Research on Sensorless Composite Position Identification Method for PMSM

Gao-yang CHEN and Hong LI
College of Automation, Beijing Information Science & Technology University, Beijing, China

Keywords: Sensorless, High frequency injection, Permanent magnet synchronous motor, Sliding mode observer.

Abstract. Both the high frequency pulsation signal injection method and sliding mode observer (SMO) can be utilized to identify the position of permanent magnet synchronous motor (PMSM) sensorless system. When the speed of motor runs high, the high-frequency signal injection method is no longer utilized. When the speed of motor runs low, the rotor position signal cannot be accurately detected because of the weak back-EMF signal, so SMO is not used. For the sake of effectively identify the rotor position in full speed range, a composite identification method combining the high frequency injection and SMO is adopted. Therefore, two identification methods need to be switched in the full speed range. The weighted switching function is used to identify the rotor position, so that that rotor position error estimate by the sensorless PMSM in the full speed range is smaller. The results of simulation verify the availability of this method.

Introduction

Compared with the traditional electrically excited synchronous motor, PMSM has many advantages, such as simple structure, high power density, high power factor and so on. It has attracted more and more attention in wind power generation, electric vehicle, high-speed train and other fields [1]. Typically, the information of rotor speed can be obtained by a photoelectric encoder. However, because of the existence of sensors, the electromechanical equipment with motor as the core has disadvantages in cost, reliability and other aspects. The concept of sensorless is put forward in the 1970s and has been applied to modern high performance electric drive systems including PMSM [2]. The basic principle of sensorless technology is based on motor parameters, using direct calculation, state estimation, indirect measurement to estimate or identify the rotor speed.

One of the commonly used sensorless position identification methods is high frequency voltage signal injection method [3]. The fundamental theory of this method is to add a three-phase voltage signal whose frequency is different from the base frequency of the motor, superimpose the three-phase voltage signal with the base frequency signal and input the three-phase winding of the motor. The rotor speed information in the corresponding high frequency current is extracted by band-pass filter, and processed to estimate the rotor position. Another commonly used sensorless position identification method is based on the fundamental wave model, which estimates the stator phase induced back electromotive force (EMF) from the stator current and voltage measurements, and then estimates the rotor position [4]. The disadvantage of this method is that it is difficult to accurately estimate the induced back EMF at low speed because the amplitude of the voltage signal is very small, so it is not suitable for position estimation at low speed.

For the sake of make the PMSM control system have good position identification accuracy and dynamic response over full speed, the advantages of two methods can be synthesized: The high frequency pulse voltage signal injection method is utilized at low speed, and SMO is utilized at high speed.

Sensorless Technology

Mathematical Model of Three-phase PMSM

In order to facilitate the later design, the mathematical model in synchronous rotating coordinate
system \( d - q \) is selected, and the stator voltage equation can be expressed as

\[
\begin{align*}
    u_d &= R_i_d + \frac{d}{dt} \psi_d - \omega_e \psi_q \\
    u_q &= R_i_q + \frac{d}{dt} \psi_q + \omega_e \psi_d
\end{align*}
\]  

(1)

The stator flux equation is

\[
\begin{align*}
    \psi_d &= L_i d_d + \psi_f \\
    \psi_q &= L_i d_q
\end{align*}
\]  

(2)

By substituting equation (2) into equation (1), that voltage equation can be written as

\[
\begin{align*}
    u_d &= R_i_d + L_i \frac{d}{dt} i_d - \omega_e L_i i_q \\
    u_q &= R_i_q + L_i \frac{d}{dt} i_q + \omega_e (L_i d_d + \psi_f)
\end{align*}
\]  

(3)

Where, subscripts \( d \), \( q \) denote \( d - q \) axis components. Respectively, \( u, i, \psi, L \) are stator electricity, current, flux linkage and inductor. \( R \) is the stator resistance; \( \omega_e \) is the electrical angular velocity and \( \psi_f \) is permanent magnet flux [5].

