Price Discount Coordination Mechanism for CHR Express Dual-Channel Supply Chain

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Abstract. Under the set of the cooperation that CHR express company treats traditional express companies as its agents, the providers of CRH express service come from two different channels, which are CRC (hereinafter referred to as CRC) and traditional express company. As there were Double Marginal Effect among the two channels, so it is essential to resolve the conflicts between two channels. In order to meet the goal, the price discount mechanism was designed to achieve the perfect coordination of the combination of dual-channel CHR express supply chain. The mainly conclusions of this paper are as shown below: when the price discount was equal to the cross-price coefficients between two channels, meanwhile traditional express enterprises needs to pay one part of its profit to CHR express company, the quantity discount contract can make CRH express duel-channel supply achieve perfect coordination. This article provides reference for the price decision of two sides.

1. Introduction

In recent years, with the rise and development of e-commerce, Chinese express industry has entered a stage of rapid growth. According to the data in the "2016-2021 China Express Market Requirement and Investment Consultation Report" issued by the China Reporting Hall, the national express delivery volume in 2016 was 31.35 billion and the business income was 400.5 billion Yuan. 80% of Chinese express industry was completed by truck transport, the freight volume of air transport was 15% and the freight volume of railway transport was less than 5% [1]. In order to meet the needs of the continuous growth of the market, high-speed rail Express, built by China Railway Corporation, was formally launched on April 1, 2014. The so-called high-speed rail Express refers to the express service products launched by CRC under the China Railway Corporation. The specific transportation process is that China Railway Corporation uses high-speed rail capacity (usually through the use of no-load high-speed railway passenger cars or express freight dedicated high-speed trains) to undertake the express transport of the main line, the two ends of the transport outside the main transport (door to door collection Parts and shipments are completed by other means of transport, such as roads. After the launch of high-speed rail express delivery, traditional express delivery companies (such as STO Express, YTO Express, ZTO Express, S. F. Express, etc.) have sought cooperation. For example, during the period of "Double 11" (which is a day when Chinese people go on massive Internet shopping sprees, because many online stores offer huge discounts on this day) in 2016, to promote market expansion, China Railway Corporation Hangzhou (the provincial capital of Zhejiang province) Branch developed cooperation with several traditional express companies, then kinds of CHR express service were provided. After China Railway Corporation cooperates with traditional express delivery companies, the high-speed rail express delivery supply chain formed by consumers, China Railway Corporation and traditional express delivery companies will naturally take shape.

Da Wa, who was the former vice chairman of China Express Association, figured out that CHR
The express company could develop traditional express companies as agents. Under the set of the cooperation, the providers of CRH express service are from two different channels, which are direct selling channel of CRC and retail selling channel of traditional express company. Specifically, on the one hand, China Railway Corporation can develop traditional express delivery companies into their own “agents”. The traditional express delivery companies, as agents, are responsible for providing high-speed rail express consumer terminal services at the prices they set in the market. On the other hand, China Railway Corporation can directly provide high-speed rail express services to consumers at their own prices through direct selling channels. As a result, the high-speed rail express supply chain system under retail and direct sales channels is formed (see Fig. 1), and the scale of high-speed rail express delivery business continues to expand. As there is double marginalized effect existed in the dual-channel CHR express supply chain, which results that the profits of entire supply chain in centralized situation is higher than that in decentralized situation. The conflict between direct and retail selling channels is unavoidable, because this cooperation method is still in the preliminary stage. How to coordinate the conflict between the two channels is of great significance.

Although CRC came up with the concept of CHR express in 2013, literature about reform of railway express had been established earlier. Oum and Yu (1994), which are members of Faculty of Commerce and Business Administration of University of British Columbia, investigated the economic benefits of the countries under the set of “separating of the network and transportation”, which were the members of OECD (Organization for Economic Cooperation and Development)[2]. Kang (1997) figured out that the construction of logistic information system was contribute to the promotion of railway freight capacity[3]. Stephan Miller et al. (2012) constructed a prediction model of freight flow based on economical development and employment structure[4]. It was concluded that the literature about railway freight were focusing on the traditional railway freight, while literature about high speed railway express were relatively scarce abroad. CHR express gradually became research hot-spot in 2012 at home. Since CHR express was not launched by CRC until 2014, so the literature between 2012 and 2014 were almost about the development strategy or what could learn form other countries [5,6,7,8,9,10,11]. After CHR express was published, the relatively researches were focus on how to deal with the game relationship between CHR express company and traditional express companies [12,13]. On the whole as the cooperation scale and intensity were not so large, the conception of CHR express supply chain was so unclear that researches about coordination on CHR express supply chain was scarce. But it was a fact that studies on coordination mechanism of supply chain had reached a distinct stage and had got rich achievements [14,15,16], it provided solid basis for the study on coordination mechanism of CHR express supply chain.

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![Figure 1. The model of CHR express supply chain system with retail and direct selling market demands](image-url)
mechanism is designed to achieve the perfect coordination of the dual-channel CHR express supply chain with direct selling and retail market demands.

