Cooperative Motion Planning Method for Two Anthropomorphic Manipulators

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Abstract—The development of methods for planning joint movements of two anthropomorphic manipulators in a common operational space becomes relevant. This is due to the development of anthropomorphic robots. Existing methods have relatively large computational complexity. They cannot be used on platforms with limited computing resources. This article proposes an approximate analytical method for planning of joint movements of two anthropomorphic manipulators for point-to-point movements. The analytical method is based on the geometric interpretation of the kinematic redundancy of anthropomorphic manipulators. The method includes three stages. At the first stage, the movement of the wrists of the manipulators is planned, at the second stage the elbow joints are planned, and at the third stage end effectors are planned. In the event of a possible collision of manipulators, correction of their path is carried out by “pushing”. Its computational complexity is about thousand operations. Simulation of the proposed method showed its efficiency. This method has prospects for implementation in existing manipulator control systems.

Keywords—anthropomorphic manipulator, robotic arm, cooperative motion planning, analytical method, geometric method, two manipulators.

1. INTRODUCTION

One of the most important areas of development of modern robotics is anthropomorphic robotics. Its key feature is the ability to operate the robot in a human-oriented environment without the need for its modification.

Existing anthropomorphic robots can operate in the mode of copying or automatic control [1, 2]. The main problems of automatic control are task planning and movement planning. Task scheduling determines which actions and in what sequence the robot must perform in order to achieve the goal. This task relates to the field of artificial intelligence and we do not consider it in this article. Another task is to plan movements, i.e. exactly how the robot should move in the process of performing actions. The purpose of planning movements of manipulators is to obtain the laws of motion that translate the manipulator from the initial position to the specified one. In this takes into account various limiting conditions. The resulting laws of motion are further developed by the robot's control system.

In the article we define the joint movements of two anthropomorphic manipulators as their simultaneous movement in a single operational space. This movement is aimed at achieving a common goal. Uncoordinated movements of manipulators working in a common operating space can lead to various kinds of collisions and damage to manipulators.

Joint movements of manipulators are of two types. The first type is the movement of the manipulators, together holding the object of manipulation. For the first type of movement, the trajectory is often known, along which the object of manipulation should move. There are a large number of methods designed for planning this type of movement [3-5].

Between the performance of target operations, i.e., manipulations with objects, manipulators perform transport movements, which are the second type of joint movements. The purpose of transport movements is to move the manipulators from some initial positions to the workers, from which the execution of target operations begins. In this case, the specific path of movement is not specified. This task is known as the point-to-point motion planning task [6].

To solve this problem, traffic planning methods can be used, taking into account obstacle avoidance [7–9]. For each manipulator, another manipulator can be thought of as unmanaged dynamic obstacles. Such methods require the use of developed technical means of visual observation of the environment. They have great computational complexity. These methods are designed to bypass uncontrolled objects. Since the other manipulator is controllable and predictable, the elimination of the collision problem of the manipulators can be carried out at the planning stage of the movement. The use of such methods creates an excessive feedback loop. This reduces the simplicity, cost and reliability of the system as a whole.
There are various methods for planning joint motion for planar robots [10-11]. These methods are more of academic interest, as they are difficult to scale for three-dimensional redundant manipulators.

There are various methods of planning joint movements for industrial manipulators [12, 7]. Despite their advantages, these methods are not focused on work with kinematically redundant anthropomorphic manipulators.

There are methods for planning joint movements of anthropomorphic manipulators [13]. These methods are based on numerical methods. This leads to their greater computational complexity.

This article offers an analytical method for planning joint movements of two anthropomorphic manipulators, which has a relatively low computational complexity. In Section 2, the formulation of the problem is carried out, in Section 3, the proposed method is described, and Section 4 presents the simulation results.

II. FORMULATION OF THE PROBLEM

The method is designed with the following assumptions and limitations. Movement planning is performed for two anthropomorphic manipulators with a kinematic scheme similar to the kinematic scheme of a human hand (Fig. 1).

For example, we can set the parameters of the kinematic scheme of both manipulators in the form of Denavit-Hartenberg parameters [14]. We assumed that the manipulators are at a sufficiently close distance so that their operating spaces intersect. Otherwise, joint implementation of target operations is impossible, and there is no need to plan joint movements.

