Accuracy of Determining Buildings’ Geometric Characteristics and Structures by Modern Means of Geodetic Measurements

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Abstract – The article deals with topical issues of determining the accuracy of geometric characteristics of buildings and structures by modern means of geodetic measurements. The method of testing high-precision electronic total stations under production conditions is proposed to evaluate and improve the accuracy of horizontal angle measurements. The paper presents the results of studies on the dependence of spatial coordinates error on the accuracy of measurement and the value of angles and distances using different models of electronic total stations, showing the possibility of determining the coordinates of points on the facade of the building with sufficient accuracy for practice. The necessity of using satellite levelling in solving a number of production problems in different conditions is justified. We have conducted theoretical studies of the accuracy of satellite technology exceedance detection. It is shown that it is possible to determine the exceedances by the method of satellite levelling with high accuracy, allowing to use this method in solving almost any engineering problems, provided that the distances between the observed points are small.

Keywords – spatial coordinates; vertical and horizontal angles; satellite receiver; exceedances; coordinate increments; mean square error.

I. INTRODUCTION

The most used means of geodetic measurements in all countries in the creation of reference geodetic networks, observations of deformations of the earth’s surface, buildings and structures, determining the geometric characteristics of objects and in solving other production problems are different models of electronic total stations and satellite receivers [12–14].

Let us consider spatial coordinates of the selected points on the building as geometric characteristics. At the first stage we define with what accuracy it is possible to determine spatial coordinates by electronic tachometers of various accuracy at performance of an actual production task-shooting of building facades.

In the second stage, we will assess the accuracy of satellite levelling.

II. MAIN PART

In order to obtain quality measurement results, it is necessary to study electronic total stations under production conditions. We chose the ELTAS10 No. 400396 high-precision electronic tachymeter for standardization. Precision set 1“, 1” и 1 мм. As a result of the research, the following errors were identified:

1. A cyclic error was detected in the light-distance meter by means of a comparative analysis of the measured horizontal layout and the delayed distance along the compartmentalized roulette tape measure. Measurements were made repeatedly (10 times or more) towards the prism and film. Table 1 and figure 1 show the results.

2. It is revealed that the energy axis of the rangefinder differs from the optical axis of the tube. We carried out a comparative analysis of the reports made on the horizontal and vertical circles with the circle left and right, when working in the auto-guided mode. Table 2 and figure 2 and 3 show the results.
TABLE I. RESULTS OF ERROR MEASUREMENTS OF THE ELECTRONIC TOTAL STATION METER

<table>
<thead>
<tr>
<th>Distance to target, in m</th>
<th>Cyclic error, in m</th>
<th>Distance to target, in m</th>
<th>Cyclic error, in m</th>
<th>Distance to target, in m</th>
<th>Cyclic error, in m</th>
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<td>-0.0007</td>
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TABLE II. RESULTS OF STUDIES ON ENERGY AXIS OF RANGEFINDER PARALLEL TO OPTICAL TELESCOPE AXIS

<table>
<thead>
<tr>
<th>Distance to target, in m</th>
<th>Discrepancy, in °</th>
<th>Distance to target, in m</th>
<th>Discrepancy, in °</th>
<th>Distance to target, in m</th>
<th>Discrepancy, in °</th>
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<td>HOR                    VER</td>
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<td>10.382                  10.7  -0.9</td>
<td>18.951 13.7        -8.5</td>
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<tr>
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<td>10.914                  11.5  -9.8</td>
<td>19.209 14.7        -9.1</td>
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<tr>
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<td>16.111                  13.0  -10.7</td>
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<td>17.917                  14.1  -7.2</td>
<td>22.794 14.8        -10.5</td>
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</table>

Fig. 1. Graph of cyclical error as a function of measured distance

Fig. 2. Graph of the energy axis deviation from the design position in the horizontal plane (collimation)

Fig. 3. Scheme of determination of spatial coordinates (x, y, z) made with a tacheometer
A number of papers [7–11] focus on the accuracy of obtaining coordinates by electronic tachometry. Our objective is to conduct accuracy studies of geometric characteristics determination by electronic total stations of different accuracy, namely, to study the dependence of spatial coordinates determination error on measurement accuracy and angles and distances. Accuracy of horizontal and vertical angles and distance were measured respectively: 1° 1 mm; 1" 2 mm; 2" 2 mm; 3" 3 mm; 5" 3 mm.

When taking pictures of facades using spatial tachymetry (figure 4) creates a spatial digital model of the object. The electronic tachymeter measures the horizontal αi, vertical vi angles and the inclined distance Si from the device to the spot.

\[ m_i^2 = m_i^2 \cdot \sin^2 \alpha + s_i^2 \cdot \cos^2 \alpha \cdot \frac{m_\alpha^2}{\rho^2} \]

where \( m^2 \) – mean square error detection \( h, \Delta x, \Delta y \).

Further, using the above formulas, a graph of the dependence of the error of spatial coordinates on the accuracy of measurement and the magnitude of angles and distances (figure 5) was plotted (horizontal and vertical angles changed within the range from 0° to 180°, the distance was taken 50 m). Formulas (2) show that the formulae for obtaining \( m_i \) and \( m_i \Delta \) are identical with a vertical angle of 0° (differ only in the type of angle being measured). A joint chart is created for them, shown in figure 3. Formulas (2) also show that the mean square error \( m \Delta \) is inversely proportional to \( m_i \Delta \).

Further we have conducted theoretical studies of the accuracy of geometric characteristics’ satellite definitions. Whereas the accuracy of coordinates of points determined by satellite methods is studied in detail [1–6], the accuracy of the determination of exceedances by satellite methods is not sufficiently studied. Satellite technologies are more and more used in solving a number of important production tasks, including those related to the growing compactness of the built-up areas in large cities and the lack of direct visibility of the observed points, the absence of a favorable environment for measurements by other geodetic means (bad weather conditions, dark time of day, etc.)

