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# Distortions Introduced by the Cartographic Projection of Gauss at a Distance of the Construction Site From the Axial Meridian

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***Abstract* – The article analyzes the distortions introduced by the cartographic projection of Gauss. The authors justify that modern high precision measuring instruments should be accompanied by qualitative methods of measurement results processing in the mountainous areas. It is shown that the operation of transfer of the axial meridian should be carried out practically at each object under construction at a distance from the axial meridian by more than 15-20 km. The maximum limits of the work area distance from the axial meridian are determined when performing high-precision geodetic operations on the territories of the area and linear objects. It is grounded that for engineering and geodetic works the three-degree width of the zone is more preferable than the six-degree width of the zone. Errors caused by the influence of geodetic height are analyzed. It is noted that the use of the auxiliary surface of relativity is a prerequisite for modern engineering and geodetic work. We have identified the main factors that distort the strict transition from Cartesian coordinates to the Gaussian coordinate system and then to the surface of the relativity of the object under construction.**

***Keywords* – artographic projection; axial meridian; distortion; auxiliary surface; Gauss projection; geodetic height; coordinates.**

## I. INTRODUCTION

Except for exclusively convenient and useful properties of the cartographic (conformal) projection of the Gauss, unfortunately, it is also inherent which it is necessary to take into account, and to make corrective amendments or to use the additional methods of calculation, allowing to compensate distortions. This has become particularly important because the arsenal of geodetic measuring instruments has been expanded to include satellite receivers, which make it possible to perform measurements with extremely high accuracy. In addition, the instrument-making industry began to produce electronic tacheometers, the measurement accuracy of which is tens of times less than the value of possible distortions of the Gauss conformal projection.

## II. MAIN PART

Gauss's cartographic projection distorts due to the distance from the axial meridian, as the scale in the equilateral transverse-cylindrical projection of Gauss in any point of the map and in any direction is expressed by the formula

$$m=\sec\varphi, \qquad (1)$$

where $\varphi$ is the central angle that characterizes the almuncantarate of the arc.

If the length of the arc is marked as $\varphi_0$, then, neglecting the ellipticity of the Earth, at small distances compared to the radius of the Earth R:

$$\varphi=\frac{\varphi_0}{R}$$

and formula(1) is reducible to

$$m=\sec\frac{\varphi_0}{R}\approx 1+\frac{\varphi_0^2}{2R^2}+\frac{5\varphi_0^4}{24R^4}+....$$

This means that the length of the S line will be distorted and the amount of distortion can be estimated by the formula (without taking into account small terms of expansion).

$$\Delta S=S\frac{Y_m^2}{2R^2}, \qquad (2)$$

where $Y_m$ is the mean value of the ordinates of the ends of the S-length line.

Figure 1 shows the distortion graph for the length of the line equal to 1 km, depending on the distance of this line from the axis meridian. As can be seen from the graph, even if the construction site is removed at a distance of 20 km, the distortions reach 5 mm, i.e. already exceed the accuracy achieved by modern measuring instruments.

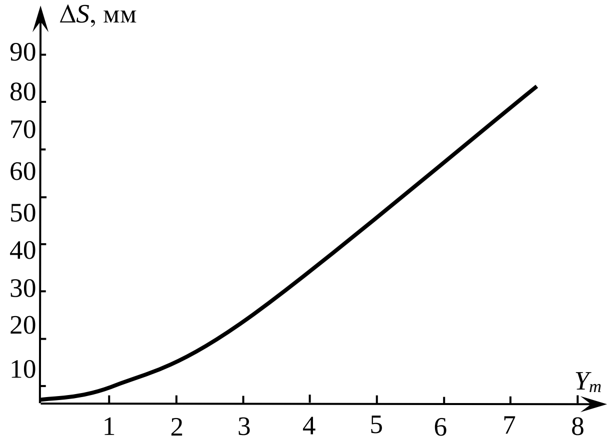


Fig. 1. Dependence of line length distortions equal to 1 km at a distance of S

It is possible to calculate more precisely by transforming the formula to the form given above

$$Y_m = R\sqrt{\frac{2\Delta S}{S}}.$$

If we assume that the permissible distortions ΔS = 5 mm, at distances $S$ = 1 km = 106 mm, we get

$$Y_m = \frac{R\text{ km}}{\sqrt{10^5}} = \frac{R\text{ km}}{316} = 20{,}16\text{ km}.$$

This means that the area of work should not be more than 15 km away from the axial meridian when performing high-precision geodetic work. In the construction of linear structures, such as roads, railways, gas and oil pipelines, high accuracy is not required and such works can be carried out at a distance of 50 km from the axial meridian.

