

Place Values and Three-Digit Subtraction by Hand-on Activities

Jackson Pasini Mairing

Department of Mathematics Education, University of Palangka Raya

Palangka Raya, Indonesia

jacksonmairing@gmail.com

Abstract—The second graders of elementary school can only learn mathematics using manipulative materials. This research aimed to describing hand-on activities of a second grader to understand place values and three-digit subtraction operations using manipulative materials. The subject was Geget (pseudoname), a second grader of a state elementary schools in Palangkaraya, Indonesia. The materials used were sticks and three boxes. The sticks could be tied into small bunches of ten sticks representing place value of the tens, and large bunches of ten small bunches representing the hundreds. The boxes were inscribed with “the units”, “the tens” and “the hundreds” representing the corresponding place values. The researcher guided the subject by instructing her to perform some tasks, and to solve some questions (problems). The guidance was based on enactive, iconic and symbolic stages. The results showed that using manipulative materials, sticks and boxes, could help the subject meaningfully understand the concept of place values and three-digit subtraction operations. The subject determined the subtraction result and started by checking the sufficiency of the digits of each place value to be taken, then subtracted it from the front.

Keywords—enactive, manipulative materials, place values, subtraction operations, construction understanding.

I. INTRODUCTION

Place value concepts play an important role for students to understand numbers and their operations. The operations to master are addition, subtraction, multiplication and division. There are two main ideas of place values, namely grouping/exchanging and positioning a digit determining the number it represents. The place values of 3, 7 and 2 in 372 respectively are the hundreds, the tens, and the units. Moreover, the place value of 3 has different place value in number of 732. The digit of 3 in 732 has a place value of the tens. The ten place values of the units can be grouped into the tens, and ten place values of the tens can be grouped into the hundreds [1, 2, 3].

The subtraction operations can be defined as taking or throwing some or all objects from a given set [1]. To teach the second graders, the definition should be guided using manipulative materials. For example, the result of $7 - 5$ can be determined by preparing seven sticks on the table, then taking five of them away. The remaining sticks are the result of the subtraction. In three-digit numbers, the second graders can subtract the digits in each place value.

Some current textbooks explain three-digit subtractions by using the term of “borrowing”. For example in determining $137 - 18$, the explanation will be as follows.

Step 1

$$\begin{array}{r} 2 \quad 10 \\ 1 \cancel{3} \quad 7 \\ - \quad 1 \quad 8 \\ \hline \quad \quad 9 \end{array}$$

We cannot do $7 - 8$, then we borrow 1 from 3 tens, become $(10+7) - 8 = 9$

Step 2

$$\begin{array}{r} 2 \\ 1 \cancel{3} \quad 7 \\ - \quad 1 \quad 8 \\ \hline 1 \quad 1 \quad 9 \end{array}$$

The 1 tens have borrowed from 3 tens, the remaining is 2 tens, $2 - 1 = 1$, we write in the tens place value. We write down 1 hundreds.

$$\text{Thus, } 137 - 18 = 119$$

Such explanation cannot help the students answer the questions such as “what is the meaning of borrowing?” or “why does the units become $(10 + 7) - 8 = 9$ after borrowing 1 from 3 of the tens?”. Students, who learn from such books, understand the operation as an unmeaningful rule. They will have paradigm that mathematics is difficult because there are lot of areas of unmeaningful rules. The paradigm makes students unmotivated in learning mathematics, whereas motivation is an important factor that affects the ability of students to solve mathematical problems [4].

Such problematic condition needs to be solved by implementing the learning that is emphasizing the meaning of learning itself. The learning should promote students’ ability to construct meaningful mathematical concepts and it should also be integrated with the manipulative materials. Previous research results showed that such learning could help students meaningfully understand the concepts of place values, or addition and subtraction operations. The understanding was indicated by the increase of students’ learning outcomes and their enthusiasm and joy in learning mathematics [5].

The manipulative materials in this research were the sticks and the boxes (see Figure 1). They give chance for students to count using their hands and to group ten sticks into a small bunch representing the tens, or to group ten small bunches into a large bunch representing the hundreds. Students can also untie the bunches when moving from the boxes of the hundreds to the tens, or from the boxes of the tens to the units. The hand-on activities of grouping and untying will give students the meaning of the subtraction. Students could answer the questions such as “why are there more sticks in the boxes of the units if a small bunch is moved from the boxes of the tens to the units?”. Similarly, it also happen when a big bunch is moved from the boxes of the hundreds to the tens.



