On Teaching Special Relativity as Part of a School Course of Physics

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Abstract-The study of Special Relativity (SR) in contemporary school is not satisfactory. It may be noted that special relativity is perceived by school students as difficult, unusual and separate from everything studied previously. Such a perception leads to not understanding Special Relativity, doubting its correctness, and at times even attempts to denounce it (as is witnessed nowadays). In fact, this is not true. Special Relativity is so elegant and such an intrinsic part of the theory of physics that upon its emergence it lead the theory of physics out of crisis at the turn of the 20th century. This should find its reflection in methodology of teaching physics, as it contributes to a deeper understanding of the theory, increased fundamentality in the study of physics, and development of content-related lines of fundamental physical formation, i.e. methodology and shaping the worldview.

Keywords-special relativity, physics education fundamentality, content-related lines of fundamental physical formation, transformations of coordinates and time, invariability

I. INTRODUCTION

When covering mechanics during the first stage of teaching, special attention of school students should be directed to the principle of Galilean relativity, more precisely its experimental aspect. When we verify the validity of the laws of mechanics, for example of the second Newton’s law in an inertial frame of reference $K$ and we determine that $F=ma$, then after the same experiments in another inertial frame of reference $K'$ that moves uniformly and linearly at velocity $V$ (for example, on board a ship that moves uniformly and linearly), we obtain the same results, i.e. $F'=m'a'$ (refer with: Fig.1). At this stage, the main idea of mechanics that all laws of mechanics are the same in any frame of reference must be understood and then consolidated during the second stage of learning. This principle is an experimentally established fact.

II. ORGANIZATION OF THE TEXT

If all laws of mechanics function equally in any inertial frame of reference, then it is impossible to determine by any mechanical experiment whether we are at rest or in uniform and linear motion. This is yet another wording of the principle of Galilean relativity which is often found in textbooks and helps to deepen the understanding thereof. All inertial frames of reference are indistinguishable in terms of their mechanical properties. It may be added, that in any inertial frame of reference space is homogeneous and isotropic, while time is homogenous. These are the main tenets of classical mechanics and they are easily learned by school students.

Figure 1. $K'$ moves along the $x$-axis of frame of reference $K$

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At the second learning stage, it is necessary to convince students, that if the laws of mechanics are the same in any inertial frame of reference and it is experimentally proven, then the equations describing them should be the same. This is obvious. It may be done by the means of the Galilean transformations given in their simplest form, when frame of reference $K'$ moves along the $x$-axis of frame of reference $K$, i.e.
It should be explained that these mathematical expressions demonstrate a relation between the coordinates and time of events in frames of reference \(K\) and \(K'\). Knowing the coordinates and time of an event in one inertial frame of reference, these expressions provide an opportunity to define the coordinates and time of that event in another inertial frame of reference. Using the Galilean transformations it is simple to show theoretically that if in one inertial frame of reference \(K\) the second Newton’s law is expressed as \(F=ma\), then in another inertial frame of reference \(K’\) it is expressed as \(F’=m’a’\) [1,2]. At this point, it is important to draw the students’ attention to the creation of equal initial conditions (initial coordinates and initial velocities), as well as to show what their difference leads to.

Before proceeding with the theory of relativity, we enunciate the principle of relativity once again and determine two parts thereof: (a) the laws of mechanics are the same in all inertial frames of reference; (b) equations describing them don’t change when passing from one inertial frame of reference to another. We point out to students that part (a) of the two aforementioned points can be called an experimental part, while part (b) a theoretical one. If point (a) is valid, then point (b) must also be valid, which means that the theory agrees with the experiment. It has to be like this and it is like this in classical mechanics. An arising question is ‘Is the principle of relativity valid in physics in general, for example in electrodynamics?’. It turns out that in electrodynamics the principle of relativity is valid only in terms of point (a), which means that the laws of electrodynamics, as shown by experiment, are valid in any inertial frame of reference, while the point (b) is not. When passing from one inertial frame of reference to another (by means of the Galilean transformations) the electrodynamics equations change their form. In the early 20th century physics was deemed to be in a state of a crisis because it basically meant disagreement between theory and experiment in electrodynamics.

By that time another set of transformations was known [1], the Lorentz transformations, that transformed electrodynamics equations from one inertial frame of reference to another without changes [2,3,4] (in this case, it is said that electrodynamics equations are invariable relative to the Lorentz transformations, as well as mechanics equations are invariable relative to the Galilean transformations.). The Lorentz transformations take the following form:

\[
\begin{align*}
(K \rightarrow K') & : \begin{cases} 
  x' = x - Vt \\ 
  y' = y \\ 
  z' = z \\ 
  t' = t 
\end{cases} & \quad (K' \rightarrow K) & : \begin{cases} 
  x = x' + Vt' \\ 
  y = y' \\ 
  z = z' \\ 
  t = t' 
\end{cases}
\end{align*}
\]

where \(\gamma = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}}\).