2. Literature Review

At present, there have been some studies on the decision-making problem of direct marketing and retail transportation, mainly focusing on air cargo and shipping. Zhong-kai WU (2014) proposes the real option pricing contract for air cargo when there are two markets in contract and retail, and verifies the validity of the contract[17]. Yun RU (2012) compares the fixed commitment contract and the option contract from the perspective of freight forwarding, in the coexistence of contract market and retail market[5]; Shu-zhen CHEN et al., (2009) studies the problem of contract pricing when there is asymmetric information between air cargo carrier and freight forwarder[14].

Researches on railway transportation are mostly focused on the study of traditional rail freight, so there is a lack of relevant foreign documents for high-speed rail express. Xiao-jie WEl (2012) starting from the national railways in the Organization for Economic Co-operation and Development, a comparative study of the economic benefits of railways in various countries under the conditions of “separation of network operations” was carried out[6]. The improvement of the operational capacity of high-speed rail logistics requires the construction of logistics information. Li-Min XU (2012) based on factors such as economic development and employment structure, a cargo flow forecast model was constructed[8]. Around 2012, the domestic high-speed rail express delivery research was gradually called the hot spot. At this time, the total iron has not yet officially launched high-speed rail express delivery. Therefore, the research before 2014 is mostly concentrated on development countermeasures and experience from developed countries for reference [15]. After high-speed rail Express officially entered the express delivery market in 2014, research on the game relationship between China Railway Corporation and traditional express delivery companies and their pricing strategies have gradually emerged. For example, Hua-wei DUAN et al., (2015) constructed two Stackelberg game models under the random market demand, and calculated the high-speed rail express delivery and traditional express pricing strategies in each case, and verified the results through calculation examples[12]. Lv PU et al., (2015) studied the China Railway Corporation facing the contract and retail dual-market high-speed rail express pricing and joint pricing under the background of the gradual marketization of rail freight[13]. In general, due to the cooperation between China Railway Corporation and traditional express delivery companies, the coordinated research of high-speed rail express delivery chain has also attracted the attention of scholars. However, the research on the coordination of supply chain has had more abundant results. Scholars have studied and designed a variety of supply chain coordination mechanisms. For example, Shu-zhen CHEN et al., (2009) constructed a dual-channel supply chain model under conditions of promotional price-sensitive demand and promotion compensation incentives, and found that the use of two pricing contracts alone could not achieve perfect coordination of the dual-channel supply chain[14]. When the compensation contract is combined with two pricing contracts at the same time, the supply chain can be perfectly coordinated. Qi-xiang ZHOU (2015) conducted a study on the coordination of risk-averse dual-channel supply chain and designed a set of revenue-sharing contracts[15]. It was verified that the contract can ensure the perfect coordination of the supply chain. Ya-jun GUO and Li-Qiang ZHAO (2008) designed a set of transfer payment contracts to coordinate the two-channel supply chain[18]. Zhong-kai XIONG and Gen-dao LI (2007) pointed out that the combined use of revenue sharing contracts and rebates can achieve dual channel supply chain coordination[19].

The above studies have a good reference for the study of this article. However, because of the incompetent advantages of rail transportation, such as air transportation and shipping, the research on railway transportation was mainly focused on the study of its development strategies and the experiences of developed countries. In China, there is only one company in rail transportation, so there is no direct competitor, so the railway company has a strong pricing power. Based on this, this paper studies the pricing decisions of China Railway Corporation in the dual channels of direct sales and retail selling channels. Due to the existence of double marginal effect, the conflict
between retail and direct selling channels is unavoidable. In order to coordinate the conflict between
the two channels in CHR express supply chain under the dual channels of retail and direct sales, this
paper will test whether it can be achieved by designing the price discount contract. The perfect
coordination of high-speed rail express supply chain.

3. Model and Framework

This section presents and analyses the model of CHR express supply chain system with retail and
direct selling market demands and is divided into two parts. The first part presents model
description and assumptions; the second part presents introduction of parameters and model.

3.1. Model Description and Assumptions

According to literature Guang-ye XU and Bin DAN (2012) [16], this paper designed a high-speed
rail express supply chain system model under retail and direct sales channels (as shown in Fig. 1),
which is consist of costumers, China Railway Corporation and traditional express delivery
companies (such as STO Express, YTO Express, ZTO Express, S. F. Express, etc.), are as follows:
CRC develops traditional express delivery companies as their own agents after signing high-speed
rail express service sales contracts with traditional express delivery companies in advance; as agents,
CRC is not responsible for providing high-speed rail express delivery services, but is only
responsible for providing high-speed rail express consumer terminal services at the prices set by
itself in the market; on the other hand, China Railway Corporation directly provide high-speed rail
express delivery to consumers at their own prices through direct channels. To the CHR express
company, it needs to fulfill two kinds of orders: one from the direct selling channel of CRC and
another from retail selling channel of traditional express company. Under the set of this cooperation,
CHR express supply chain system with retail and direct selling market demands is formed. Both
companies are independent entities and they compete on the market price.