Fig 1. The manipulative materials (sticks and boxes)

This research aimed to describing hand-on activities done by a second grader using the sticks and the boxes to construct meaningful understanding of the place values and the subtraction operations. The results of this research can be used by teachers to develop an integrated learning with the learning materials. The learning can help the second graders have a meaningful understanding of the concepts. The understanding helps students construct number and operation senses.

The rest of this paper is organized as follow: Section 2 describes proposed research method of this work. Section 3 presents the obtained results and following by discussion in section 4. Finally, Section 5 concludes this work.

II. METHOD

The research was a basic interpretative study by implementing qualitative approach. The research was conducted in six stages [1]. Firstly, the researcher determined the focus of the research to the activities which a second grader should do in constructing meaningful understanding of place values and subtraction operations.

Secondly, the researcher developed additional instruments to collect the data which were in activities scenario, boxes inscribed with “the hundreds”, “the tens”, and “the units”, the sticks, and the rubber bands. The scenario contained the researcher's guidance to the research subject in constructing the understanding. The guidance was open-ended questions or tasks for the subject that should be done by her hands. The questions were emerged based on the subject’ answers during the interview. The boxes represented the corresponding place values. The rubber bands were used to group 10 sticks into a small bunch representing the tens, and 10 small bunches into a larger bunch representing the hundreds. The numbers of the sticks, the small bunches, and the large bunches represented the digits of corresponding place values.

Thirdly, the researcher chose a research subject. The researcher came to one of the elementary schools in Palangka Raya, Central Kalimantan, Indonesia. The researcher observed the second graders learning activities of mathematics class in the school. The subject was a student having fair ability and being active when the learning was conducted in the classroom. The selection of the subject was also based on her teacher suggestion. The subject was a female students named Geget (psedoname). Finally, the researcher asked permission to subject’s parents to conduct in-depth interviews with the subject at her home.

Fourthly, the researcher collected the data by asking some questions and requesting the subject did hand-on activities using the prepared materials. The activities were based on the enactive, the iconic and the symbolic stages. Mathematical learning which begins with manipulating the materials by hands is called the enactive or concrete stage. Learning

activities using the images is called as the iconic or image stage. Furthermore, the symbolic stage is the learning activities using the symbols or the words [6] [7]. The researcher used handycam and audio recorder to record the activities.

Fifthly, the researcher analyzed the data started by transcribing the recorded data. Then, the data was reduced by adding the code on the transcript. The code consisted of four digits. The first digit indicated the enactive, the iconic, or the symbolic stages respectively symbolized by E, I or S. The second to the fourth digits showed the sequence of the activities on the transcript. For example, code I012 showed the subject performing the 12th level of the iconic activities [8].

Sixthly, the researcher interpreted the results and performed verification. The interpretation was done by giving meaning and explaining to the result of data presentation. The meaning acquired through analyzing of words/phrases/sentences. The analysis was done by reading the transcripts, focusing on certain words/phrases/sentences, listing possible meanings arised in the researcher’s mind, and returning to the transcripts to determine the appropriate meaning [8].

The verification process was conducted by evaluating the criteria of credibility, dependability, and transferability. The credibility in the reseach was accomplished by prolonged engagement with the subject, persistent/consistent observation, triangulation, structural relationships, and member checks. The triangulation was conducted by checking the data and the interpretations of the subject’s writings on the transcripts (method triangulation), or checking the writings and the transcripts of an interview with the other interviews at different times (time triangulation). The dependability was accomplished by creating complete and detailed documentation of collecting and analyzing data (leaving an audit trail). The transferability was accomplished by providing complete description of the subject, and context where research took place [9] [10].

III. RESULTS

A. Hand-on Activities of Place Value

Previous experiences of the researcher showed that the second graders could not understand the meaning of the place value concepts by introducing 100 sticks tied as the hundreds, or 10 sticks tied as the tens. At the time, the researcher showed 2 large bunches and asked “How many the hundreds do we have?”, the students answered “two hundreds” instead of answering “two”. They refered to number of the sticks, not to number of digits in place values of the hundreds. Therefore, the researcher guided the subject in learning the place value concept started by introducing three boxes inscribed with words “the units”, “the tens” and “the hundreds”. Furthermore, the researcher introduced notion of “small bunch” representing concept of the tens, and “large bunch” representing the hundreds. The guidance was as follows.

