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# Strength Assessment Modeling for Reinforced Concrete Beams in Transverse Forces Zone

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***Abstract* – The existing computational model to determine the strength of inclined sections of bending reinforced concrete elements, adopted in the regulatory documents of the Russian Federation, due to the complex stress-deformed state has significant differences with the actual nature of structure work and requires further development. When comparing calculated dependencies to determine the strength of an inclined section under the action of transverse forces with experimental data, the author found that in reinforced concrete beams without distributed reinforcement, the strength over an inclined section under the action of transverse ones does not in some cases reflect the actual operation of the structures under consideration. A mathematical model is proposed to estimate the effect of geometric characteristics of reinforced concrete beams on strength in transverse forces zone when the relative span of *a/h*₀ is changed from 1 to 3 based on the experimental data.**

***Keywords – reinforced concrete beams; strength; modeling; geometric characteristics; relative span of cut.***

## I. Introduction

In modern conditions, the development of new constructive forms directly depends on the calculation methods that ensure maximum material savings at a given level of reliability and durability. The solution to this problem is based on the improvement of calculation methods.

In the Russian Federation, according to the Regulatory Approach, the calculation of bending reinforced concrete elements by strength is made according to normal sections (under the action of bending moments and longitudinal forces); on inclined sections (under the action of transverse forces) [1].

When calculating by the model of inclined sections, the element strength along the strip between inclined cracks and inclined section for the effect of moments and transverse forces should be ensured. The strength of an inclined concrete strip between cracks is determined by empirical dependence and is determined by the strength of concrete, intensity of transverse reinforcement, thickness of a beam, and working height of the section. When calculating the strength of oblique sections for a bending moment, the same as when calculating normal sections, the height of the compressed zone is determined and strength condition is checked relative to application point of resultant force — tensile forces in reinforcement and compressive forces in concrete (in compressed reinforcement, if any). The strength of the inclined section to the effect of transverse force is determined by the empirical formula and depends on carrying capacity of transverse and bent reinforcement crossing the inclined crack, as well as the transverse force perceived by concrete of the compressed zone (the equation contains an unknown value – the length of projection of the inclined section). The greatest discrepancies between the theoretical and experimental values are observed with a relative span of the slice $a/h_0 \leq 3$.

In the current Code of Rules SP 6313330.2012, the strength calculation of short reinforced concrete elements (with a relative span of a slice of $a/h_0 \leq 1$a) is recommended to be made on the basis of a frame-core model. Thus, the considered Normative calculation methods do not fit together and a whole class of beams (with an average span of a slice) does not have a calculation methodology that would reflect the physical work of these structures.

## II. Methods and Materials

During the destruction of short reinforced concrete beams, two failure patterns were revealed: along a compressed concrete strip and along a normal section [2, 3, 4].

The calculation of short reinforced concrete beams on the action of transverse force to ensure strength along an inclined compressed strip located between the load and the support should be made from the condition

$$Q = 2S \sin\alpha , \tag{1}$$

where $S$ – compressive force in inclined element of the model;
$\alpha$ – tilt angle of compressed concrete strip;

$$S = S_c + S_{sw} , \tag{2}$$

where $S_c$ – force perceived by oblique compressed strip of concrete;

$$S_b \leq \gamma_b \varphi_c R_b b l_b , \tag{3}$$

$\varphi_c$ – coefficient taking into account the effect of concrete surrounding the calculation strip;
$\gamma_b$ – coefficient derived from experiments;
$R_b$ – design concrete resistance to axial compression;
$b$ – width of rectangular section of the beam;
$l_b$ – width of the calculated concrete strip;
$S_{sw}$ – effort perceived by distributed reinforcement.

The frame-rod model reflects the actual work of short concrete elements, uniquely determines the length of the projection of the inclined section. The calculated dependences for the evaluation of strength are consistent with experimental data, which is confirmed by experimental and theoretical studies [4–6]. The borders of use of frame-rod model are limited by relative cut span $a/h_0 \leq 1$.