Based on the model (ie, shown in Fig. 1), for the convenience of the study, the following
conditional assumptions are purposefully proposed:

Conditional Assumption 1. With monopoly position of high railway, CHR express company has
enormous control over CHR express business, so it is assumed that CHR express company acted as
a leader in the supply chain, which is to say that traditional company makes pricing decision after
CHR express company.

Conditional Assumptions 2. CRC and traditional express delivery companies are rational, and all
information on both sides is common knowledge.

Conditional Assumptions 3. The demand for both channels is affected both by the price of the
channel and by the crossover of the price of another channel.

Conditional Assumptions 4. Demands are the linear functions of the direct and retail selling
prices.

Conditional Assumption 5. High-speed rail express companies and traditional express companies
have loyal old customers.

Conditions Assumption 6. China Railway Corporation and traditional express delivery companies
do not consider fixed costs, because fixed costs only affect the profits of the two, does not affect the
decision-making.

3.2. Introduction of Parameters and Model

The significance of parameters in this paper are shown in Table 1.
Table 1. Significance of the symbols in the text

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Π&lt;sub&gt;e&lt;/sub&gt;</td>
<td>profits of traditional express company</td>
</tr>
<tr>
<td>Π&lt;sub&gt;h&lt;/sub&gt;</td>
<td>profits of CHR express company</td>
</tr>
<tr>
<td>Q&lt;sub&gt;e&lt;/sub&gt;</td>
<td>demands of retail selling channel</td>
</tr>
<tr>
<td>Q&lt;sub&gt;h&lt;/sub&gt;</td>
<td>demands of direct selling channel</td>
</tr>
<tr>
<td>C&lt;sub&gt;0&lt;/sub&gt;</td>
<td>the unit cost of CHR express service provided by CHR express company</td>
</tr>
<tr>
<td>W</td>
<td>the wholesale price</td>
</tr>
<tr>
<td>P&lt;sub&gt;h&lt;/sub&gt;</td>
<td>market price of direct selling</td>
</tr>
<tr>
<td>P&lt;sub&gt;e&lt;/sub&gt;</td>
<td>market price of retail selling</td>
</tr>
</tbody>
</table>

Note: C<sub>0</sub> is constant, and C<sub>0</sub>&gt;0; besides P<sub>e</sub>&gt;W because the profits of traditional express company must be bigger than zero.

According to the reference (Xu Guangye and Dan Bin, 2012) and the assumptions, CHR express company and traditional express company, respectively, are facing the following demand functions in the model:

\[
Q_e = \theta K - P_e + bP_h
\]

\[
Q_h = (1-\theta)K - P_h + bP_e
\]

Where parameters \(Q_e\) and \(Q_h\) are, respectively, the demand of retail selling channel and direct selling channel; parameter \(K(K>0)\) is the potential valuation of the whole market, \(\theta(0<\theta<1)\) is the market share of retail selling channel, then \((1-\theta)\) is the market share of direct selling channel; parameter \(b\) is the degree of substitution of two channel (or the cross-price coefficients) \((1>b>0)\); \(0K-aP_1>0\) and \((1-0)K-aP_2>0\), which means that both channels has their own loyal customer.

The profits of CHR express company is consisted of two parts: one from direct selling channel and another from wholesaling CHR express service; To traditional express company in this model, its profits come from retailing CHR express service. So the profits function of traditional express company and CHR express company, respectively, are as follows:

\[
\Pi_e = (P_e - W)Q_e
\]

\[
\Pi_h = (P_h - C_0)Q_h + (W - C_0)Q_e
\]

The whole profits of this supply chain come from the total of two channels. The profits function of the supply chain system is as follow:

\[
\Pi = \Pi_e + \Pi_h = (P_h - C_0)Q_h + (P_e - C_0)Q_e
\]

It is necessary to consider the following reasonable constrains into model to ensure that the obtained prices are feasible. (1) Wholesale and direct selling prices should not be less that the unit cost of CHR express service, and (2) wholesale price should not be higher than retail selling prices. These constrains are shown as follows:

\[
W, P_e \geq C_0; W \leq P_e, P_h
\]

3.3. Benchmark Model

This section presents and analyses results of two decisions models and is divided into two parts. The first part presents centralized decision on CHR express supply chain with direct and retail
selling market demands; the second part presents decentralized decision on CHR express supply chain with direct and retail selling market demands.

In order to reveal the reasons for the imbalance in the supply chain system of high-speed rail express delivery under retail and direct sales channels, the centralized decision model and decentralized decision model in the dual-channel supply chain of high-speed rail express will be separately analyzed. The point to be specifically pointed out is that centralized decision-making is based on the ideal situation (that is, China Railway Corporation and traditional express delivery companies make decisions with the goal of maximizing the profits of the entire high-speed rail express supply chain system), while decentralized decision-making is ubiquitous in reality. By comparing the two decision-making situations, it reveals the causes of system misalignment and provides support for the establishment of a coordination mechanism for the entire high-speed rail express supply chain system under retail and direct sales channels.