Researcher : I have three boxes to keep sticks, small bunches, and large bunches. They are sticks, small bunches, and large bunches. Please, Geget, count the number of the sticks in a small bunch and a large bunch.

Geget : (she counted the sticks with her hands)

The researcher asked the subject to place some large bunches in the box of the hundreds, some small bunches in the box of the tens, and some sticks in the box of the units. The subject counted number of all sticks (enactive stage). The results were written on a table (see Figure 2). Furthermore, the researcher mentioned number of the large bunches, the small bunches and the sticks in each box, and the subject determined the number of all sticks without counting (the iconic stage).

The researcher mentioned a certain number and the subject placed some large bunches, small bunches and sticks to each box. The interview transcript was as follows.

Hundreds box	Tens box	Units box	Number of sticks
Kotak RATUSAN	Kotak PULUHAN	Kotak SATUAN	Banyak lidi
4	7	9	479
5	8	6	586
3	7	2	372

Fig. 2. The subject determined the sticks in the enactive stage

Researcher : Geget, please write 237.
 Geget : Two hundred and thirty seven (while writing)
 Researcher : Now, Geget, please place the sticks, the small bunches and the large bunches into the boxes of the hundreds, the tens and the units.
 Geget : Two hundreds (taking two large bunches and placing them into the box of the hundreds), thirty (taking three small bunches and placing them into the box of the tens), seven (untying small bunches then counting them by the voice) one, two, three, four, five, six, seven.
 Researcher : Now in the box of the hundreds, how many big bunches are there?
 Geget : Two (writing the “2”)
 Researcher : In the box of the tens, how many small bunches are there?
 Geget : Three (writing the “3”)
 Researcher : In the box of the units, how many sticks are there?
 Geget : Seven (writing the “7”)

The researcher wrote a number in the column of number of sticks (see Figure 2). The subject determined the number of the sticks, the small bunches and the large bunches in each box without using the materials (the symbolic stage).

Researcher : Now, if you write four hundred seventy two (while writing), how many big bunches are in the box of the hundreds?
 Geget : Four (writing the “4”)
 Researcher : How many small bunches are in the box of the tens?
 Geget : Seven (writing the “7”)
 Researcher : How many sticks are in the box of the units?
 Geget : Two (writing the “2”)

Thus, the subject was able to determine the digits of each place value of a particular number and vice versa.

B. Hand-on Activities of Substraction

The second graders are able to understand a concept if it is related to the context in daily life, which is represented by the manipulative materials. The concept of subtractions can be understood as “taking” or “throwing” in daily life context. The researcher guided the subject to understand the meaning.

Researcher : Today we will learn about subtraction, for example $9 - 5$ (writing). The “minus” in everyday context means being taken or thrown away. Firstly, Geget, please prepare 9 sticks.
 Geget : (the subject took the sticks and started to count) One, two, three, four, five, six, seven, eight, nine.
 Researcher : Then “minus” 5 means we take five sticks from the nine. Geget, please take five sticks away.
 Geget : (taking sticks and counting) One, two, three, four, five.
 Researcher : It means 9 minus 5, what is the result? We should count the remaining sticks.
 Geget : Four (writing the “4”)

The same meaning was used to determine the result of a three-digit subtraction. The guidance was given in six stages. Firstly, the subject used the materials to subtract numbers to the tens without untying or borrowing the bunches. Secondly, the subject subtracted numbers to the hundreds without borrowing using the rectangular images. Thirdly, the subject used the materials to subtract numbers to the tens by borrowing. Fourthly, the subject used the rectangular images to subtract numbers to the hundreds by borrowing from the hundreds only or the tens only. Fifthly, the subject used the images to subtract numbers to the hundreds by borrowing from both the hundreds and the tens. Sixthly, the subject used formal mathematical symbols to subtract numbers.

hundreds	tens	units
ratusan	puluhan	satuan
7	6	
2	1	
<hr/>		
5	5	

Fig. 3. The subtraction of the tens without borrowing

The following was guidance for the first stage (see Figure 3).

