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Standard and Set-theoretic Analysis Strategies of a Scientific Theory

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***Abstract*—The so-called standard interpretation of a scientific theory, proposed in the framework of the program of logical positivism, suggested the possibility of formalizing of some scientific theory in the language of first-order predicate logic. This method of logical analysis of the structure of a scientific theory does not allow one to sufficiently and accurately describe the class of its proposed models, which to some extent discredits the use of formal methods in the philosophy of science. A natural alternative to the standard formalization of the theory is its set-theoretic formalization, which uses the semantic concept of the theory model in the sense of A. Tarsky as the main one and lacks the program of logical positivism. The set-theoretic strategy for analyzing scientific knowledge more closely matches the actual practice of scientific knowledge, allowing us to explicitly and unambiguously describe the class of the proposed models of the theory. The article includes the basic principles of data analysis strategies of scientific theories.**

Keywords—theory; syntax; semantics; model; formalization; axiomatization

I. INTRODUCTION

The program for the substantiation of scientific knowledge, proposed by logical positivism, suggested the solution of a number of fundamental epistemic and formal logical tasks. The main epistemic tasks include the detection of elementary and reliable fundamentals of scientific knowledge, a strict distinction between analytical and synthetic statements that are part of scientific knowledge, the formulation of procedures for substantiating theoretical knowledge (reducing theoretical knowledge to empirical), which at the same time turned out to be a solution to the problem of strictly determining the criteria for the meaningfulness of statements and criteria for the demarcation of scientific and non-scientific knowledge [1]. A prerequisite for the successful implementation of this methodological project in the framework of the program of logical positivism was the successful solution of the formal-logical problem of strict explication of these problems in some artificial language. In the modern historical-philosophical and logical literature, it is accepted to call the formalized language as the language of logic of first-order predicates with equality. At the same time, the scientific theory itself, which is to be analyzed, was formulated as a partially interpreted axiomatic system, the axioms of which were the fundamental laws of the corresponding theory, expressed in a certain theoretical language L_T. Observed consequences of these laws were formulated in a separate language of observation L_O, and the connection between concepts from languages L_T, L_O was carried out using the rules of conformity C (in later editions of this program — with the help of reduction offers R). In this case, only concepts from the language had direct semantic meanings L_O. The theory itself was viewed as a set of sentences — fundamental postulates T and compliance rules C, ordered by formal deductive hatchability. This interpretation of scientific theory is usually called the standard.

Attempts to implement the methodological program of logical positivism revealed a number of its significant shortcomings, which cast doubt on the very possibility of effectively applying logical-mathematical methods to analyzing the structure of scientific knowledge [2]. An alternative set-theoretic (or semantic) strategy for analyzing scientific knowledge proposed in the 50s of the 20th century neutralized or weakened most of the problems that arose during the implementation of a logical-positivist formal program.

II. SYNTACTIC AND SEMANTIC STRATEGIES OF ANALYSIS OF A SCIENTIFIC THEORY

Accumulated knowledge within the framework of the logical-positivist program of analysis of scientific, led to a radical revision of a number of basic "program tenets" of logical positivism or direct rejection of them. Among the most common counter-arguments put forward against the methodology of logical positivism, it is necessary to name the following:

Firstly, in actual practice, scientific knowledge (natural science) theories are almost never constructed as axiomatic systems in the above sense. The standard formalization of such theories (that is, their exhaustive axiomatization in the language of logic of first-order predicates with equality) either is not feasible in principle, or leads to an excessive and unnatural technical complication of the theory’s apparatus, which makes it impossible to work with it [3].

Secondly, in the general case, a strict distinction between analytical and synthetic statements (and, as a consequence, theoretical terms and terms about observable quantities) is impossible when using standard formalization of the theory. Consequently, the proposed criteria for the empirical meaningfulness of statements are unsatisfactory.

Thirdly, the so-called "rules of conformity" which compare theoretical and observational dictionaries, often represent a heterogeneous mixture of semantic concepts, elements of experimental procedures and measurement procedures. Since the meanings are attributed to theoretical terms only with the help of correspondence rules (sentences), and the theory as a whole is identified with its linguistic formulation, any progressive change in the organization of an experiment or measurement procedure technique inevitably leads to a change in the theory itself, which is absurd from the point of view of the actual practice of scientific knowledge.

Fourthly, the very syntactic interpretation of theories is unjustifiably narrow: theories are not (only) linguistic objects — sets of well-formed sentences of some formalized language, connected by a relation of logical deducibility or closed relative to this relation. Since, according to the traditional principles of logical semantics, out of all categories of linguistic expressions, only statements (sentences expressing judgments) can be evaluated as true or false, the category of truth in this case turned out to be fundamentally inapplicable to the analysis of the real structure of scientific knowledge. Both the question of the conformity of scientific theories of reality and the traditional (correspondent) theory of truth that lies at its basis turned out to be irrelevant to the practice of scientific knowledge [4].