### 3.3.1. Centralized Decision Model

Under the centralized situation, two firms are vertically integrated equivalently, the decision is maximized the total profits of the whole supply chain. The relevant function is as follow:

\[
\text{max } \Pi^C = (P_h - C_0)[(1-\theta)K - P_e + bP_e] + (P_e - C_0)(0K - P_e + bP_e) \quad (7)
\]

From Equation (7), the Hessian Matrix for \( P_h \) and \( P_e \) is established:

\[
H_1 = \begin{bmatrix}
-2 & 2b \\
2b & -2
\end{bmatrix}
\quad (8)
\]

As mentioned above, the parameter \( b \) meets the condition \( 1 > b > 0 \), we can conclude that Hessian Matrix \( H_1 \) is negative definite. The profits model of the whole supply chain is concave and can reach the global maximum value when the two companies make decisions in accordance with the rules as follows:

\[
\frac{\partial \Pi^C}{\partial P_e} = 0K + 2bP_e - 2P_e + (1-b)C_0 = 0 \quad (9)
\]

\[
\frac{\partial \Pi^C}{\partial P_h} = (1-\theta)K + 2bP_e - 2P_e + (1-b)C_0 = 0 \quad (10)
\]

By solving the Equations (9) and (10), we can get the best reply functions of retail and direct selling price as follows:

\[
P^*_e = \frac{(0+\theta)bK+(1-b^2)C_0}{2(1-b^2)} \quad (11)
\]

\[
P^*_h = \frac{(1-\theta)+\theta)bK+(1-b^2)C_0}{2(1-b^2)} \quad (12)
\]

Through Equations (1) (2) (11), and (12), we can conclude functions of demands of direct and retail selling channels.

\[
Q^*_e = \frac{0K+(b-1)C_0}{2} \quad (13)
\]
Through Equations (7) (11) and (12), we can conclude the function of the overall profits of supply chain.

\[
Q^*_h = \frac{(1-0)K+(b-1)C_0}{2}
\]  

Through Equations (7) (11) and (12), we can conclude the function of the overall profits of supply chain.

\[
\Pi^*_h = \frac{[1-(1-b)(1-0)b]K^2+(b^2-1)KC_0+(b^3-b+1)C_0^2}{4(1-b^2)}
\]  

3.3.2. Decentralized Decision Model

In the decentralized dual-channel supply chain, CHR express and traditional express company are rational units. The objective of both are to maximize expected profits. CHR express company is considered as a leader, because of its monopoly position of high railway, it has enormous control over CHR express business. In the decentralized situation, CHR express company declares the direct selling price \( P_h \) and the wholesale price \( W \), initially. Then traditional express company announces its retail selling price decision.

The maximum profits functions of CHR express company and traditional express company are shown as follows:

\[
\max \Pi_e = (P_e - W)(\theta K - P_e + bP_h)
\]  

\[
\max \Pi_h = (P_h - C_0)(\theta K - P_h + bP_e) + (W - C_0)(\theta K - P_e + bP_h)
\]

As traditional express company had known the strategy of CHR express company before making decision, and giving that \( \frac{\partial^2 \Pi}{\partial^2 P} = -2a < 0 \), we can conclude that the profits model of traditional express company is concave and could reach the global maximum value when it makes decision in accordance with the rule as follow:

\[
P_e(W, P_h) = \frac{\theta K + bP_h + C_e}{2} = \frac{\theta K + bP_h + W}{2}
\]

To CHR express company the optimal conditions are as follows:

\[
\frac{\partial \Pi_h}{\partial W} = \frac{1}{2}[-2W + 2bP_h + 0K + (1-b)C_0] = 0
\]  

\[
\frac{\partial \Pi_h}{\partial P_h} = \frac{1}{2}[2bW + 2(b^2 - 2)P_h + (2-20+b\theta)K + (1-b-b^2)C_0] = 0
\]

Combined Equations (19), (20) with (18), we get the best reply functions of the wholesale price, retail and direct selling price as follows:

\[
P_{h}^{b^*} = \frac{(1-0+b0)K+(1-b^2)C_0}{2(1-b^2)}
\]  

\[
W^{b^*} = \frac{(b-b0+0)K+(1-b^2)C_0}{2(1-b^2)}
\]
Through above Equations, we can conclude the function of demands of direct and retail selling channels:

\[ Q^{D*}_e = \frac{0K + (b-1)C_0}{4} \]  \hspace{1cm} (24)

\[ Q^{D*}_h = \frac{[2(1-0)b]K-bC_0+(b^2-2)C_0}{4} \]  \hspace{1cm} (25)

Through above Equations, we can conclude that the maximum profits of traditional express company and CHR express company, respectively, are as follows:

\[ \Pi^{D*}_e = \frac{(0K+bC_0-C_0)^2}{16} \]  \hspace{1cm} (26)

\[ \Pi^{D*}_h = \frac{X_1K^2-Y_1K+Z_i}{8(1-b^2)} \]  \hspace{1cm} (27)

Note: \( X_1 = 0^2b^2 + 4(1-0)0b + 30^2 - 40 + 2 \), \( Y_i = 2(1-b^2)(2-0+0b)C_0 \), \( Z_i = (1-b^2)(3-2b+b^2)C_0^2 \).