**SMO Based on PLL**

The phase-locked loop (PLL) system is used to extract the position information of the rotor. The principle is shown in Fig. 1.

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Assuming \( k = (L_d - L_q)(\omega_i d - p_i q) + \omega_i \psi_f \), when \( |\hat{\theta}_e - \theta| < \pi/6 \), \( \sin(\theta_e - \hat{\theta}_e) = \theta_e - \hat{\theta}_e \) is considered valid. The following relationship can be obtained from Figure 2.

\[
\Delta E = -\hat{E}_\alpha \cos \hat{\theta}_e - \hat{E}_\beta \cos \hat{\theta}_e \\
= k \sin \theta_e \cos \hat{\theta}_e - k \cos \theta_e \sin \hat{\theta}_e \\
= k \sin(\theta_e - \hat{\theta}_e) \approx k(\theta_e - \hat{\theta}_e)
\]  

(4)

The rotational speed can be obtained by the position derivative after the position estimation error converges to zero.

**Fluctuating High Frequency Signal Injection Method**

For the sake of accurately estimate the rotor speed, the coordinate relationship is established as shown in Fig. 2, where \( \hat{d} - \hat{q} \) and \( d - q \) are estimated and actual synchronous rotating coordinate systems respectively [6].
In Fig. 2, \( \alpha - \beta \) is a two-phase stationary coordinate system; \( \hat{\theta} \) and \( \theta \) are the estimated and actual values of the rotor position angle; \( \Delta \theta \) is the estimated error angle of the rotor between the coordinate systems \( \hat{d} - \hat{q} \) and \( d - q \).

Since the two coordinate systems have the spatial relative position shown in Fig. 2, the voltage and current between coordinate systems \( \hat{d} - \hat{q} \) and \( d - q \) have the following relations:

\[
\begin{bmatrix}
  u_d \\
  u_q
\end{bmatrix} =
\begin{bmatrix}
  \cos \Delta \theta & \sin \Delta \theta \\
  -\sin \Delta \theta & \cos \Delta \theta
\end{bmatrix}
\begin{bmatrix}
  \hat{u}_d \\
  \hat{u}_q
\end{bmatrix}
\]  
(5)

\[
\begin{bmatrix}
  i_d \\
  i_q
\end{bmatrix} =
\begin{bmatrix}
  \cos \Delta \theta & \sin \Delta \theta \\
  -\sin \Delta \theta & \cos \Delta \theta
\end{bmatrix}
\begin{bmatrix}
  \hat{i}_d \\
  \hat{i}_q
\end{bmatrix}
\]  
(6)

Where, \( \hat{u}_d \), \( \hat{u}_q \) are the voltage components and \( \hat{i}_d \), \( \hat{i}_q \) are the current components in the \( \hat{d} - \hat{q} \) coordinates.

![Figure 2. Spatial Position Relation between Estimation and Actual Rotating Coordinate System](image)

Using the transformation matrices in equation (5) and equation (6), the high frequency response output of the pulsed high frequency voltage injection method as shown in equation (7) can be obtained:

\[
\begin{bmatrix}
  \hat{i}_d \\
  \hat{i}_q
\end{bmatrix} =
\begin{bmatrix}
  \cos \Delta \theta & -\sin \Delta \theta \\
  \sin \Delta \theta & \cos \Delta \theta
\end{bmatrix}
\begin{bmatrix}
  1/L_d & 0 \\
  0 & 1/L_d
\end{bmatrix}
\begin{bmatrix}
  \cos \Delta \theta & \sin \Delta \theta \\
  -\sin \Delta \theta & \cos \Delta \theta
\end{bmatrix}
\begin{bmatrix}
  \hat{u}_d \\
  \hat{u}_q
\end{bmatrix}
\]  
(7)