We will examine the accuracy of satellite measurements on the example of transferring the height to the top of the building.
Geodetic heights differ from normal heights by the value of quasigeoid heights

\[ h_{A_i} = H_{tA_i} + \xi_{A_i}; \]
\[ h_{B_i} = H_{tB_i} + \xi_{B_i}, \]

where \( H_{tA_i}, H_{tB_i}, H_{tA_i} \) and \( H_{tB_i} \) are geodetic and normal heights of some points A and B; \( \xi_{A_i} \) and \( \xi_{B_i} \) are quasigeoid heights above the reference ellipsoid.

Let us assume that the distance between the items is small and \( \xi_{A_i} = \xi_{B_i}. \) then we can write

\[ h_{AB} = H_{tA_i} - H_{tB_i} = H_{tA_i} - H_{tB_i}. \]

Exceeding \( h_{AB} \) will depend on the accuracy of the determination of the coordinate differences \( \Delta X_{AB}, \Delta Y_{AB}, \Delta Z_{AB}. \)

We use the following formula to estimate the accuracy of the excess calculation:

\[ H = \sqrt{X^2 + Y^2 + Z^2} \cos B + Z \sin B - a\sqrt{1 - e^2 \sin^2 B}. \]  \hspace{1cm} (3)

This formula is derived from the identity

\[ H = (N + H_e) \cos^2 B + (N + H_e) \sin B - e \sin 2N \sin 2B - N(1 - e^2 \sin^2 B), \]

where \( X, Y, \) and \( Z \) are Cartesian coordinates of the points where satellite receivers are placed; \( B \) is the geodetic latitude (in the ellipsoidal coordinate system) of the points of installation of satellite receivers; \( a, e^2 \) are correspondingly, a large semi-axis of Krasovsky’s ellipsoid and the first eccentricity of the ellipsoid;

\[ N = \frac{a}{\sqrt{1 - e^2 \sin^2 B}} \] — radius of curvature of the first vertical;

\( NG \) are geodetic heights of points.

Taking into account formula (3) (168), the difference in heights \( h_{AB} \) will be presented as

\[ h_{AB} = H_{AB} + \xi_{A_i} - \xi_{B_i}; \]
\[ h_{AB} = \sqrt{X^2 + Y^2} \cos B + \sqrt{X^2 + Y^2} \cos B_i + Z_{A_i} \sin B - \]
\[ - Z_{B_i} \sin B - a\sqrt{1 - e^2 \sin^2 B} - a\sqrt{1 - e^2 \sin^2 B_i}. \]  \hspace{1cm} (4)

In case of small distances between satellite receivers for accuracy estimation, we will assume that

\[ \cos B_A = \cos B_B, \sin B_A = \sin B_B. \]  \hspace{1cm} \text{Formula (4) is reducible to}

\[ h_{AB} = \sqrt{(X_1 + \Delta X_i)^2 + (Y_1 + \Delta Y_i)^2 + (Z_1 + \Delta Z_i)^2} \cos B_i + \Delta Z \sin B, \]  \hspace{1cm} (5)

where \( \Delta X, \Delta Y, \Delta Z \) is the increment of coordinates between points.

We will do the expansion in a Taylor series and stick to the first order members. Then, formula (5) is reducible to

\[ h_{AB} = \frac{X \Delta X + Y \Delta Y}{\sqrt{X^2 + Y^2}} \cos B + \Delta Z \sin B. \] \hspace{1cm} (6)

Previously, we carried out studies that showed that at small distances between the points the error of excess calculation depends only on the errors of coordinate increments. At the same time, there is almost no dependence on point coordinates errors. Assuming that the latitudes of the claims are practically equal, we obtain formula (6)

\[ dh_{AB} = \frac{X d(\Delta X) + Y d(\Delta Y)}{\sqrt{X^2 + Y^2}} \cos B + d(\Delta Z) \sin B. \] \hspace{1cm} (7)

We get

\[ m_{h_{AB}}^2 = \frac{X^2 m_{\Delta X}^2 + Y^2 m_{\Delta Y}^2}{X^2 + Y^2} \cos^2 B + m_{\Delta Z}^2 \sin^2 B. \] \hspace{1cm} (8)

Considering that satellite receivers have equal average square errors

\[ m_{\Delta X}^2 = m_{\Delta Y}^2 = m_{\Delta Z}^2 = m_{\Delta}^2 \]

Formula (8) is reducible to

\[ m_{h_{AB}}^2 = m_{\Delta}^2 \cos^2 B + m_{\Delta}^2 \sin^2 B = m_{\Delta}^2, \]

or

\[ m_{h_{AB}} = m_{\Delta}. \]

The average square error of determination of coordinates by modern satellite receivers of coordinates is \(~3 \text{ mm}.\) It is possible to determine the excesses of satellite receivers at small horizontal distances between the points of 100-200 m with the same accuracy. It is within these distances that satellite receivers are installed for engineering tasks.

IV. CONCLUSION

The performed researches show that the accuracy of geodetic measurements with the use of electronic total stations of different models and accuracy characteristics quite corresponds to the required accuracy of determination of geometric characteristics of buildings and structures when solving a number of engineering tasks, including surveying of facades, determination of deformed condition of buildings and
Accuracy of satellite leveling (~3 mm) also corresponds to the required accuracy in determining the geometric characteristics of buildings and structures, namely the definition of exceedances between the points located at a distance of 100-200 m, as well as to solve a number of other problems.

References