Considering that the destruction of reinforced concrete beams (with a relative cut span of $a/h_0 \leq 1$) along an inclined section occurs according to one of the following schemes: crushing the inclined strip between the cracks; the shift of two parts of the bent element caused by transverse force; rotation of two parts of the bent element relative to the center of gravity of the compressed concrete zone over the crack, caused by the action of the bending element, the strength is calculated along the inclined strip between the inclined sections, along the inclined section of the moments and shear forces.

Under the Regulatory approach, the lateral force ($Q_b$) in the inclined section of a bending reinforced concrete element without distributed reinforcement perceived by concrete is determined depending on the design concrete resistance to tension ($R_{bt}$), working height ($h_0$), width *(b)* and length of the projection of the inclined section to the axis beams *(s)* by the empirical formula

$$Q_b = \frac{1.5 R_{bt} b h_0^2}{c}, \tag{4}$$

which has a number of restrictions: $0.5R_{bt}bh_0 \leq Q_b \leq 2.5R_{bt}bh_0$; inclined section projection length $h_0 \leq c \leq 2h_0$ according to [1] (or $0.6h_0 \leq c \leq 3h_0$ according to [8]).

The existing computational model to determine the strength of inclined sections of bending reinforced concrete elements, adopted in the regulatory documents, due to the complex stress-strain state has significant discrepancies with the actual nature of the structures and requires further development [7-10].

When comparing the calculated dependence (4) with experimental data carried out at American Concrete Institute (ACI) [11], the author found that in reinforced concrete beams without distributed reinforcement, the strength in oblique section under the action of transverse forces in some cases does not reflect the real work of the structures under consideration.

Four series of tests of reinforced concrete beams, conducted by Professor G.N.J. Kani (ACI). All prototypes have a number of constant parameters, such as concrete strength (280 kg/cm$^2$), strength and percentage of longitudinal reinforcement ($\mu_s$=2.8%), beam width (b=5.25 cm). Variable factors are: height of beams ($h_0$=13.5 cm in series I, $h_0$=27 cm in series II, $h_0$=54 cm in series III, $h_0$=108 cm in series IV) and the distance from the support to the line of action of the concentrated load. The prototype circuit is shown in Figure 1.

According to recommendations [1], for the projection length of the inclined section $h_0 \leq c \leq 2h_0$, the results are shown in Figure 2. The largest discrepancies between the experimental and calculated values are traced in reinforced concrete beams with a relative cut span ($a/h_0$) less than 3.

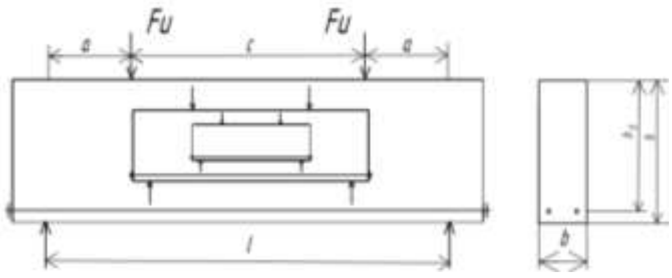


Fig. 1. Prototypes scheme of reinforced concrete beams.

The calculation of short reinforced concrete beams on the basis of the skeleton-core model is limited by the relative span of the slice $a/h_0 \leq 1$, the calculation by the oblique section under the action of transverse forces does not fully reflect the actual work of flexural concrete elements at $1 \leq a/h_0 \leq 3$. In addition, the considered calculation methods are not compatible with each other and a whole class of beams (one can say with an average span of a slice) does not have a calculation methodology that would reflect the actual operation of these constructions.

Below is the relationship between the strength of the inclined section of a bending reinforced concrete element and factors such as the height of the beam ($h_0$), the distance from the support to the line of action of the concentrated load *(a)* when changing the relative cut span ($a/h_0$) from 1 to 3 data from the American Concrete Institute (ACI) [11].