For each manipulator, the initial and final positions specified in the space of generalized coordinates must be known. We denote the initial position as \( \mathbf{q}_{i,i} \), and the final position as \( \mathbf{q}_{f,i} \), where \( i = \{1; 2\} \) is the identifier of the manipulator.

The purpose of joint motion planning is to obtain the trajectory of the manipulators in the space of generalized coordinates. This trajectory translates the manipulators from the initial position to the final one under the condition that there are no collisions between the manipulators.

III. MATERIALS AND METHODS

The proposed method is based on the following features:

1) Specific kinematic redundancy of anthropomorphic manipulators.

2) Planning of movements is performed not from the gripper of the manipulator, but from its wrist joint.

3) All links and joints of the manipulator are represented as spheres and cylinders with a certain radius.

If we consider the Cartesian coordinates of the joints of the manipulator, the given position and orientation of the hand will correspond to an infinite number of possible positions of the elbow joint. This fact testifies to kinematic redundancy. The set of possible positions of the elbow joint lies on the circle formed by rotating the elbow joint \( \mathbf{P}_e \) around an axis passing through the humeral \( \mathbf{P}_h \) and radiocarpal \( \mathbf{P}_w \) articulation (Fig. 2). Based on this feature, some methods of solving the inverse problem of kinematics, methods of planning the movement of one anthropomorphic manipulator [15-16] and the proposed method of planning joint movements of two anthropomorphic manipulators are built.

In many motion planning methods for industrial manipulators, motion planning is performed from the working end of the manipulator. The second feature of the proposed method is the planning of movements not from the working end of the manipulator, but from his wrist joint. This allows us to simplify the analytical representation and reduce the computational complexity of the formation of the laws of motion of the joints of manipulators.

The third feature of the proposed method is the representation of links and joints of the manipulator in the form of cylinders and spheres of a certain radius \( r \). \( r \) is a safe distance between the parts of different manipulators and is selected on the basis of the dimensions of a particular manipulator.

The proposed method consists of the following steps:

- Planning joint movements of the wrist joints.
- Planning joint movements of the elbow joints.
- Planning joint movements of the wrist links.

This article does not cover motion planning an anthropomorphic gripe [17,18]. The following describes the steps of the proposed method.

B. Joint movements planning of the wrist joints

The general methodology for planning the movement of wrist joints is similar to the method of planning the movement of an anthropomorphic manipulator. In this takes into account the circumvention of the obstacle proposed in [19].
Let us designate for the i-th manipulator the radius-vector of the wrist joint in the initial position as \( p_{wi,i} \), and in the final position as \( p_{wi,e} \).

The generalized coordinates of the manipulators in the initial and final positions, \( q_{ki,i} \) and \( q_{ei,i} \) uniquely determine the position of the wrist joints \( p_{wi,i} \) and \( p_{wi,e} \) that can be found by solving the direct problem of kinematics. Denote the points at which the wrist joints in the initial and final positions of the manipulators as terminal.

In this method, it is proposed to perform the movement of the wrist from the initial position to the final one along the path, which is a straight line in Cartesian coordinates.

We introduce the conditional time \( t \in [0; 1] \) to describe the process of motion of the manipulators. The moment \( t = 0 \) corresponds to the beginning of the movement, and the moment \( t = 1 \) corresponds to the end of the movement of the manipulator.

Since the movement of the wrist joints is carried out in a straight line, we can describe their movement in a parametric form:

\[
p_{wi}(t) = p_{wi,i} + q_{wi}(t) \ast t, \quad (1)
\]

where \( q_{wi}(t) \) is the velocity profile of the wrist joint represented by some polynomial.

In this case, we can express the distance between the wrist joints as follows:

\[
r_w(t) = \left\| p_{wi,2}(t) - p_{wi,1}(t) \right\| =
\]

\[
= \left\| p_{wi,2,s} + q_{wi,2}(t) \ast t - p_{wi,1,s} - q_{wi,1}(t) \ast t \right\| =
\]

\[
= \left\| \Delta p_w + \Delta q_w(t) \ast t \right\|. \quad (2)
\]

where \( \Delta p_w = p_{wi,1,s} - p_{wi,2,s} \) is the initial difference in the position of the wrist joints; \( \Delta q_w(t) = q_{wi,2}(t) - q_{wi,1}(t) \) - the relative speed of movement of the wrist joints.