The given analysis shows that the three-degree width of the zone is preferable to the six-degree width of the zone for geodetic engineering works. However, this operation does not solve the problem either.

These are very strict conditions, but when performing many types of engineering and geodetic work, they must be performed, otherwise the expediency of using high-precision measuring instruments and methods is lost. In addition, the experience of geodesy development has shown that improving the accuracy of geodetic measurements is always accompanied by an increase in the economic efficiency of objects under construction.

In order to meet these very strict requirements, the operation of transfer of the axial meridian to the average longitude of the object under construction should be carried out on a mandatory basis at almost every construction site of the responsible structure, as reducing the size of the zone even to 1.5 º will not eliminate these distortions to acceptable values for high-precision engineering and geodetic works.

Another equally important source of distortion is the removal of the construction site from the reference ellipsoid by height. The magnitude of these distortions is expressed in a formula:

$$\Delta S = S\frac{H}{R}, \tag{3}$$

where H is the geodetic height of the object.

The value of these distortions for the 1 km segment is presented in figure 2. As can be seen from the graph, the distortions caused by the removal of the construction site from the reference ellipsoid, become unacceptable even at a geodetic height of 200 m, and at high geodetic heights – catastrophic, and it is impossible not to take into account these distortions, as shown in figure 2.

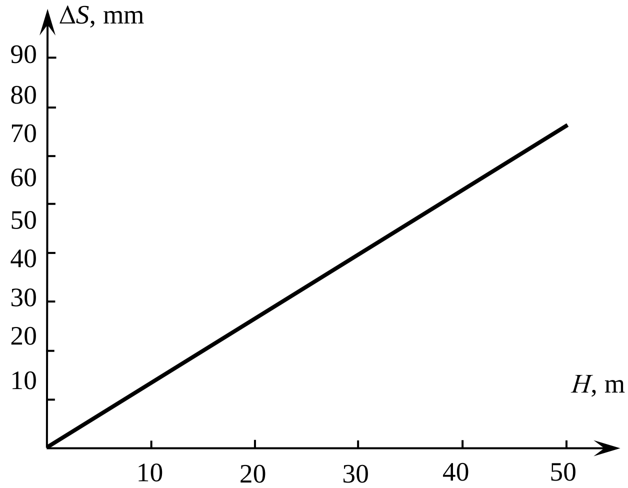


Fig. 2. Dependence of line length distortions equal to 1 km on the geodetic height

Due to the fact that geodesists, as a rule, know only the normal height, the geodetic height can be calculated by the formula:

$$H = H^{\gamma} + \zeta,$$

where $H^{\gamma}$ is a normal height;

$\zeta$ – an anomaly of height.

On the territory of Russia, the height anomaly varies widely from -8 m in the Caspian depression to +50 m in Vladivostok. This means that in Vladivostok, even on the coast of the ocean, where the normal height is close to zero, distortions can reach 7 mm per each kilometer of the line length.

In order to reduce the distortion it is useful to know the value of the height anomaly in the territory of engineering and geodetic works. Due to the fact that real objects, as a rule, are built on a significant geodetic height, it is necessary to have a special method of processing the results of geodetic measurements, which would allow to combine the results of calculations of geometric parameters of objects on the coordinates on the surface of the reference ellipsoid with field measurements of the same parameters by geodetic devices. The coordinates of the points are fixed with geodetic signs on the surface of the ground, and the coordinates of these points refer to the surface of the reference ellipsoid. As a result, the real horizontal layout, which registers the AB's light-distance meter, is not equal to the distance A′B′ calculated from the coordinates (figure 3)

$$S_{A'B'} = \sqrt{\Delta x_{A'B'}^2 + \Delta y_{A'B'}^2}.$$

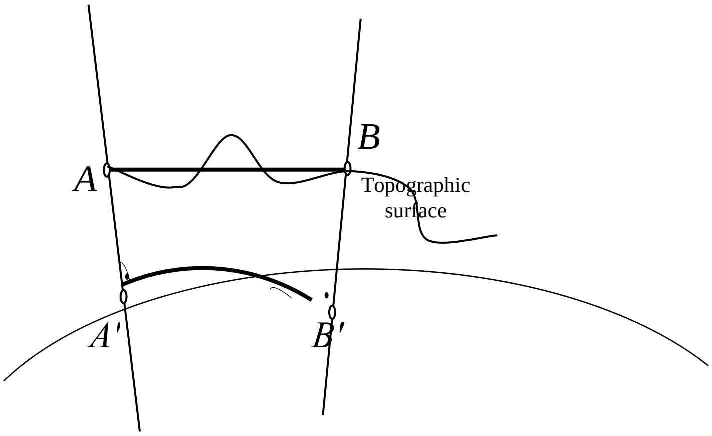


Fig. 3. Horizontal placement of AB is not equal to the distance A′B′ on the ellipsoidal surface