TABLE I. GEOMETRIC CHARACTERISTICS OF EXPERIMENTAL SAMPLES OF REINFORCED CONCRETE BEAMS

| N series | N sample | $h_0$ (m) | *a* (m) | *l* (m) | $a/h_0$ | *Ftest* (kN) |
|---|---|---|---|---|---|---|
| I | 1 | 0.144 | 0.340 | 1.137 | 2.41 | 102.887 |
| | 2 | 0.133 | 0.272 | 1.001 | 2.04 | 129.176 |
| | 3 | 0.136 | 0.272 | 1.001 | 2.00 | 138.073 |
| | 4 | 0.132 | 0.136 | 0.728 | 1.03 | 310.486 |
| | 5 | 0.136 | 0.136 | 0.728 | 1.00 | 315.379 |
| | 6 | 0.135 | 0.406 | 1.270 | 3.02 | 65.166 |
| | 7 | 0.140 | 0.373 | 1.203 | 2.67 | 100.307 |
| | 8 | 0.139 | 0.407 | 1.271 | 2.94 | 78.600 |
| II | 9 | 0.271 | 0.814 | 2.542 | 3.00 | 129.888 |
| | 10 | 0.266 | 0.271 | 1.457 | 1.02 | 719.277 |
| | 11 | 0.273 | 0.543 | 1.999 | 1.99 | 221.077 |
| | 12 | 0,275 | 0.678 | 2.271 | 2.46 | 145.457 |
| | 13 | 0.276 | 0.815 | 2.545 | 2.95 | 124.995 |
| | 14 | 0.275 | 0.679 | 2.273 | 2.47 | 152.574 |
| | 15 | 0.272 | 0.679 | 2.273 | 2.50 | 154.353 |
| | 16 | 0.270 | 0.544 | 2.002 | 2.02 | 223.746 |
| III | 17 | 0.542 | 1.085 | 3.186 | 2.00 | 326.499 |
| | 18 | 0.528 | 0.543 | 2.101 | 1.03 | 1095.60 |
| | 19 | 0.542 | 0.543 | 2.101 | 1.00 | 1170.77 |
| | 20 | 0.544 | 1.628 | 4.271 | 2.99 | 204.173 |
| | 21 | 0.549 | 1.087 | 3.190 | 1.96 | 393.668 |
| | 22 | 0.523 | 1.631 | 4.277 | 3.12 | 215.294 |
| | 23 | 0.524 | 1.631 | 4.277 | 3.11 | 215.739 |
| | 24 | 0.518 | 1,3589 | 3.734 | 2.62 | 229.528 |
| IV | 25 | 1.097 | 2.195 | 6.421 | 2.00 | 652.109 |
| | 26 | 1.095 | 2,737 | 7.506 | 2.50 | 473.735 |
| | 27 | 1.092 | 3.277 | 8.585 | 3.00 | 330.058 |