Since the distance is a non-negative value, for convenience we will consider the square of the distance between the wrists:

\[
r_w^2(t) = \Delta p_w^2 - 2\Delta p_w \Delta q_w(t) \ast t + \Delta q_w^2(t) \ast t^2. \quad (3)
\]

Given that \( \Delta p_w(t) \) constitutes a polynomial of some degree, \( \Delta q_w(t) \) is also a polynomial, which allows us to find the points of its extremum in an analytical form.

In this case, from the condition that the derivative is zero we can find one or several minima of the distance between the wrists.

\[
\frac{dr_w^2(t)}{dt} = 0. \quad (4)
\]

Thus, we can find and select the minimum points of the square of the distance belonging to the segment \( t \in [0; 1] \).

Let a certain threshold safe distance \( r_t \) between the wrists of both manipulators be given. As such a distance, we can use the doubled radius of spheres and cylinders, approximating the links and articulation of manipulators according to the assumption made earlier:

\[
r_t = 2r. \quad (5)
\]

Thus, the problem of collision between the manipulators is formulated as follows: it is necessary that at any time the distance between the wrists is not less than the threshold:

\[
\| r_w^2(t) \| \geq r_t, t \in [0; 1]. \quad (6)
\]

Let’s call the problem points where the distance between the wrist joints is minimal, and where the condition is not met

\[
r_w^2(t) \geq r_t^2. \quad (7)
\]

In the absence of problem points, we can consider the planning of the movement of the wrist joints as complete and proceed to the planning stage of elbow joints movements.

To eliminate problem points it is proposed to use the following recursive algorithm:

1) From all the problem points are the points with the smallest distance between the wrist joints.

2) At this point, the "wringing" of the wrist joints (Fig. 3) is performed to prevent a collision. We denote the points to which the problem points after the "repulsion" moved as intermediate.

3) The analysis given above for the possibility of movement of the wrist joints without a collision and "pushing" is performed recursively for two segments connecting the terminal and intermediate points.

We denote the moment in time in which the distance between the wrists is the shortest, as \( t_c \). Problem points for which you need to perform "pushing" will be \( p_{wi,1}(t_c) \) and \( p_{wi,2}(t_c) \).

Denote the center of the segment, connecting \( p_{wi,1}(t_c) \) and \( p_{wi,2}(t_c) \), as the center of "pushing" of the wrist joints:

\[
p_{wpc} = \frac{p_{wi,1}(t_c) + p_{wi,2}(t_c)}{2}. \quad (8)
\]
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“Pushing away” of problem points is performed from the center of “pushing away” of wrist joints for the value of the threshold distance:

\[
P_{w,i,m} = \frac{p_{w,i}(t_c) - p_{wpc}}{\|p_{w,i}(t_c) - p_{wpc}\|} \times \frac{r_i}{2}
\]

(9)

where \( p_{w,i,m} \) is the intermediate point of the \( i \) manipulator.

If the modulus of the vector \( p_{w,i}(t_c) - p_{wpc} \) is close to zero, we can find the points \( p_{w,i,m} \) by the formulas:

\[
\mathbf{n} = q_{w,1}(t_c) \times q_{w,2}(t_c)
\]

(10)

Fig. 3. "Pushing" problem points

\[
P_{w,1,m} = p_{wpc} + \frac{\mathbf{n}}{||\mathbf{n}||} \times \frac{r_1}{2}
\]

(11)

\[
P_{w,2,m} = p_{wpc} - \frac{\mathbf{n}}{||\mathbf{n}||} \times \frac{r_2}{2}
\]

(12)

C. Planning for elbow joints movements

The strategy for planning joint movements of the elbow joints is based on the kinematic redundancy of anthropomorphic manipulators. The set of possible positions of the elbow joint for a given position of the wrist joint is a circle. Therefore, it is proposed that the elbow joints of the anthropomorphic manipulators are positioned so that the distance between them is as large as possible when the manipulators move (Fig. 4). This prevents the elbow joint from colliding with the shoulder or forearm of another manipulator.