Nowadays the following practice of performance of engineering-geodetic works at construction of various objects is widely spread. First, the main or main axes of the object are moved according to the project coordinates. In this case geodetic points should be located around the object, creating a so-called control network. The coordinates of the control network are defined in the local coordinate system. It is allowed to combine only one point of the local network with the point of the State Geodetic Network and the direction angle of only one side. This method is used to avoid distortion of the field measurements as a result of network equalization, as full-scale measurements are made at the real geodetic height, and the coordinates of the State Geodetic Network points are determined on the surface of the reference ellipsoid, where the geodetic height is zero. Figure 4 shows that.

Not taking into account the real geodetic height in the construction of extended objects, there may be mutual economic claims, as the real length of the extended object will always be greater than the design length, which is calculated by coordinates. Thus, for example, the real length of the road of 1000 km, laid at an average geodetic height of 300 m, will be almost 50 m longer than the design length. In the mountains, these distortions will be even more noticeable. At an average height of 1,000 m, the real length of the road, which is only 100 km long, will be almost 16 m longer than its design length. In case of expensive construction works in the mountains, the cost of such an "additional" section will be significant. At the same time, there may be difficulties in the construction of high-rise objects, processing the results of high-precision measurements, creation of geodetic networks [2, 5–11].

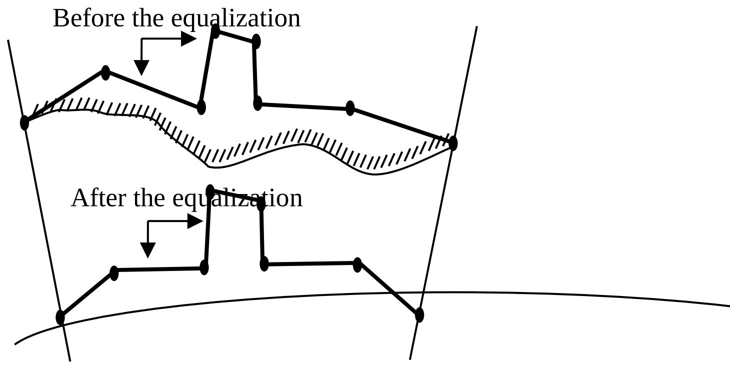


Fig. 4. Transfer of coordinates of polygonometric stroke points to the surface of relativity as a result of equalization

In modern practice of engineering and geodetic works both electronic total stations and satellite receivers are used equally. However, the account of distortions inherent in the conformal projection of Gauss should have different methodical methods. In the following sections of the dissertation will be justified methodical techniques that allow to significantly reduce the negative properties of the cartographic projection of Gauss.

Average square errors of coordinate differences determined by the results of phase measurements are much smaller and sometimes an order of magnitude smaller than the average square errors of coordinate differences of the same points in the state geodetic coordinate systems [4]. The result is a problem with coordinate transformation – more accurate measurements are based on a less accurate network. It is not recommended to change the coordinates of the points of the State Geodetic Network, as a huge amount of topographic surveys, breaking down works, executive surveys, cadastral documents, etc. performed by the State Geodetic Network. In the case of satellite measurement methods, the high accuracy of determination of point differences, as stated by the manufacturer of satellite receivers, is observed only in the Cartesian coordinate system of PZ-90 or WGS-84, and engineering and geodetic networks are created in other, significantly different coordinate systems. In the transition from Cartesian coordinates to geodetic network coordinates, the transformed coordinates can be negatively affected by inaccurate knowledge of point height anomalies, errors in determining point coordinates based on the results of code measurements, and errors in point coordinates of geodetic networks [3].

As it was already noted earlier, in case of thickening of planned geodetic network points, for example, by polygonometry method, reduction of measured values to the surface of relativity is not made even at significant geodetic heights. But if such geodetic constructions are based on the points whose coordinates are calculated on the surface of the ellipsoid, then as a result of equalization of such constructions the coordinates of the thickening network points will also be related to the surface, as shown in Figure 4. However, the severity of the alignment of measurement results will be compromised, as there will be more than just field measurement errors in the non-connections. For example, at an average geodetic height of H = 200 m, with a stroke length of S = 1 km, the error δS will exceed 30 mm.

