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Basic Math Course for Bachelors – Statistical Analysis

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***Abstract*-The results of the basic course Mathematics for Economist at the University of Economics in Prague in winter semester of the academic year 2017/2018 are statistical analysed in present paper. We shall study dependence of the results of examinations of 879 students in this course on the examiners (ten examiners took part in the examinations at this course). We shall use different statistical methods for the analysis (chi square test of independence in contingency table, analysis of variance, Scheffé's method etc.). The obtained results can be used for improving bachelor exams in mathematics at the department of mathematics in coming years.**

Keywords-bachelor exams, examiners, mathematics, statistical methods

I. Introduction

Bachelor exams in mathematics in basic course Mathematics for Economist (ident 4MM101) include mid-term test, final test and oral examination. These tests are standard test, the multiple choice question tests (see e.g. Klůfa (2015b), Klůfa (2016)) are not used. The bachelor exams of this kind are used at the Faculty of Informatics and Statistics, Faculty of Finance and Accounting, Faculty of Business Administration and at the Faculty of International Relations. Other faculties (Faculty of Economics and Faculty of Management) of the Prague University of Economics use different tests.

The number of points in the mid-term test in course Mathematics for Economist can be in interval [0,20], the number of points in the final test can be in interval [0,40] (Otavová and Sýkorová, 2014) and the number of points in the oral examination can be in interval [0,40], i.e. total number of points in mathematics in basic course Mathematics for Economist can be in interval [0, 100]. The result of examinations is as follows:

Grade		Points
Excellent	1	90-100
Very good	2	75-89
Good	3	60-74
Failed, eligible for retake	4+	50-59
Failed	4	0-49

Analysis of the results of the basic course Mathematics for Economist at the University of Economics in Prague in winter semester of the academic year 2017/2018 is provided in this paper.

The aim of this paper is to study dependence of the results of examinations in the course 4MM101 on the examiners (ten examiners took part in the examinations at this course). Similar problems are solved in Otavová and Sýkorová (2017), Kaspříková (2012), Klůfa (2015c), Kaspříková (2013), Bartoška, Brožová, Šubrt and Rydval (2013), Hrubý (2016), Ječmínek, Kukalová, Moravec and Filipová (2018), Poláčková and Svatošová (2013), Zvára and Anděl (2001), Kubanová and Linda (2012). These results will be used to further improve of the bachelor exams in mathematics at the department of mathematics.

II. Material and Methods

The analysed data are the results of 879 students in the basic course Mathematics for Economist at the University of Economics in Prague in winter semester of the academic year 2017/2018. Part of these data (results of examinations - grades) is in contingency table – see Table 1.

For study dependence of the results of examinations (grades) on the examiners we shall use χ^2 test of independence in contingency table. Statistic χ^2 is

$$\chi^2 = \sum_{i=1}^{r}\sum_{j=1}^{s}\frac{(n_{ij}-n_{ij}^o)^2}{n_{ij}^o} \tag{1}$$

where r is number of rows, s is the number of columns in contingency table and n_{ij}^o is the expected frequency in case of independence – see e.g. Anděl (1978). When

$$\chi^2 > \chi^2_\alpha((r-1)(s-1)), \tag{2}$$

where $\chi^2_\alpha((r-1)(s-1))$ is critical value of χ^2 distribution with (r-1)(s-1) degrees of freedom, hypothesis of independence is rejected at significance level, which is asymptotically equal to α.

For comparison of the examiners we shall use ANOVA and Scheffé's method. We shall verify the validity of the null hypothesis: mean number of points in mathematics is the same for all examiners. When (the test statistic F see e.g. Rao (1973))

$$F > F_\alpha(s-1, n-s) \tag{3}$$

where $F_\alpha(s-1,n-s)$ is critical value of Fischer-Snedecor distribution with (s-1) and (n-s) degrees of freedom (n =

879, s = 10 (number of examiners)), hypothesis is rejected at significance level α.

III. RESULTS

A. Dependence on the Examiners

Results of examinations in mathematics (distribution of the grades in course 4MM101) are in Table 1 and in Figuge 1. For example 4 students with examiner "C" obtained grade "excellent = 1" in exam in mathematics, i.e. 4 is frequency n_{31} in 3rd row and 1st column of the contingency table – see Table 1.

TABLE 1: DISTRIBUTION OF THE GRADES IN COURSE 4MM101 (CONTINGENCY TABLE)

Examiner	Grade				Sum
	1	2	3	4 and 4+	
A	8	13	41	27	89
B	6	29	28	33	96
C	4	23	46	30	103
D	8	11	14	18	51
E	5	20	36	22	83
F	9	9	16	38	72
G	20	23	49	11	103
H	10	20	32	41	103
I	6	35	28	20	89
J	7	20	29	34	90
Sum	**83**	**203**	**319**	**274**	**879**

Using data in Table 1 we shall study dependence of the results of examinations (grades) on the examiners. We shall test null hypothesis

H$_o$: results of examinations (grades) are not dependent on the examiners.

We shall use χ^2 test of independence in contingency table – see e.g. Anděl (1978). In the first step we calculate according to (1) statistic χ^2. For example (see Table 1)

$$n_{11} = 8, \quad n_{11}^{o} = \frac{89 \ x \ 83}{879} = 8.404$$

Because of the small expected frequencies, we combine grade "4" and "4+" in the contingency table. We have

$$\chi^2 = 88.480$$

Critical value of χ^2 distribution for 27 degrees of freedom and significance level 0.01 is

$$\chi^2_{0.01}(27) = 46.963$$

Since

$$\chi^2 = 88.480 \ > \ 46.963$$

null hypothesis Ho is rejected at approximately 1% significance level. For calculation we used MS Excel – see Marek (2013). We can say that the results of examinations (grades) depend on the examiners.