P. Suppes, one of the authors of an alternative, set-theoretic strategy for the axiomatization of a scientific theory, pointed to the following principal "technical" defect of the correspondence rules, discrediting their role as a tool for comparing observed phenomena and the fundamental tenets of the theory.

Any specific experimental data obtained in the course of the experiment cannot be directly related to the fundamental tenets of the theory in any sense.

These experimental data are pre-ordered using some functions, graphs, classification systems, resulting in fixed in the standard, "canonical" form, which can be called data model.

It is obvious that the compliance rules are directly applicable to experimental data models rather than to theories of theories. It is also obvious that the models of the experiment are of a slightly different logical type than the models of theories. For example, theory models usually contain continuous functions and infinite sequences of objects, while the experimental models are mostly discrete and finite (finite) in nature.

As a result, according to Suppes, the structure of a developed scientific theory can be distinguished (when describing "from top to bottom") the following independent hierarchical levels:

- Fundamental theory
- Models of fundamental theory
- The theory of experiment (in the framework of which the set of possible implementations / applications of the theory is determined)
- Experimental Models
- Data models
- Organization of the experiment
- Conditions ceteris paribus (allowing to neglect the change in the values of the quantities unimportant for the given theory)

At the same time, the key role in verifying predictions made on the basis of the fundamental theory is not played by "direct observational data" (we almost never deal with them in scientific knowledge), but rather by data models.

Since the stated hierarchy of models "lies" between the fundamental theory postulates and the experimental data, the relations between them in principle cannot be correctly described using the correspondence rules.

Frederic Sapp, another critic of the formal program of logical positivism and the author of the "canonical" for the English-language philosophical literature reconstruction of the main stages of the development of this program, argues that only the standard admissible formalization and axiomatization of scientific theories was the language of logic of first-order predicates with equality. By virtue of the Löwenheim — Skolem theorem on "increasing" the power that is valid for this language, it is impossible to distinguish the desired (supposed) models of the theory from undesirable ones. As a result, the axioms and theorems of the theory formulated in the language of logic of first-order predicates with equality will inevitably turn out to be true on subject domains with "paradoxical" properties, to describe which the theory was not originally intended. Such undesirable models of the theory will be its potential counter examples.

That is why, according to F. Sapp, the reference method proposed in the framework of the "standard interpretation" of the theory formalization in the language of logic of first-order predicates with equality does not solve its main task - the task of an exhaustive description and unambiguous identification of relevant theories, and, therefore, it turns out to be, with all its technical sophistication, almost meaningless [5].

However, it does not follow from this that successful formalization and axiomatization (at least of some) of natural science theories are not possible in principle, and logical-mathematical methods are not suitable for analyzing the substantive problems of philosophy and the methodology of science.

The most natural alternative to the standard axiomatization of a scientific theory has become its axiomatization by building a special set-theoretic predicate, which directly characterizes all its desired (supposed) models.

This method of characterizing a scientific theory uses as its main concept the semantic concept of a theory model in the sense of A. Tarsky, allows (with certain reservations) to interpret theories as true or false in a “correspondent” sense and is devoid of most of the technical flaws usually identified by critics of the logical positivism program.

The idea of formalizing a theory by directly describing the class of its models is common to a number of areas in modern formal philosophy of science, which are characterized as model-theoretic, semantic, set-theoretic, structuralistic.

Program ideas for all these strategies of formalization of scientific knowledge are the ideas of Patrick Suppes, who proposed at the turn of the 50s – 60s of the 20th century the initial version of the indicated axiomatization of a number of scientific theories [6] [7] [8] [9].

The essence of the set-theoretic program proposed by P. Suppes for axiomatization of a scientific theory can be expressed in the following postulates.

The specific linguistic formulation of a scientific theory (its axiomatization as a sequence of sentences of a certain formalized language) is secondary in its description and identification. The most effective way to identify a theory is to describe its structure, which characterizes the class of real and potential models of the theory (in the sense of A. Tarsky). The structure of the theory is interpreted as a set-theoretic predicate of a special type.

A model of a theory in formal semantics is usually understood as some possible realization of a theory that fulfills its axioms. In turn, a possible implementation of the theory is a set-theoretic object of the corresponding logical type — ordered sequence of elements <D, R, F>, where D — some arbitrary nonempty set of objects, R — nonempty set of relations of different locality defined on D, F — the set (possibly empty) of objective functions (operations) defined on D. This construction is a model of a theory, if only all the sentences (axioms) of the theory are true when interpreted in terms of <D, R, F>.

Since the structure of a theory in the above sense is not a linguistic object, but a set-theoretic one, the very question of its truth / falsity is incorrect. We can talk only about the truth / falsity of the following fundamental empirical statement of the corresponding theory:

There is some correspondence between the elements of the subject area of the theory (observable phenomena) and the models of the theory, determined by its laws, which provide an adequate representation of reality in the models of the theory.