This means that modern high-precision measuring instruments should be accompanied by qualitative methods of measurement results processing, especially since in mountainous conditions this error can increase many times [1, 5-7].

Due to the fact that the coordinates of points in the Gauss projection are strictly connected with the dimensions of a specific reference ellipsoid at the geodetic height equal to zero, and the construction site in general is located much higher, with the geodetic height is connected with another source of distortion of geodetic measurements: the effect of the vertical lines evasion, as the results of geodetic measurements are reduced to the ellipsoid by the normal to the ellipsoid, and the geodetic tools in the process of field measurements are set on the plumb line.

The angle between the plumb line and the normal to the surface of the reference ellipsoid is called the relative or astronomical geodetic evasion of the plumb line (absolute evasion – the angle between the plumb line and the normal to the total terrestrial ellipsoid). The magnitude of vertical avoidance depends on many reasons and varies over a wide range, and is divided into two parts:

- common, corresponding to the general geoid waves, they do not exceed 8″ and are caused by the presence of depressions and continental elevations; they are characterized by slow change and uniformity over large areas;
- local, corresponding to local geoid retreats from the ellipsoid, mainly caused by the presence of mountain ranges and deep trenches; local evasion of steep lines extends over small areas and may vary significantly over short distances.

The deviations of steep lines in Vladikavkaz in the direction of north-south at a distance of 50 km change from -18″ to +35″. On Lake Baikal, the differences in deviations of vertical lines on different banks (at a distance of 60 km) reach 40″ In Hawaii, the difference in the deviation of the steep lines at a distance of 120 km reaches almost 100″ In the above cases, the main reason for the large changes in deviations of vertical lines is a significant change in the shape of the relief. However, significant changes in deviations of steep lines can also be found in flat and calm terrain. An example is the "local Moscow Attraction", which is caused by the local uneven distribution of masses with different specific weight in the Earth's crust. In Moscow, in the direction of north-south evasion of steep lines change by 15.6″ at a distance of 25 km. In Russia, Krasovsky's ellipsoid is so oriented in the Earth's body that the evasion of steep lines is on average ±3-4″. Exceptions are local anomalies of Caucasus, Baikal, Vladivostok, Moscow, etc.

On average, in the mountainous conditions of the Caucasus, the deviation of steep lines may exceed 20″, which at a geodetic height of even 500 m shifts the projection of the point on the surface of the reference ellipsoid by 50 mm. And since in mountains the deviations of plumb lines can fluctuate in considerable limits, the amount of distortion does not remain constant. This fact is only stated in scientific and technical literature, but there are no practical recommendations for engineering and geodetic work to reduce the impact of deviations from vertical lines. This fact significantly reduces the efficiency of using high-precision measuring instruments in mountain conditions [1].

The situation is complicated by the fact that the coordinates of the points of the State Geodetic Network in the Gauss coordinate system are referred to the surface of the reference ellipsoid, the results of satellite measurements are referred to the Cartesian coordinate system PZ-90 or WGS-84, and the real object is built on the geodetic altitude, which differs significantly from the surface of the reference ellipsoid, especially in mountainous areas. There are five circumstances that prevent a strict transition from Cartesian coordinates to the Gauss coordinate system and then to the surface of the relativity of the object under construction:

- a big mistake in knowing the geodetic height of the object under construction (as a rule, it is tens of meters);
- big mistakes of mutual position of the State Geodetic Network points, which should be the basis for high-precision determination of coordinates by modern measuring instruments; usually the mutual position of the points of the geodetic network is known to be much worse than the results of modern measurements allow;
- in mountainous conditions, significant distortions occur due to significant errors caused by avoidance of plumb lines;
- there is no clearly formulated concept of surface relativity and justification of the accuracy of transition from the ellipsoidal surface to the surface relativity and vice versa;
- the problem of taking into account the Earth's curvature at high-precision geodetic engineering works by modern high-precision geodetic measuring instruments is practically not discussed.

## III. CONCLUSION

Based on the above, the following conclusions can be drawn:

1. The operation of moving the axial meridian should be carried out at almost every object under construction, as the permissible distance from the axial meridian should be 15-20 km. Otherwise, the use of high-precision measuring instruments is rendered meaningless, as the distortions will be too high.

2. The use of the auxiliary surface of relativity is a prerequisite for modern geodetic engineering works.

3. In construction norms, rules, and other instructional and methodical data, it is necessary to recommend calculating the average geodetic height of the object under construction and its distance from the axial meridian.

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