Through Equations (26) and (27), the total profits of the whole supply chain is published as follow:

\[ \Pi^{D*} = \Pi^{D*}_e + \Pi^{D*}_h = \frac{(1-b^2)(0K+bC-C_0)^2 + 2(X_1K^2-Y_1K+Z_i)}{16(1-b^2)} \]  \hspace{1cm} (28)

Combined Equations (28) and Equations (16), we can conclude that the profits gap of entire supply chain between decentralized and centralized decision is as follow:

\[ \Delta\Pi^* = \Pi^{C*} - \Pi^{D*} = \frac{(0K+bC_0-C_0)^2}{16} = \Pi^{D*}_e \]  \hspace{1cm} (29)

Combined Equations (11) and Equations (23), we can conclude that the retail selling price gap of traditional express between decentralized and centralized situation is as follow:

\[ p^{D*}_e - p^{D*}_c = \frac{0K-C_0+bC_0}{2} > \frac{0K-P_e+bC_0}{2} > \frac{0K-P_e}{2} > 0 \]  \hspace{1cm} (30)

We can find that the retail selling price in decentralized situation is larger than that in centralized decision situation. It meant that the supply chain cannot achieve perfect coordination in decentralized decision. In order to achieve perfect coordination, a price discount coordination mechanism will be designed in the next.

4. Price Discount Contract Coordination Mechanism Design

In order to achieve perfect coordination, the function of the wholesale price is designed as follow:

\[ W(P_h) = C_0 + a(P_h - C_0) \]  \hspace{1cm} (31)
Note: The letter \( a \) means the price discount rate, it meets the condition: \( 0 < a < 1 \);
The wholesale price must ensure: \( P_h > W(P_h) > C_0 \).

Then function of traditional express company profits is as follow:

\[
\Pi^{PD}_e = [P_e - W(P_h)]Q_e - [P_e - C_0 - a(P_h - C_0)](0K - P_e + bP_h)
\] (32)

With monopoly position of high railway, CHR express company has enormous control over CHR express business, so CHR express company acts as a leader in the supply chain, it means that traditional company makes pricing decision after CHR express company.

To CHR express company, the optimal conditions is as follow:

\[
\frac{\partial \Pi^{PD}_e}{\partial P_e} = 0K + (a + b)P_h - 2P_e + (1 - a)C_0 = 0
\] (33)

Through Equation (33) the best reply functions of retail selling price is as follows:

\[
P^{PD}_e = \frac{0K + (a + b)P_h + (1 - a)C_0}{2}
\] (34)

Combined Equation (34) with Equation (1) and (2), we can conclude function of demands of direct and retail selling channels:

\[
Q^{PD}_e = \frac{0K + (b - a)P_h + (1 - a)C_0}{2}
\] (35)

\[
Q^{PD}_h = \frac{(2 - 2\theta + b\theta)K + [(a + b)\theta - 2]P_h + (1 - a)\theta C_0}{2}
\] (36)

Game in the first period, the maximum profits functions of CHR express company is as follow:

\[
max_{P_h} \Pi^{PD}_h = (W(P_h) - C_0)Q^{PD}_e + (P_h - C_0)Q^{PD}_h
\] (37)

Combined Equations (35), (36) with Equation (37), to traditional express company, the optimal conditions is as follows:

\[
\frac{\partial \Pi^{PD}_h}{\partial P_h} = \frac{1}{2}[(2 - 2\theta + b + a\theta)K + (2 + a^2 - b^2 - 2ab)P^{PD}_h + (2a^2 - a - 3ab + 2b + b^2 - b^2)] = 0
\] (38)

Through the above Equation, the best reply functions of direct selling price is concluded:

\[
P^{PD}_{h*} = \frac{(2 - 2\theta + b\theta + a\theta)K + (2a^2 - a - 3ab + 2b + b^2)C_0}{2(a^2 - b^2 - 2ab + 2)}
\] (39)

After Equation (39) is substituted to Equation (34), the following function is concluded:

\[
P^{PD*}_{e} = \frac{(2 - 2\theta + b\theta + a\theta)K + (2a^2 - a - 3ab + 2b + b^2)C_0}{2(a^2 - b^2 - 2ab + 2)}
\] (40)

**Proposition 4-1**: When price discount rate \( a \) is equal to cross-price coefficients \( b \) of two channels, CHR express supply chain with retail and direct selling market demands achieves...
coordination.

In other words, after the price discount contract is carried out, the direct and retail selling price decisions must be same to centralized situation, which means the following conditions must be met.

\begin{equation}
P_{PD_c}^* = P_{C}^*
\end{equation}

\begin{equation}
P_{PD_n}^* = P_{C}^*
\end{equation}

Besides the following condition also must be met.

\begin{equation}
0 < W_{PD_c}^* < P_{PD_c}^*
\end{equation}

All in all, only when Equation (41), (42) and (43) are true at the same time, can confirm the truth of Proposition 4-1.