To find the most distant points of the circles shown in Fig. 4, it is necessary to put vectors along the axis drawn through the centers of the circles from the centers of the circles. The lengths of the vectors must be equal to the radii of the corresponding circles.

We can find the center of the circle \( p_{c,i} \) of the possible positions of the elbow joint of the \( i \) manipulator for the position \( p_{s,i} \) of the shoulder joint and the position \( p_{w,1,i} \) of the wrist joint (Fig. 2) by the formula:

\[
p_{c,i} = p_{s,i} + \frac{p_{w,1,i} - p_{s,i}}{||p_{w,1,i} - p_{s,i}||} \times s_i \times \cos(\alpha_i)
\]

(13)

where \( s_i \) is the length of the humeral link, \( \alpha_i \) is the angle between the humeral link and the axis passing through the humeral and wrist joints of the \( i \) manipulator.

The length \( s_i \) of the shoulder link is known from the Denavit-Hartenberg parameters, and we can find \( \cos(\alpha_i) \) by using the cosine theorem for the triangle formed by the shoulder, elbow and wrist joint:

\[
\cos(\alpha_i) = \frac{d_i^2 + s_i^2 - f_i^2}{2d_if_i}
\]

(14)

where \( d_i \) is the length of \( d_i = ||p_{w,i} - p_{s,i}|| \), \( f_i \) is the length of the manipulator forearm.

After transformations we get:

\[
p_{c,i} = p_{s,i} + (p_{w,i} - p_{s,i}) \frac{d_i^2 + s_i^2 - f_i^2}{2d_if_i}
\]

(15)

The vector of the axis passing through the humeral and wrist joints is given by the expression:

\[
\mathbf{o} = p_{c,2} - p_{c,1}
\]

(16)

We can find the most remote possible positions of the elbow joint by the formulas:

\[
p_{e,1} = p_{c,1} - R_i \left( \mathbf{o} - \frac{d_1}{d_i} \frac{d_2}{d_i} \mathbf{o}_i \right)
\]

(17)

\[
p_{e,2} = p_{c,2} + R_2 \left( \mathbf{o} - \frac{d_1}{d_i} \frac{d_2}{d_i} \mathbf{o}_i \right)
\]

(18)

where \( p_{c,i} \) is the location of the elbow joint, \( R_i \) is the radius of the circumference of the \( i \) manipulator, calculated by the formula:

\[
R_i = s_i \left( \sqrt{1 - \cos^2(\alpha_i)} \right)
\]

(19)

In the initial positions of the anthropomorphic manipulators, the position of the elbow joints may not correspond to the proposed strategy. In this case, you must first transfer the manipulators to a position consistent with the chosen strategy.

D. Planning of joint movements of the wrist links

When planning joint movements of the wrist links, it is necessary that the wrist link of each anthropomorphic manipulator does not collide with the rest of the links of the same manipulator and another one. For solving this problem it is proposed to use the following strategy.
We take for the direction of the wrist link vector connecting the wrist joint and the working end. Let the shoulder and forearm of the first manipulator lie in the same plane $\beta_1$. Then, from all possible directions of the wrist link $\mathbf{h}_2$ of the second manipulator, the one at which the wrist link will be directed from the plane $\beta_1$ at an angle as close as possible to the straight line is selected. This will ensure the absence of the wrist link of the second manipulator with the links of the first one.

We impose restrictions on the movement of the wrist link, corresponding to the mobility of the human hand. This eliminates the collision of the carpal link of the second manipulator with the remaining links of the second manipulator. The angle between the wrist link and the forearm should not be less than 90°.

In this case, we can find the direction of the wrist link of the second manipulator as a projection of the normal vector to the plane $\beta_1$ on a plane $\gamma_2$ perpendicular to the forearm of the second manipulator (Fig.5). The normal vector should be directed from the plane $\beta_1$ to the wrist joint.