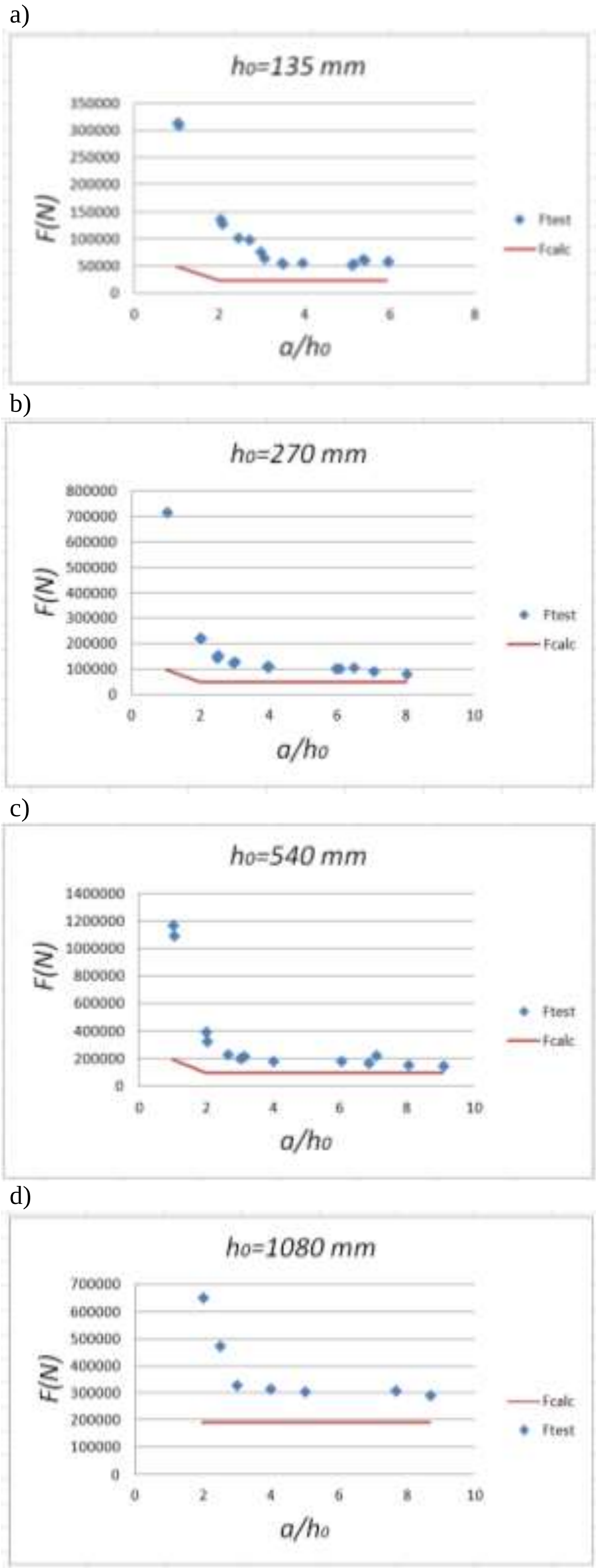


Fig. 2. Comparison of experimental data with the calculated dependence (4), when $h_0 \leq c \leq 2h_0$ [1].

To establish the functional dependence of strength on the geometric characteristics, 27 samples were considered (when the relative cutoff span $a/h_0$ was varied from 1 to 3) that collapsed in transverse forces zone.

Geometric characteristics of the samples under consideration are given in Table 1 (the scheme of prototypes is shown in Figure 1).

## III. Results

On the basis of the correlation analysis (Fig. 3), a connection was established between the resultant attribute (strength) and the factors under study (height of the beam, the span). The level of all identified bonds is different and consists of the following: the bond is the strength of a beam - the height is straight and noticeable; connection height of the beam - the span of the cut is straight and high; bond strength of the beam - the span of the cut is straight and weak

| | Ftest | h o | a |
|---|---|---|---|
| Ftest | 1 | | |
| h o | 0.43354100195413 | 1 | |
| a | 0.09875563264798 | 0.9225496466891 | 1 |

Fig. 3. The results of correlation analysis.

Based on the initial data (Table 1), using the software add-on “Analysis Package”, “Regression” (Microsoft Excel), we obtained a linear regression between $y=lnF_{test}$ and $x_1=lnh_0$, $x_2=lna$. The results of the regression statistics are presented in Figure 4.