Definition: \( L = (L_d + L_q)/2 \), \( \Delta L = (L_q - L_d)/2 \). Equation (7) can be rewritten as:

\[
\begin{bmatrix}
  \hat{i}_d \\
  \hat{i}_q
\end{bmatrix} =
\frac{1}{L^2 - \Delta L^2}
\begin{bmatrix}
  L + \Delta L \cos(2\Delta \theta) & \Delta L \sin(2\Delta \theta) \\
  \Delta L \sin(2\Delta \theta) & L - \Delta L \cos(2\Delta \theta)
\end{bmatrix}
\begin{bmatrix}
  \hat{u}_d \\
  \hat{u}_q
\end{bmatrix}
\]  
(8)

Inject a high frequency sinusoidal signal \( \begin{bmatrix}
  \hat{u}_d \\
  \hat{u}_q
\end{bmatrix} = \begin{bmatrix}
  U_m \cos(\omega_h t) \\
  0
\end{bmatrix} \) into the \( \hat{d} \) axis of the \( \hat{d} - \hat{q} \) coordinate system. Where \( U_m \) and \( \omega_h \) represent the amplitude and angular frequency of the injected signal, respectively. In this case, the high-frequency current on the \( \hat{d} - \hat{q} \) axis can be simplified as:

\[
\begin{bmatrix}
  \hat{i}_d \\
  \hat{i}_q
\end{bmatrix} =
\frac{U_m \sin(\omega_h t)}{\omega_h (L^2 - \Delta L^2)}
\begin{bmatrix}
  L + \Delta L \cos(2\Delta \theta) \\
  \Delta L \sin(2\Delta \theta)
\end{bmatrix}
\]  
(9)

Equation (9) shows that in the \( \hat{d} - \hat{q} \) coordinate system, the amplitudes of the high frequency
current components are related to $\Delta \theta$. When $\Delta \theta = 0$, the high frequency current $\hat{i}_q = 0$. According to this feature, $\hat{i}_q$ can be input to the rotor position tracking observer after processing, and the position and speed of the rotor can be obtained after further estimation [7].

The quadrature axis current component in equation (9) is amplitude modulated and passed through a low-pass filter to obtain an input signal of the rotor position tracking observer, namely

$$f(\Delta \theta) = \text{LPF}(i_q \sin \omega_b t) = \frac{U_m \Delta L}{2 \omega_b (L^2 - \Delta L^2)} \sin(2\Delta \theta)$$

If the error of rotor position estimation is small enough, the error signal can be linearized to obtain

$$f(\Delta \theta) = \frac{U_m \Delta L}{2 \omega_b L_d L_q} \sin(2\Delta \theta) \approx 2k_e(\Delta \theta)$$

Where $k_e = \frac{U_m \Delta L}{2 \omega_b L_d L_q}$. Equation (11) shows that if $f(\Delta \theta)$ is adjusted to zero, the estimation error of rotor position angle is zero, that is to say, the estimated value of rotor position may follow the actual value.

In the process of rotor position identification and speed information extraction, a band-pass filter (BPF) is used to extract high frequency current signal. Equation (11) is then filtered using a low-pass filter (LPF) to obtain error information for the rotor, and finally converges to zero with PI adjustment.

**Composite Position Identification Method Based on SMO and High Frequency Voltage Injection**

Neither high frequency voltage signal injection method nor SMO is suitable for position identification of PMSM in full speed range independently. Combining the pulse high frequency voltage injection method with SMO, the high frequency pulse voltage signal injection method is used when the motor speed is low, and SMO is used when the motor speed exceeds a certain speed after start-up. The weighted combination of the two methods is used to identify the position of the rotor, and a composite observer of rotor speed and position in full speed range is established. The block diagram of the sensorless system based on the weighted composite identification method is shown in Fig. 3.

![Figure 3. Block diagram of sensorless system based on weighted composite identification method](image-url)
high frequency injection method. With the motor speed increases, for the sake of obtain the rotor position information of PMSM and calculate the speed, SMO being used. To achieve positional observation in full speed range, it is necessary to find a way to make the two methods can be switched smoothly. According to the characteristics and applications of the two position identification methods, the weighted composite method is used to realize the connection of the position identification method when the motor speed rises gradually from low to high. The basic principle of designing the weighted function is shown in Fig. 4 [8].