The degree of “strictness” of the mentioned correspondence relationship (isomorphism, partial isomorphism, homomorphism, weaker variants of the “similarity” relationship) can vary in different versions of the model-theoretic program.

In the natural and social sciences, a somewhat different understanding of the model as a certain way (symbolic or visual) of the area of reality under study is common. [10]

According to P. Suppes, the formal-semantic definition of the model concept covers all non-trivial variants of the “representative” use of this concept. Moreover, the difference between the models of these types is found only when the ontology of the domain of the model D is clearly fixed.

One can speak of the truth of a theory with its set-theoretic axiomatization, at least in two different senses:

- Truth can be understood purely formally as the feasibility of the theory’s sentences in some model in the sense of Tarski. Since this understanding of truth is not directly related to an extra-linguistic reality, it can be interpreted as an “inner”;
- The truth / falsity of a theory can be understood as the truth / falsity of its above-mentioned fundamental empirical statement declaring some structural correspondence between models of the theory and models of observed phenomena (data models). This concept of the truth of the theory can be viewed as a clarification of the classical (correspondent) concept of truth and in this sense is “external” (fixes the degree of conformity of the models of the theory of the observed region of reality).

The “external” concept of the truth of natural science theories involves the use of the mathematical theory of measurement and the proof of the axiomatization theory of the representation theorem (representation), which is fundamental for this method of axiomatization

Within the framework of measurement theory, the exact way is established for the transition from qualitative in nature observations to quantitatively accurate statements of empirical sciences. The way of such a transition is fixed with the help of axiomatization of the corresponding algebra of experimental procedures [11].

Based on the axiomatized theory of measurement of a certain empirical quantity, a representation theorem is proved for the empirical and numerical models of this theory, establishing the fact of structural conformity of empirical models of the theory to its numerical models, as well as the presence of common invariant characteristics of all models of the theory, allowing to describe the class of possible models of the theory in terms of a unified set-theoretic structure.

In general, the representation theorem means the following statement.

Let M be the set of all models of a certain theory, A is some distinguished (finite) subset of M. The proof of the representation theorem for M with respect to A consists in the proof of the statement that for an arbitrary model of a given theory from M there exists an isomorphic model belonging to A. In other words, any possible model of a given theory is represented by a certain model from a limited (finite) set A (the number of possible types of models of a theory is finite). If the set A turns out to be singleton, this means that any two models of the theory are isomorphic to each other. In this case, the theory is called categorical. (An example of a categorical theory is elementary number theory, which uses the standard set definition.)

Actually, the axiomatization of the theory consists in the construction of a set-theoretic predicate with parameters, through the specification of which all possible models of the corresponding theory can be described.

The process of axiomatization of a branch of natural science through the formulation of the set-theoretic predicate includes the following stages:

- Definition of the list of laws belonging to other theories, the truth of which is implied in the axiomatization of this theory (for example, in the axiomatization of solid mechanics, the basic laws of particle mechanics and classical mathematical analysis will belong to such a list)
- Enumeration of the initial concepts of the theory and specification of their set-theoretic specificity (in classical mechanics, these are the concepts “set of particles”, “time interval”, “particle coordinate”, “particle mass”, etc.)
- Inclusion of the obtained set-theoretic concepts into the composition of axioms that allow verification of some deductive consequences from the original definitions. At this stage, the set-theoretic reconstruction (reformulation) of special theorems of the empirical discipline under study is also carried out. And finally, at this stage, a representation theorem is formulated, describing the general structure of this empirical theory (characterizing the set of its models) and allowing us to answer the question about the mutual immersion of this theory and some related ones.
- Construction of a set-theoretic predicate axiomatizing theory.

For example, P. Suppes proposed the natural set-theoretic axiomatization of classical mechanics, carried out on the basis of six initial concepts: P— nonempty set of particles; T— the set of real numbers corresponding to the moments of time; s — coordinate function defined on Cartesian product P×T; g — force function, also defined on P×T; m — mass function; defined on the set P; f — force function defined on the Cartesian multiplication of elements P; T and sets of natural numbers. A possible implementation of classical mechanics as a theory in this case would be a set of six ordered elements (P, T, s, m, f, g) of the corresponding logical type, and a model of this theory would be a set of six ordered elements (P, T, s, m, f, g) fulfilling all axioms (basic laws) of classical mechanics.

III. Conclusion

The set-theoretic strategy for analyzing scientific knowledge turned out to be much more flexible than the standard axiomatization of theories in the language of the first-order predicate logic. This strategy is more consistent with the theory building procedures used in real scientific practice, and allows (with certain reservations) the interpretation of the theories as true or false in the "correspondent" sense [12].

That is why the set-theoretic strategy of analyzing scientific knowledge is “leading” in modern formal philosophy of science and is fruitfully used in analyzing the structure and dynamics of both natural science and social theories.

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