**Mathematical proof:**

After setting a equals b in the Equation (40), we can get the function as below:

\begin{equation}
P_{PD_c}^* = \frac{30a^2 + 20 - 2(1 + b)0a - (b^2 - 2(1 - 0)b - 40)K}{4(a^2 - b^2 - 2ab + 2)} + \frac{[(1 + 3b)a^2 - 2(1 + b)^2a + 4b - b^3]C_0}{4(a^2 - b^2 - 2ab + 2)}
\end{equation}

\begin{equation}
= \frac{(1 + b - b)K + (1 - b^2)C_0}{2(1 - b^3)}
\end{equation}

Compared Equation (11) with Equation (44), it is conclude that Equation (41) is true.

As the same, after setting a equals b in the Equation (39), we can conclude get the function as below:

\begin{equation}
P_{PD_n}^* = \frac{2 - 20 + b0 + a0K + (2a^2 - 3ab+2 + b^3)C_0}{2(a^2 - b^2 - 2ab + 2)} = \frac{(1 - 0b)K + (1 - b^2)C_0}{2(1 - b^3)}
\end{equation}

Compared Equation (12) with Equation (39), it is conclude that Equation (42) is true.

All the same, after setting a equals b in the Equation (31), the function of wholesale price is as follow:

\begin{equation}
W_{PD_c}^* = \frac{[0 + (1 - 0b)K + (1 - b^2)(2 - b)C_0]}{2(1 - b^3)}
\end{equation}

Combined Equation (44) and Equation (46), the following function is calculated:

\begin{equation}
P_{PD_c}^* = \frac{(1 - 0b)K + (1 - b^2)C_0}{2(1 + b)}
\end{equation}

According to the assumption e, the inequation \(0K - P_{PD_c}^* > 0\) is true. As the inequation \(P_{PD_c}^* > C_0\) is true, it is obvious that the inequation \(0K - C_0 > 0\) is true. Then there is not doubt that Equation (43) is true.

In all, it had been proved that Proposition 4-1 is right.

**Proposition 4-2:** Under the above coordination mechanism, the China Railway Corporation will not claim to be the leader of the entire supply chain.
After plugging Equations (44), (45) and (46) into profits functions, the profits functions in price discount contract situation can be calculated:

\[
\Pi_e^{PD^*} = \frac{(0K + bC_0 - C_0)^2}{4} \quad (48)
\]

\[
\Pi_h^{PD^*} = \frac{[(1-0+0b)K-(1-b^2)C_0]^2}{4(1-b^2)} \quad (49)
\]

Then the function of entire supply chain is as follow:

\[
\Pi^{PD^*} = \Pi_e^{PD^*} + \Pi_h^{PD^*} = \frac{[1-(1-b)(1-0)\theta K^2+(b^2-1)KC_0+(b^3-b+1)C_0^2]}{4(1-b^2)} = \Pi^C^* \quad (50)
\]

Through Equations (50), it is found that the profits of entire supply chain in price discount contract situation is equal to that in decentralized situation, which means the price discount contract can make the supply achieve coordination.

It is worth that only when the profits two companies are larger than decentralized situation can the price discount contract be carried out. So it is necessary to analyze the change of the profits. Combined Equations (26) and (27) with Equations (48) and (49), the following function is calculated:

\[
\Delta \Pi^* = \Pi^{PD^*} - \Pi^D^* = \frac{(0K + bC_0 - C_0)^2}{16} > 0 \quad (51)
\]

\[
\Delta \Pi^*_e = \Pi^{PD^*}_e - \Pi^D^*_e = \frac{3(0K + bC_0 - C_0)^2}{16} > 0 \quad (52)
\]

\[
\Delta \Pi^*_h = \Pi^{PD^*}_h - \Pi^D^*_h = -\frac{(0K + bC_0 - C_0)^2}{8} < 0 \quad (53)
\]

Through Equations (51), (52) and (53), we can get three conclusions:

A. Compared with decentralized situation, the profits of entire supply chain is increased in price discount situation.

B. Compared with decentralized situation, the profits of CHR express company is decreased in price discount situation.

C. Compared with decentralized situation, the profits of traditional express company is increased in price discount situation.

Through the above conclusions, there are no doubt that CHR express company shall be not willing to carry out the present price discount contract, because, compared with decentralized situation, the profits of CHR express company is decreased. So Proposition 2 is come up.

**Proposition 4-3:** After the above price discount contract is implemented, if traditional express company can transfer part of its profits (regarded it as the letter T) to CHR express company, and makes sure \( T \in \left[ \frac{(0K + bC_0 - C_0)^2}{8}, \frac{3(0K + bC_0 - C_0)^2}{16} \right] \), the profits of two companies can achieve pareto improvement.

**Mathematical proof:**

Only when the following conditions are true in the same time, can confirm the truth of Proposition 2.
\[ \Pi^{PD*} > \Pi^{D*} \]  
(Note: As Equations (51) shown, it is obvious that Equations (51) is true)

\[ \Pi^{o} - T > \Pi^{o} \]  
\[ \Pi^{h} + T > \Pi^{h} \]

Combined Equations (55) with Equations (56) the value ranges of \( T \) is as follow:

\[ \Pi^{o} - \Pi^{o} < T < \Pi^{o} - \Pi^{o} \]

Through Equations (52) and (53), the following result is calculated:

\[ \frac{(0K+bC_0-C_0)^2}{8} < T < \frac{3(0K+bC_0-C_0)^2}{16} \]

In conclusion, Proposition 4-2 is right.