Researcher : Now we have an operation of $76 - 21$ by arranging. Firstly, Geget, please prepare 76.
 Geget : (taking small bunches and counting) One, two, three, four, five, six, seven (placing them in the box of the tens, then taking the sticks and counting) one, two, three, four, five, six (placing them in the box of the units).
 Researcher : The term “minus” means to take or to throw away. On the question, “minus 21” means, take

2 small bunches away from the box of the tens. Take 1 stick away from the box of the units.

Geget : (taking 2 small bunches from the box of the tens while counting) One, two (taking 1 stick from the box of the units while counting) One.

Researcher : How many small bunches in the box of the tens remain?

Geget : (counting the small bunches in the box of the tens) One, two, three, four, five.

Researcher : Good, write 5 here (pointing to the tens). How many sticks are in the box of the units?

Geget : (counting the sticks in the box of the units) One, two, three, four, five.

Researcher : Write here (pointing to the units). What is the result of the subtraction?

Geget : 55.

In the initial of second stage, the researcher guided the subject to subtract numbers to the hundreds without using the sticks and the boxes. Furthermore, the subject determined the subtraction result without using the materials. She used the rectangular images (see Figure 4).

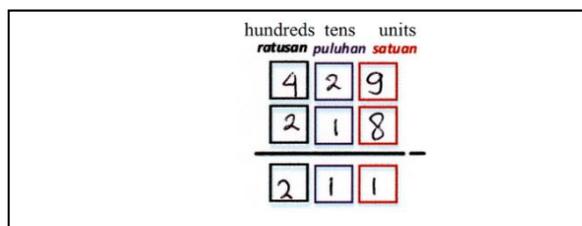


Fig. 4. The subtraction of the hundreds without borrowing

Researcher : Now I will give you a problem, Geget, please solve the problem but do not use the sticks (emptying the sticks in each box and writing a problem of $429 - 218$).

Geget : (the subject started to subtract beginning from the hundreds using her fingers) The remain is 2 (subtracting the tens using her fingers again and writing the "1"), 9 minus 8 (using her fingers) is 1.

In the third stage, the researcher guided the subject to use the materials for subtracting numbers to the tens by borrowing (see Figure 5).

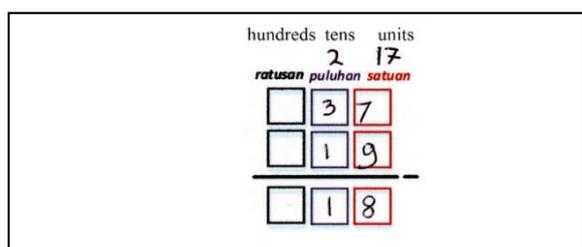


Fig. 5. The subtraction of the tens with borrowing

Researcher : Now I will give you a problem (writing the "37-19"). Geget, please give the solution of the problem.

Geget : (taking 1 out of 3 in the box of the tens, then taking 9 out of 7 in the box of the units) I can not.

Researcher : If you can not, there are 3 small bunches in the box of the tens previously. How many sticks are in 1 small bunch?

Geget : Ten

Researcher : You can untie it, then moving the sticks into the box of the units (untying 1 small bunch and moving the sticks into the box of the units), Now, please count, how many the small bunches in the box of the tens remain?

Geget: : (counting, one two) Two

Researcher : Write the result here (pointing over the tens and writing the "2"). Thus, previous number of the small bunches in the box of the tens was 3, now after being untying 1 small bunch, it becomes 2. Continuing in the box of the units, how many are the sticks now?

Geget : (the subject counted: one, two, three to seventeen) Seventeen.

Researcher : In the box of the units, previously there were 7, after untying it, how many sticks are there?

Geget : 17

Researcher : Write 17 over the units (writing the "17"). Now, can you take 9 out of 17?

Geget : No, I can not.

Researcher : Now try to take 9 out of 17 using the sticks.

Geget : 9 should be taken from 17 (taking the sticks while counting).

Researcher : Can you take 9 from 17?

Geget : Yes, I can.

Researcher : Initially, we can not take 9 out of 7, 1 small bunch is untied and moved into the box of the units, now we can count it. Now, count the remaining sticks.

Geget: (counting while talking) Eight.

Researcher : We have taken 1 small bunch in the box of the tens, now how many are the remaining sticks?

Geget : 1

Researcher : Write here (pointing to the tens and writing the "1"). Now, in the box of the units after taking 9, how many are the remaining sticks?

Geget : 8

Researcher : Write here (pointing to the units and writing the "8"), What is the result?