These transformations should be analyzed. It is clear that they are different from the Galilean transformations when the motion velocity of frame of reference is comparable to the speed of light \(c\), i.e. when \((V \sim c)\). When the velocity is low, when \((V \ll c)\), the Lorentz transformations become the Galilean transformations.

The state of a crisis was overcome with the creation of Special Relativity (relativistic mechanics), equations of which are invariable relative to the Lorentz transformations. Table 1 can serve as a table representing a model of the crisis in physics and its resolution.

<table>
<thead>
<tr>
<th>Theory of physics</th>
<th>Transformations of coordinates and time</th>
<th>Relativity principle</th>
<th>SR Learning stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-relativistic physics</td>
<td>Mechanics Galilean transformations</td>
<td>valid</td>
<td>point (a)</td>
</tr>
<tr>
<td></td>
<td>Electrodynamics Galilean transformations</td>
<td>valid</td>
<td>point (b)</td>
</tr>
<tr>
<td>Relativistic physics</td>
<td>Relativistic mechanics Lorentz transformations</td>
<td>valid</td>
<td></td>
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Students must be explained what aspects they should take into consideration while studying special relativity within the physics course. The study starts with the theory postulates which must be presented as facts that were verified experimentally many times (as these very experimental facts must form a basis of any theory, and a
theory must be verified experimentally). If the point that all physical processes and, consequently, the laws that describe them proceed equally in any inertial frame of reference is out of question because it is the application of the mechanical principle of relativity, which is already well-known to school students, to all physical processes, then the point on the equality of the speed of light in vacuum in any inertial frame of reference requires further explanations. Indeed, if in inertial frame of reference $K$, the speed of a light particle, a photon, moving along the $x$-axis, is equal to $c$, then in reference frames of $K'$ and $K''$, moving along this axis toward the photon and in direction of its motion, the photon speed must be equal to $c + V$ and $c - V$ respectively, where $V$ is the speed of $K'$ and $K''$ in relation to $K$. It is obvious to school students. Experience shows that in all three frames of reference – in $K$, $K'$ and $K''$ – the result is the same: the light speed is equal to $c$. Here it should be recalled once again that these are experimental facts that must form a basis of a theory, not the things that seem obvious to us. The equal value of $c$ in any inertial frame of reference is called invariance of the light speed. It is necessary to clearly mark out the following facts about the speed of light signal passage. The finitude of the light passage speed means that it is not infinitely great. It is finite and equal to $c \approx 3 \cdot 10^8 \text{m/s}$. School students are very interested in numerous historical experiments on measuring the light speed, carried out at different times by different scientists, from Galileo Galilei to Albert A. Michelson and modern methods of calculating its numerical value. The finiteness of the light speed means its finite nature. It is the greatest speed in nature (within the framework of special relativity). It is impossible to reach it, never mind exceed it. No information, effect, material body or signal can be transferred with a speed greater than $c$. Finally, invariance, considered above, is marked out as a postulate together with the principle of general relativity. An important issue in school students’ mastering of the relativistic theory fundamentals is the idea on what is moving and what is at rest, what physical values are there and how they are denoted in respective frames of reference. For example, a segment of a certain length $\Delta x_0$ that is at rest in an inertial frame of reference (for example, in $K'$), a resting point, in which events of a certain time length $\Delta t_0$ occur and a body, resting in this reference frame with the mass of $\Delta m_0$, are denoted as $\Delta x_0$, $\Delta t_0$, $\Delta m_0$ respectively. In all other reference frames (for example, in $K$), the listed bodies and points are moving (along the $x$-axis) and have the length, duration of events and mass of $\Delta x$, $\Delta t$, $\Delta m$ respectively. The connection between them is expressed by the formula (refer with: Eq. 1, Eq. 2, Eq. 3)

\[ \Delta x = \frac{1}{\gamma} \Delta x_0, \quad (1) \]

\[ \Delta t = \gamma \Delta t_0, \quad (2) \]

\[ \Delta m = \gamma \Delta m_0. \quad (3) \]

III. SUMMARY

The given information should precede the learning of Special Relativity regardless of how and where it will be studied (in a school course or in a general course of physics). This will contribute to: deeper understanding of the substance of Special Relativity, importance and need of its emergence, of fundamentality of the taught physics course, development of content-related lines of fundamental physical formation, i.e. methodology and shaping the worldview.

REFERENCES