All in all, “the price discount contract + the transfer payment mechanism” can make the supply chain achieve perfect coordination. There is one thing must be point out that the value of \( T \) has positive relationship with the negotiation ability of CHR express company. CHR express company can strengthen its negotiation ability by elevating the price discount rate.

5. Analysis of an Example

Set the potential valuation of the market \( K=200 \), the market share of retail selling channel \( \theta=0.4 \); the unit cost of CHR express service provided by CHR express company \( C_0=10 \); \( \Delta \Pi_e = \Delta \Pi_e - T \), \( \Delta \Pi_e = \Delta \Pi_e - T \).

Under the set of \( a=b \), we will analyze the relative profits in two different situations:

**Situation one**: The cross-price coefficient is a variable, \( T \) is a constant.

Through the software EXCEL and Matlab, the relative profits value are as shown in Table 2.

<table>
<thead>
<tr>
<th>( b )</th>
<th>( \Pi_e^{PD*} )</th>
<th>( \Pi_h^{PD*} )</th>
<th>( \Delta \Pi_e^{PD*} )</th>
<th>( \Pi_h^{o} )</th>
<th>( \Delta \Pi_h^{o} )</th>
<th>( \Pi^{PD*} )</th>
<th>( \Delta \Pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>560.25</td>
<td>315.06</td>
<td>245.19</td>
<td>3332.43</td>
<td>3262.55</td>
<td>69.88</td>
<td>3892.68</td>
</tr>
<tr>
<td>0.20</td>
<td>596.00</td>
<td>324.00</td>
<td>272.00</td>
<td>3020.67</td>
<td>2968.67</td>
<td>52.00</td>
<td>3616.67</td>
</tr>
<tr>
<td>0.30</td>
<td>632.25</td>
<td>333.06</td>
<td>299.19</td>
<td>2774.62</td>
<td>2740.74</td>
<td>33.88</td>
<td>3406.87</td>
</tr>
<tr>
<td>0.40</td>
<td>669.00</td>
<td>342.25</td>
<td>326.75</td>
<td>2585.76</td>
<td>2570.26</td>
<td>15.50</td>
<td>3254.76</td>
</tr>
<tr>
<td>0.50</td>
<td>706.25</td>
<td>351.56</td>
<td>354.69</td>
<td>2452.08</td>
<td>2455.21</td>
<td>-3.13</td>
<td>3158.33</td>
</tr>
<tr>
<td>0.60</td>
<td>744.00</td>
<td>361.00</td>
<td>383.00</td>
<td>2381.00</td>
<td>2403.00</td>
<td>-22.00</td>
<td>3125.00</td>
</tr>
</tbody>
</table>

The following conclusions are drawn from the above table:

A. When the value of \( T \) is fixed, the profits of CHR express company and the entire supply chain have positive relationship with the cross-price coefficient.

B. When the value of \( T \) is fixed, there is a negative correlation between the profits of traditional express company and the cross-price coefficient.

C. The extra profits of CHR express company (it means the value of \( \Delta \Pi_h^{o} \)) has negative relationship with the cross-price coefficient.
D. The extra profits of traditional express company (it means the value of $\Delta \Pi'$) and entire supply chain (it means the value of $\Delta \Pi$) have positive relationship with the cross-price coefficient. There is negative number in the seventh rank of the table, when $b=0.50$ is true, following function is concluded: 

$$\frac{(0K+bC_0-C_0)^2}{8} = 703.125 \neq 700$$

which is inconsistent with Proposition 4-2. In this situation, CHR express company shall be not willing to implement the mechanism unless the value of $T$ is promoted.

**Situation two:** The cross-price coefficient is a constant, $T$ is a variable. Through the software EXCEL, the relative profits value are as shown in Table 3.

**Table 3.** Comparison of the profits of supply chain members between pricing discount coordination mechanism and decentralized decision

<table>
<thead>
<tr>
<th>T</th>
<th>$\Pi^D*$</th>
<th>$\Pi^D_2$</th>
<th>$\Delta \Pi'$</th>
<th>$\Pi^D_1$</th>
<th>$\Delta \Pi_1$</th>
<th>$\Pi^D_0$</th>
<th>$\Delta \Pi_0$</th>
<th>$\Delta \Pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td>732.25</td>
<td>333.06</td>
<td>399.19</td>
<td>2874.62</td>
<td>-66.12</td>
<td>3506.87</td>
<td>3173.81</td>
<td>333.0625</td>
</tr>
<tr>
<td>650</td>
<td>682.25</td>
<td>333.06</td>
<td>349.19</td>
<td>2774.62</td>
<td>-16.12</td>
<td>3456.87</td>
<td>3123.81</td>
<td>333.0625</td>
</tr>
<tr>
<td>700</td>
<td>632.25</td>
<td>333.06</td>
<td>299.19</td>
<td>2740.74</td>
<td>33.88</td>
<td>3406.87</td>
<td>3073.81</td>
<td>333.0625</td>
</tr>
<tr>
<td>750</td>
<td>582.25</td>
<td>333.06</td>
<td>249.19</td>
<td>2724.62</td>
<td>83.88</td>
<td>3356.87</td>
<td>3023.81</td>
<td>333.0625</td>
</tr>
</tbody>
</table>

The following conclusions are drawn from the above table:

A. When the value of $b$ is fixed, the profits of CHR express company and the entire supply have positive relationship with transferred fee of traditional express company (it means the value of $T$).