Geget : 18

Researcher : If we can not take any sticks in the box of the units, we can untie 1 small bunch in the box of the tens then we move them to the box of the units. But if it can be taken, we do not have to untie 1 small bunch.

In the fourth stage, the researcher guided the subject to use the rectangular images to subtract numbers to the hundreds by borrowing from the hundreds only or the tens only as follows (see Figure 6).

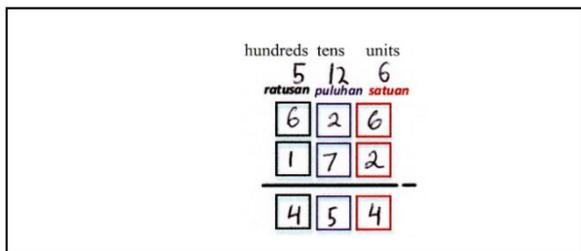


Fig. 6. The subtraction to the hundreds by borrowing from the hundreds only or the tens only

Researcher : Now, we should solve this problem (writing the “626- 172”)

Geget : It is possible for 6 minus 1. It is not for 2 minus 7. It means we should borrow 1 from the hundreds, the remainings are 5 (writing the “5” over the hundreds). After that, we have already joined 10, count 11, 12, twelve (writing the “12” over the tens). It is possible for 6 minus 2. It means 6 should stay as 6 (writing the “6” over the units). So, for 5 minus 1, it remains 4. For 12 minus 7 (counting), it remains 5. For 6 minus 2, it remains 4. The result is 454.

In the fifth stage, the researcher guided the subject to use the images to subtract numbers to the hundreds by borrowing from the hundreds and the tens, as follows (see Figure 7).

Researcher : Let’s try another one: 421- 167.

Geget : The sticks should not be used, should them?

Researcher : No, the sticks should not be used.

Geget : From the hundreds, 1 can be taken away from 4, 6 can not be taken away from 2, meaning that you need to borrow 1 from here (pointing the hundreds), it remains 3, and put it here (writing the “3” over the hundreds).

Researcher : Then in the box of the tens, initially 2, we borrow 1 from here (pointing the hundreds) then untying it and adding it here (the tens). It is added by 10, then it becomes ...?

Geget : 12

Researcher : Then, in the units, we can not take 7 from 1, so we borrow 1 from 12, it remains 11, writing it here (writing above the “12”), then in the units, it is added by 10, then it becomes....?

Geget : 11, writes here (above the units).

Researcher : Now, 1 is taken from 3. It becomes ...?

Geget : 2 (writing the result in the hundreds), 11 minus 6 (counting), 5 (writing it in the tens), then 11 minus 7 (counting), 4 (writing it in the units). Two hundred and fifty four.

Researcher : So, how we can determine the result is firstly to check whether there are digits that can not be taken, if it can not, you can borrow it from the previous digits.

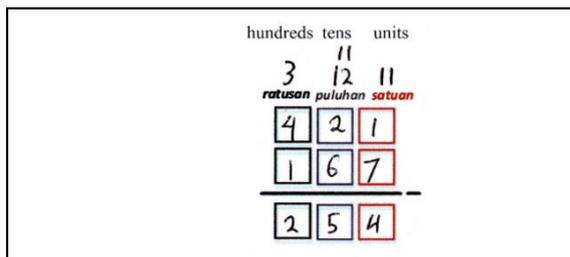


Fig. 7. The subtraction to the hundreds by borrowing from the hundreds and the tens

In the sixth stage, the subject used formal mathematical symbols to determine the subtraction result (see Figure 8).

Geget : It is possible for 9 minus 2. It is not for 0 minus 8. Borrowing from here, it remains 8 (writing the “8” over the hundreds). In the tens, 10 (writing the “10” over the tens). It cannot be for 3 minus 7. I can borrow 1 from here, it remains 9 (writing the “9” above the “10”). It was 3 that was borrowed from the previous (counting). It is 13 (writing the “13” over the units). Then, for 8 minus 2, it remains 6 (writing the result in the hundreds). For the operation of 9 minus 8, it remains 1 (writing in the tens). I will count for 13 minus 7 (counting), it remains 6 (writing in the units). The result is 616.

Thus, the research results showed that manipulating the sticks and the boxes through the enactive, the iconic and the symbolic stages helped the subject construct meaningful understanding in determining the result of subtraction operations using formal mathematical symbols.