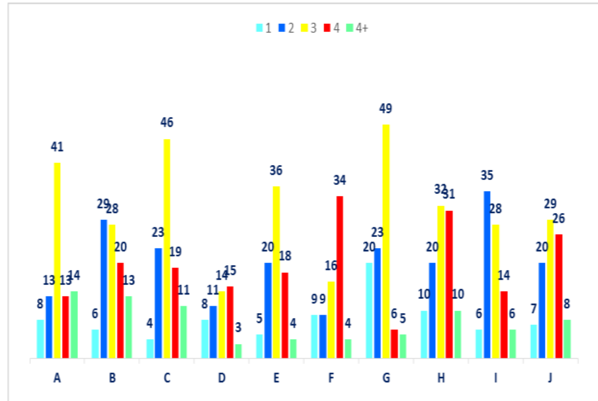

Figure 1: Distribution of the grades in course 4MM101 according to the examiners

B. Differences Between the Examiners

For more detailed analysis we now compare the number of points in mathematics (instead of grades). Basic descriptive statistics for number of points in the course 4MM101 are in Table 2. For comparison of the examiners we shall use ANOVA. We shall test null hypothesis

H$_o$: mean number of points in mathematics is the same for all examiners,

i.e. the differences between average number of points in Table 2 are not statistically significant (number of points in mathematics in course Mathematics for Economist can be in interval [0, 100]).

TABLE 2: BASIC DESCRIPTIVE STATISTICS FOR NUMBER OF POINTS IN THE COURSE 4MM101

Examiner	Frequency n_i	Sum	Average number of points	Variance
A	89	5423	60.93258	425.9954035
B	96	5907	61.53125	468.7148026
C	103	6304	61.20388	378.8697887
D	51	3272	64.15686	497.4149020
E	83	5053	60.87952	451.3755510
F	72	3848	53.44444	739.9687011
G	103	7214	70.03883	309.4494575
H	103	6033	58.57282	618.1490577
I	89	5865	65.89888	490.7510215
J	90	5215	57.94444	591.6935081

Results of ANOVA we got with MS Excel – see Table 3. Since

$$F = 3.709 \ > \ 2.428$$

null hypothesis is rejected at 1% significance level. There are some differences between the examiners (the differences between average number of points in Table 2 are statistically significant).

TABLE 3: RESULTS OF ANOVA

Source of variability	Sum of Squares	Degrees of freedom	Fraction	*F*	*P* value	*F* crit
Examiners	16347.87	9	1816.42957	3.709322	0.00014	2.427724
Residual	42554.4	869	489.693209			
Sum	441891.3	878				

Remark.

Moreover we used Bartlett's test to verify the assumption of ANOVA (the same variance of number of points for all examiners). Test statistic *B* (see e.g. Anděl (1978)) is *B*=25.097. Critical value of χ^2distribution for 9 degrees of freedom and significance level 0.01 is 21.666. Since *B*>21.666, the assumption of ANOVA is not met. Therefore we also used the corresponding nonparametric Kruskal-Wallis test. This test confirmed the results of ANOVA, different variance did not affect the ANOVA result.

Finally we shall study which pairs of averages differ significantly. We use Scheffé's method – see e.g. Anděl (1978). Pairs of averages differ significantly if absolute value of difference in averages exceeds critical value (see Table 2 and Table 3)

$$\sqrt{\left(\frac{1}{n_i}+\frac{1}{n_j}\right)\ x\ 9\ x\ 489.693209\ x\ 2.427724}$$

From Table 4 it is seen that a significant difference at 1% significant level is only between examiner "F" and examiner "G". All other pairs of averages are not significantly different.

Similar problem as in present paper was solved in Otavová and Sýkorová (2016). There are studied differences between results of examinations in mathematics obtained by students of different faculties of the University of Economics in Prague. Relation between results of the entrance exam test and university study results at Czech University of Life Sciences we can find in Kučera, Svatošová and Pelikán (2015). The same problem at Charles University was solved in Zvára and Anděl (2001). Dependence of the results of examinations in mathematics on test variants was investigated in Klůfa (2015a). Analysis of the entrance examinations in mathematics at University of Pardubice we can find in Kubanová and Linda (2012). Results of the entrance tests in mathematics at Comenius University in Bratislava were analysed in Kohanová (2012). The aim of these papers in comparison with present paper was a little different.

IV. CONCLUSION

Ten examiners took part in the examinations at the basic course Mathematics for Economist at the University of Economics in Prague in winter semester of the academic year 2017/2018. Differences between these examiners were analysed in present paper.

From chi square test of independence in contingency table it follows that the results of examinations (grades) depend on the examiners. Based on a detailed analysis of the differences between examiners (Scheffé's method) we can say that significant difference is only between examiner "F" and examiner "G". The differences between other examiners are not statistical significant. The results obtained in this paper will be used to further improve of the bachelor examinations in mathematics at the department of mathematics.

TABLE 4: ABSOLUTE VALUE OF DIFFERENCES BETWEEN AVERAGE NUMBER OF POINTS (SEE TABLE 2)

Examiner	A	B	C	D	E	F	G	H	I	J
A		0.60	0.27	3.22	0.05	7.49	9.11	2.36	4.97	2.99
B			0.33	2.63	0.65	8.09	8.51	2.96	4.37	3.59
C				2.95	0.32	7.76	8.83	2.63	4.69	3.26
D					3.28	10.71	5.88	5.58	1.74	6.21
E						7.44	9.16	2.31	5.02	2.94
F							16.59*	5.13	12.45	4.50
G								11.47	4.14	12.09
H									7.33	0.63
I										7.95
J										

* Significant difference for α=0.01 (Scheffé's method)

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