B. When the value of $b$ is fixed, there is a negative correlation between the profits of traditional express company and transferred fee of traditional express company (it means the value of $T$)

There are negative number in the first and second line of the table, according to the Proposition 4-2 Equation $\frac{(0K+bC_0-C_0)^2}{8} = 666.125$ is calculated, so it is obvious that $T=600$ and $T=650$ are inconsistent with Proposition 2. In this situation, CHR express company shall be not willing to implement the mechanism unless the value of $T$ is promoted.

All in all, in the above two situations, the price discount coordination mechanism always can make the CHR express supply chain achieve the perfect coordination.

6. Conclusion

This article is devoted to the study of whether the retail and direct-sale dual-channel high-speed rail express supply chain achieves perfect coordination. By comparing the centralized decision model with the decentralized decision model, it is found that the overall benefit of the express delivery chain under the decentralized decision is less than the centralized decision. This shows that under the decentralized decision-making model, the high-speed rail express delivery supply chain under the dual channels of retail and direct sales has not reached the coordination status. Therefore, the design price discount contract is used to coordinate the channel conflicts in the high-speed rail express supply chain under both retail and direct sales channels and proves that Proposition 4-1, Proposition 4-2 and Proposition 4-3 are all established. The establishment of Proposition 4-1 means that in order to achieve high-speed rail express delivery supply chain coordination under China Railway Corporation, it is only necessary to set the price discount rate as the cross elasticity coefficient of the two channels. In other words, the operability of the high-speed rail express companies to implement the price discount contract coordination mechanism is very strong. From Proposition 4-2 we can see that, under the existing coordination mechanism, the China Railway Corporation will not claim to be the leader of the entire supply chain because the China Railway Corporation have reduced their profits compared with the pre-coordinated decentralized decision-making. Therefore, in order to ensure the effective implementation of the price discount
contract, CRC can then design a transfer payment mechanism so that both parties can achieve Pareto improvement. Proposition 4-3 is a true proposition that express high-speed rail express companies are setting the price discount rate equal to the cross-elasticity coefficient of two channels, although the overall profits and traditional express delivery of the high-speed rail express supply chain under both retail and direct sales channels Corporate profits have increased compared to decentralized decision-making, but the profits of high-speed rail express companies have also been reduced compared to those of centralized decision-making; if at this time, traditional express companies can transfer to high-speed rail express companies within a certain range. Paying T’s profit can make the China Railway Corporation and traditional express delivery companies in the high-speed rail express supply chain of retail and direct sales channels achieve a win-win situation. Under this price discount mechanism, as the profits of express delivery companies are greatly increased, traditional express delivery companies have sufficient incentive to pay part of their profits to China Railway Corporation, but it is necessary to ensure that the ultimate profits of traditional express delivery companies are greater than those under decentralized decision-making. In other words, the above-mentioned transfer payment mechanism can reasonably exist. It should be pointed out that the specific transfer payment profit can be adjusted within a reasonable range, and the magnitude of the adjustment depends on the negotiation ability (or bargaining power) of both parties in this matter. Finally, the effectiveness of the price discount contract mechanism is tested through an example.

7. Limitations and Future Research Directions

The research results of this article will help promote the development of high-speed rail express and help the entire express delivery industry to upgrade and transform. Because the development of high-speed rail express delivery not only meets the requirements of the national sustainable development strategy (high-speed rail transport is more energy-efficient and environmentally friendly than road transport and air transport, but also can effectively reduce the emissions caused by the express delivery industry and help ease the country's governance. The pressure of haze will increase the level of logistics services in the express delivery industry and thus promote the economic development of the entire express delivery market.

But there are also deficiencies: First, this article assumes that the demand function is a linear demand function, and in reality, the demand function is mostly a nonlinear demand function. And in the operation and development of the express delivery industry, prices are sometimes not the direct and main influencing factors. Consumer preference is also one of the important influencing factors. Second, this article assumes that the high-speed express delivery companies and traditional express delivery companies are symmetrical in information. Third, this paper only designed a coordination mechanism of price discount contract. Therefore, in the future research, we need to consider that the demand function is a nonlinear demand function and the information asymmetry between the two parties; besides, in the future research on the high-speed rail express supply chain under the dual channels of retail and direct sales, we can try to design other coordination mechanisms (such as revenue sharing mechanism, quantity flexibility mechanism, etc.).

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9. References