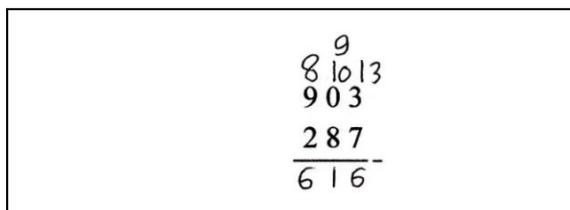


Fig. 8. The subtraction using formal mathematical symbols

IV. DISCUSSION

The activities of using manipulative materials should be well-planned to make the learning effective. The learning activities using the materials need to be related to the concepts constructed by the students. The students could not make the relation [11]. The first activity of the constructing process was that the student was introduced with the sticks, the small bunches and the large bunches. A small bunch was formed by counting the sticks up to ten and tying them with a rubber band. A large bunch was formed by counting the small bunches up to ten and tying them. The researcher also introduced the boxes of the hundreds, the tens and the units to represent the corresponding place values. The boxes of the units, the tens and the hundreds were used as a place to keep the sticks, the small bunches, and the large bunches respectively. The second activity was that the student used the sticks, the small bunches and the large bunches to represent a

certain number from two- to three-digit numbers. The third activity was that the student did subtraction of one- or two-digit numbers by preparing the sticks or the small bunches in the corresponding box, then took them away (the enactive stage). The fourth was that the student did subtraction of two- or three- digit numbers, then taking them without borrowing or moving. The student did not use the sticks and the boxes, but they used rectangular images inscribed with words “the hundreds”, “the tens”, and “the units” to determine the result (the iconic stage). The fifth was that the student did three-digit subtraction by moving some sticks from the hundreds only or the tens only. The sixth was the student did three-digit subtraction by moving the sticks from both the hundreds and the tens. Finally, the student did three-digit subtractions by using formal mathematical symbols to determine the result (the symbolic stage).

Therefore, the sticks and the boxes in this research were related to the mathematical concepts. The activities of relating started from concrete representations to abstract concepts. This relating activities could help students have meaningful understanding of the place values and the operations. The understanding could help students improve ability to solve problems, creative thinking, and learning outcomes [12] [13].

The previous research results showed that using manipulative materials could help students understand the concepts of place values, addition and subtraction operation of numbers meaningfully. Learning using the materials of “dekak-dekak” could improve the learning outcomes of the third graders in addition and subtraction operations [5]. The cards of numbers and the bags of numbers could also be used to improve learning outcomes of the second graders learning place value concepts [14], [15]. In general, researchers demonstrated the positive effects of using the manipulative materials toward students’ outcomes [16].

V. CONCLUSION

The place value concept in this research was represented by the boxes inscribed with words “the hundreds”, “the tens”, and “the units”. The subject gained the understanding of the concept by counting the sticks from one to ten then tying them by using a rubber band into a small bunch representing the tens and placing it in the box of the tens. She counted small bunches from one to ten then tying them into a large bunch representing the hundreds and placing it into the box of the hundreds. Student should use the contextual terms of the small and large bunches rather than the abstract terms of the tens or the hundreds respectively when learning the place values and the operations. For example, the researcher asked the subject to count the small bunches in the box of the tens. The research results indicated the subject was able to write the place values of a certain number, and vice versa.

The subject gained the understanding of the subtraction as taking, throwing or borrowing activity. The construction of the understanding started from the hand-on activities by manipulating the sticks, the small bunches, and the large bunches (the enactive stage). Then, the subject used the rectangular images inscribed with words “the hundreds”, “the tens”, and “the units” to determine the subtraction result (the iconic stage). Finally, the subject used formal-mathematical symbols to determine the the result (the symbolic stage).

To determine the operation result for the research, the subject checked the adequacy of the hundreds which was indicated by the sufficiency of the large bunches to be taken in the box of the hundreds. If it was sufficient, the subject checked the adequacy of the tens indicated by the sufficiency of the small bunches to be taken in the box of the tens. If it was not sufficient, the subject moved and untied a large bunch in the boxes of the hundreds then placed it into the box of the tens. The subject wrote a number of the large and small bunches now. The subject checked the adequacy of the units indicated by the sufficiency of the sticks to be taken in the box of the units. If it was not sufficient, the subject untied a small bunch and performed as before. Finally, the subject could already subtracted the digits in each place value represented by taking the objects (sticks or bunches) in each box. The result of research showed that the subject could construct meaningful understanding of the subtraction concept.

