematical model of the gas-dynamic effect on the near-well formation zone.
The studied physic-mathematical model of the gas-dynamic effect on the reservoir by the combustion products of solid fuel charges through the formed perforation channels is an element of the integrated technology that takes into account interdependent physical processes occurring in the ZGRP-1-01 generator, in the well and in the reservoir. Our findings do not contradict the studies performed [4–10].

The considered model can serve as a basis for the development of a software simulator. The simulator will help more accurately determine the relationship between the operation parameters of the device, which ensures powder action of the ballistic solid fuel, and the parameters of the well and the reservoir. In addition, the simulator will provide basic equations describing the gas-dynamic effect of the solid fuel charge on the reservoir.

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