The calculated regression coefficients enable to obtain an equation expressing the dependence of the strength of reinforced concrete beams ($F_{test}$) on the height ($h_0$) and the span *(a):*

$$y = 7.46 + 2.27x_1 - 1.50x_2 . \quad (5)$$

The value of the multiple coefficient of determination $R^2$=0,993 shows that about 99% of the total variation of the resultant trait is explained by the variation of the factor signs $x_1$, $x_2$. This means that the selected factors influence on the strength of the beams over an inclined section, which confirms the correctness of their inclusion in the constructed model. The calculated level of significance $\alpha_p$=6.05E–23<0,05 confirms the significance $R^2$.

| Regression Statistics | |
|---|---|
| Plural R | 0.992937413568957 |
| R-squared | 0.98592470726501 |
| Normalized R-squar | 0.98475178620376 |
| Standard error | 0.09271463428656681 |
| Observations | 27 |

Analysis of variance

| | df | SS | MS | F | Significance F |
|---|---|---|---|---|---|
| Regression | 2 | 14.4508746884 | 7.225437 | 840.557757 | 6.0462937E-23 |
| Remainder | 24 | 0.20630408186 | 0.008596 | | |
| Total | 26 | 14.6571787703249 | | | |

| | Coefficients | Standard error | t-statistics | P-Value | Lower 95% | Upper 95% |
|---|---|---|---|---|---|---|
| Y-intersection | 7.4571893735 | 0.05227828599 | 142.644106074 | 1.15E-36 | 7.34929229 | 7.565086452 |
| Variable X 1 | 2.2712103124 | 0.05656095505 | 40.1550912648 | 1.60E-23 | 2.15447423 | 2,38794638 |
| Variable X 2 | -1,74744957 | 0.046995632488 | -31,82626915 | 3.85E-21 | -1,39870143 | -1,39870143 |

Fig. 4. The conclusion of regression statistics.

Testing the significance of regression coefficients shows that the absolute values of the coefficients $a_0$, $a_1$, $a_2$ are greater than their standard errors. Moreover, these coefficients are significant, which can be judged by the values of the $P$ – value indicator (less than a given level of significance $\alpha = 0.05$).

Thus, the computational model to determine the strength of reinforced concrete beams at destruction along an inclined section reflects the influence of the studied factors $x_1$, $x_2$ on the resultant feature.

Let us transform equation (5) into a power function (to clarify the combined influence of the factors $h_0$, when calculating bent elements from the inclined section on the effect of the transverse force):

$$\ln F_{test} = \ln e^{7.46} + \ln h_0^{2.27} + \ln a^{-1.5}, \quad (6)$$

$$F_{test} = e^{7.46} \frac{h_0^{2.27}}{a^{1.5}}. \quad (7)$$

The resulting expression enables to quantify the influence of each of the factors $h_0(m)$, $a(m)$ on the strength of reinforced concrete beams $F_{test}(kN)$ in transverse forces zone.

Equation (7) can be reduced to the form:

$$F_{test} = e^{7.46} \frac{h_0^{0.77}}{(a/h_0)^{1.5}}, \quad (8)$$

which enables to estimate the effect of the height of the beam on the strength of the inclined section for a given relative span of the shear. The experimental confirmation of the effect of the joint increase in the height of the beam and span cut (2, 4, 8 times) on the strength of the wall-beams when $a/h_0$ varies from 1 to 3 is shown in Figure 5.

Increasing the height of the beam and the span of the cut provides the maximum increase in strength with a relative cut span of $a/h_0$=1 (safety margin of more than 35% compared to beams with $2 \le a/h_0 \le 3$), which indicates the peculiarities of short reinforced concrete beams-walls.

## IV. Conclusion

1. An analytical expression was obtained to determine the strength of reinforced concrete beams when the height (from 13 cm to 108 cm) and the span of the slice (from 13 cm to 328 cm) change.