![Figure 4. Weighting function of compound identification method](image)

According to Fig. 4, the output value of the composite observer is determined by both the pulse high frequency injection method and MRAS when the speed $\omega_e$ is between $\omega_{e1}$ and $\omega_{e2}$.

When the rotor speed is lower than $\omega_{e1}$, the rotor speed at this time is the speed $\omega_1$ observed by the pulse signal injection method. When the speed of the motor is higher than $\omega_{e2}$, the rotor speed at this time is the speed $\omega_2$ observed by SMO. When the rotational speed is between A and B, the rotor information of rotor position and rotational speed needs to be obtained through the weighting function. The implementation block diagram of this weighting algorithm is shown in Fig. 5.

The weighting function of the estimated rotational speed is shown in equation (12):

$$\omega_{re} = \alpha \omega_1 + (1-\alpha) \omega_2 \quad (12)$$

Where, $\omega_1$ is the estimated rotational speed using the pulsed high-frequency injection; $\omega_2$ is the estimated rotational speed using SMO; and $\alpha$ is the weighted coefficient of speed. Define $\omega_{e1}$ and $\omega_{e2}$ as upper and lower limits of the corresponding speed switching interval, then the value of $\alpha$ is shown in equation (13).

$$\alpha = \begin{cases} 
1 & \omega_{re} \leq \omega_{e1} \\
\frac{1}{\omega_{e2} - \omega_{e1}} (\omega_{re} - \omega_{e2}) & \omega_{e1} \leq \omega_{re} \leq \omega_{e2} \\
0 & \omega_{re} > \omega_{e2}
\end{cases} \quad (13)$$

### Simulation Results and Analysis

For the sake of verify the effectiveness of the weighting function designed in the compound position identification method, simulation experiments were carried out in MATLAB/SIMULINK.
environment. The PMSM parameters in the experiment were: \( p=4; \) \( n=750\, \text{r/min}; \) \( R=0.33\, \Omega; \) \( L_d=5.2\, \text{mH}; \) \( L_q=17.4\, \text{mH}; \) \( \psi_f=0.646\, \text{Wb}; \) \( J=8\times10^{-3}\, \text{kg}\cdot\text{m}^2; \) \( B=0. \) Besides, \( T_L=2\, \text{N}\cdot\text{m}. \) The parameters of high frequency voltage signal were: \( U_m=20\, \text{V}; \) \( f_h=1000\, \text{Hz}. \) The speed switching value of the weighting function is set to 20% and 40% of the rated speed, respectively, that is, \( \omega_{e1}=150\, \text{r/min}, \) \( \omega_{e2}=300\, \text{r/min}. \)

When the weighted control function is switched, the estimated rotor position converges to the actual value except for the glitch caused by the small phase delay. The estimated rotor speed of the motor has a large deviation when it is just started, and it can basically track the actual rotational speed in real time when it is switched to the weighted control mode with a small amplitude fluctuation. The composite method also has some position observation errors, especially the steady-state errors after the reference speed has been reached, which affects the position identification accuracy of the motor and the speed control accuracy.

**Conclusion**

In this paper, a sensorless compound control technology is studied, which combines the pulse high frequency voltage injection method and SMO. This composite position observation technique is suitable for the position identification of PMSM in the full speed range. Combining the advantages of the two methods, the smooth transition of the motor transition region is achieved by controlling the weighted switching function and the upper and lower limits of the weighted function when switching from the low speed region to the high speed region. Finally, the simulation model is built in MATLAB/SIMULINK according to the structure block diagram, and the simulation results verify that the method has good speed observation effect.

**Acknowledgements**

This work is supported by Beijing Natural Science Foundation (3172015).
References