The results of the research were obtained by in-depth interviews conducted to a subject. Further research can be directed to develop some lesson plans using the sticks and the boxes and following the activities conducted in this research. The plans should be designed and implemented in the classroom to give the students the guidance in constructing the meaning of the place value concepts and the subtraction operations. Furthermore, the materials can also be used to help the students construct the meaning of the addition operation. Research aiming at involving manipulative materials and the addition operation should be conducted in the future.

REFERENCES

- [1] Bennet, B. a., & Nelson, T. L. (2004). *Mathematics for elementary teachers: A conceptual approach* (6 ed.). New York, NY: The McGraw-Hill Companies, Inc.
- [2] Reys, R., Lindquist, M. M., Lambdin, D. V., & Smith, N. L. (2009). *Helping children learn mathematics* (9 ed.). Hoboken, NJ: John Wiley & Sons, Inc.
- [3] Van De Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2010). *Elementary and middle school mathematics: Teaching developmentally* (7th ed.). Boston, MA: Ally & Bacon.
- [4] Ary, D., Jacobs, L. C., & Sorensen, C. (2006). *Introduction to research in education* (8th ed.). Belmont, CA: Wadsworth.
- [5] Hidayati. (2014). Meningkatkan hasil belajar matematika dengan menggunakan alat peraga dekak-dekak pada siswa kelas III SD Negeri Sardonoarjo 2 Kecamatan Ngaglik. Yogyakarta, Indonesia: Universitas Negeri Yogyakarta.
- [6] Hoong, L. Y., Kin, W. H., & Pien, C. L. (2015). Concrete-Pictorial-Abstract: Surveying its origins and charting its future. *The Mathematics Educator*, 16(1), 1-18. Retrieved from http://math.nie.edu.sg/ame/matheduc/tme/tmeV16_1/TME16_1.pdf
- [7] Kennedy, L. M., Tipps, S., & Johnson, A. (2008). *Guiding children’s learning of mathematics* (11 ed.). Belmont, CA: Thomson Wadsworth.
- [8] Strauss, A., & Corbin, J. (1998). *Basics of qualitative research, techniques and procedures for developing grounded theory*. : Sage Publications Inc. Thousand Oaks, CA: SAGE Publications, Inc.
- [9] Bloomberg, L. D., & Volpe, M. (2008). *Completing your qualitative dissertation: A roadmap from beginning to end*. Thousand Oaks, CA: Sage Publications, Inc.
- [10] Lodico, M. G., Spaulding, D. T., & Voegtle, K. H. (2006). *Method in educational research: From theory to practice*. San Francisco, CA: John Wiley & Sons, Inc.
- [11] Marshall, L., & Swan, P. (2008). Exploring the use of mathematics manipulative materials: Is it what we think it is? *EDU-COM International Conference* (pp. 338-350). Perth, Australia: Edith Cowan University. Retrieved from <http://ro.ecu.edu.au/ceducom/33>.

- [12] Department of Education Science and Training Australia. (2004). Understanding place value: A case study of the base ten game. Canberra, Australia: Australian Government's Numeracy Research and Development Initiative.
- [13] Laski, E. W., Jor'dan, J. R., Daost, C., & Murray, A. K. (2015). What makes mathematics manipulatives effective? Lesson from cognitive science and Montessori Education. *SAGE Open*, 1-8. doi:10.1177/2158244015589588
- [14] Tiamin. (2012). Meningkatkan hasil belajar siswa pada materi menentukan nilai tempat ratusan, puluhan dan satuan pada pelajaran matematika dengan menggunakan kartu bilangan di kelas II SDN Sememi I Kecamatan Benowo Surabaya. Surabaya, Indonesia: Universitas Negeri Surabaya.
- [15] Yanti, I. D. (2015). Peningkatan hasil belajar matematika matematika nilai tempat melalui metode bermain dengan media kantung bilangan kelas II MIN Dalam tahun pelajaran 2014/2015. Salatiga, Indonesia: STAIN, Salatika.
- [16] Cope, L. (2015). Math manipulatives: Making the abstract tangible. *Delta Journal of Education*, 5(1), 10-19.