2. Based on the correlation analysis, a link was established between the resultant attribute (strength) and the factors under study (beam height, span span). The level of all identified bonds is different and consists of the following: the bond is the strength of a beam - the height is straight and noticeable; connection height of the beam - the span of the cut is straight and high; bond strength of the beam - the span of the cut is straight and weak

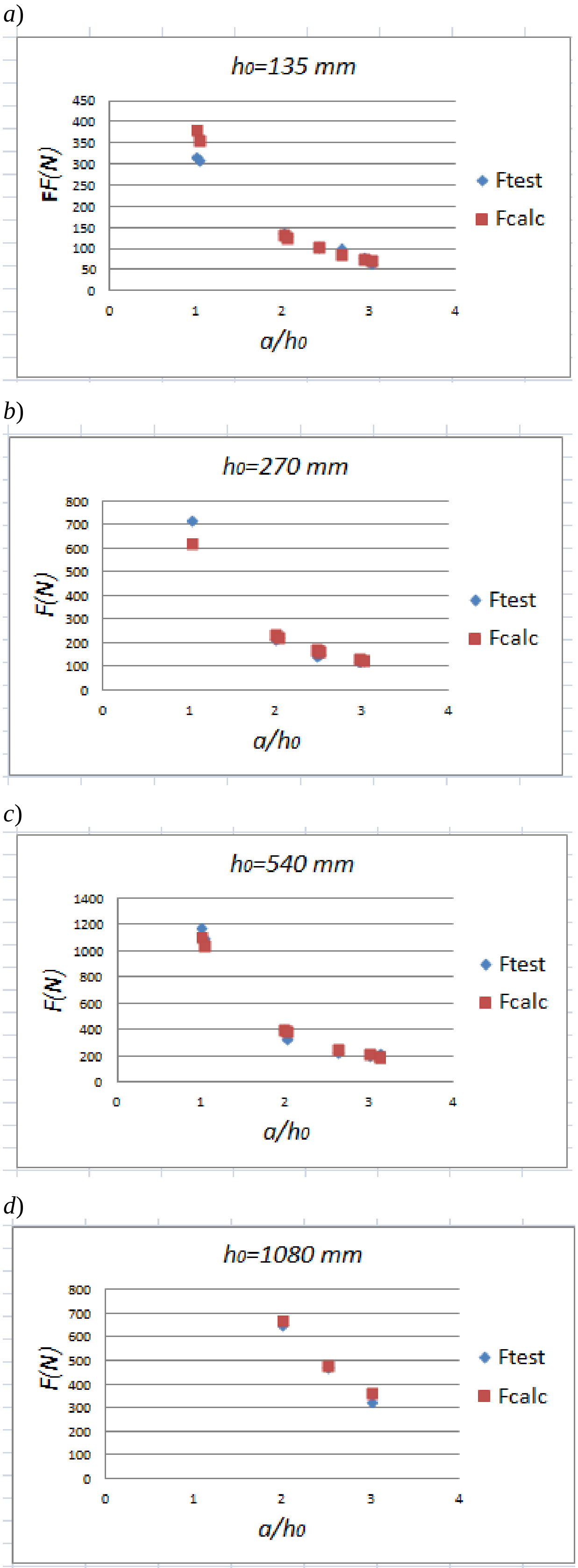


Fig. 5. Comparison of experimental and calculated values (dependence 8) of the strength of beams when changing the height and span of the cutoff.

3. It was established:

- at a relative cut span of $a/h_0$=1a, the beam height increases by 2 times (series II) increases the strength by 2.3 times; an increase in beam height by 4 times (series III) increases the strength by 3.7 times
- with a relative cut span of $a/h_0$=2, an increase in the beam height by 2 times (series II) increases the strength by 1.7 times; an increase in the height of the beam by 4 times (series III) increases the strength by 2.7 times; an increase in the height of the beam by 8 times (series IV) increases the strength by 4.9 times;
- with a relative cut span of $a/h_0$=3, an increase in beam height by a factor of 2 (series II) increases the strength by a factor of 1.8; increasing the height of the beam 4 times (series III) increases the strength by 2.8 times; increasing the height of the beam 8 times (series IV) increases the strength by 4.6 times;
- proportional increase in the height of the beam and the span of the cut provides the maximum increase in strength with a relative cut span of $a/h_0$=1 (safety margin of more than 35% compared to beams with $2 \leq a/h_0 \leq 3$), $2 \leq a / h0 \leq 3$) walls.

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