We can find the plane $\beta_1$ at known points of the shoulder, elbow and wrist joints. The formula of this plane in general:

$$ A_\beta x + B_\beta y + C_\beta z + D_\beta = 0. $$

Two normal vectors can be drawn to the plane $\beta_1$:

$$ \mathbf{n}_{\beta_1} = \left[A_\beta B_\beta C_\beta\right]^T, $$

$$ \mathbf{n}_{\beta_2} = \left[-A_\beta - B_\beta - C_\beta\right]^T. $$

The choice of one of them can be made on the basis of the distance from the wrist joint $\mathbf{p}_{w,1}$ to the plane $\beta_1$:

$$ d = \frac{\mathbf{n}_{\beta_1 \cdot \mathbf{p}_{w,1}} + D_\beta}{\sqrt{A_\beta^2 + B_\beta^2 + C_\beta^2}}. $$

If $d > 0$, then it should be used as a normal $\mathbf{n}_\beta$, otherwise $\mathbf{n}_{\beta_2}$.

Normal to the plane of the forearm of the second manipulator corresponds to the vector directed from the elbow joint to the radiocarpal:

$$ \mathbf{n}_\gamma = \mathbf{p}_{w,2} - \mathbf{p}_{e,2}. $$

We can find the direction of the wrist link by the formula (Fig. 5):

$$ \mathbf{n}_h = \mathbf{n}_\beta \times \mathbf{n}_\gamma \times \mathbf{n}_\gamma. $$

According to the received direction vector of the wrist link, the position of the working end of the manipulator can be found:

$$ \mathbf{p}_{h,2} = \mathbf{p}_{w,2} + h\mathbf{n}_h, $$

where $h$ is the length of the wrist link.

The laws of motion of anthropomorphic manipulators in generalized coordinates can be obtained by solving the inverse problem of kinematics using one of the existing methods[20].

**IV. RESULTS**

To test the effectiveness of the proposed method, a simulation was performed in the Matlab package. The simulation showed the efficiency of this method and its low computational complexity. The following is an example of simulation results. Table 1 shows the initial data for the calculations. Table 2 shows the results of motion planning. The visualization of the results of motion planning is shown in Figure 6.

**TABLE I. INITIAL DATA**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{w,1}$</td>
<td>(0.00, 0.00, 0.00)</td>
</tr>
<tr>
<td>$p_{e,1}$</td>
<td>(-66.06, 44.77, -5.60)</td>
</tr>
<tr>
<td>$p_{w,2}$</td>
<td>(0.00, 80.00, -10.00)</td>
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<tr>
<td>$p_{e,2}$</td>
<td>(0.00, 82.48, 9.85)</td>
</tr>
<tr>
<td>$p_{h,2}$</td>
<td>(100.00, 101.99, -9.90)</td>
</tr>
<tr>
<td>$s_1$</td>
<td>80.00</td>
</tr>
<tr>
<td>$f_1$</td>
<td>75.00</td>
</tr>
<tr>
<td>$h_1$</td>
<td>20.00</td>
</tr>
<tr>
<td>$r_1$</td>
<td>10.00</td>
</tr>
</tbody>
</table>
Figure 6 shows the initial (P_s;i,m, P_e;i,e, P_w;i,w, P_h;i,h) mediates (P_s;i,m = P_s;i,s, P_e;i,e, P_w;i,w, P_h;i,h) and ending (P_s;i,e = P_s;i,e, P_e;i,e, P_w;i,e, P_h;i,e) positions of the manipulators. The links of the first manipulator are marked in green and links of the second manipulator in blue. With the independent movement of the manipulators from the initial position to the final position, they would collide in the middle of the path. The application of the developed method made it possible to adjust their movement so that a collision does not occur.

As follows from the simulation results, the proposed method allows planning the joint movements of two anthropomorphic manipulators.

The computational complexity of the proposed method has about thousand of operations. It is relatively low compared to the computational complexity of analogues. Therefore, the goal of developing a method for planning joint movements of two anthropomorphic manipulators can be considered achieved.

For use in existing control systems of anthropomorphic robots, more in-depth testing and refinement of the proposed method is required. For example, it is necessary to add the ability to bypass obstacles, the object of manipulation and other parts of an anthropomorphic robot.

At the same time, this method allows to obtain repeatability of results, in contrast to numerical methods that are poorly designed to work with kinematically redundant anthropomorphic manipulators.

V. CONCLUSION

The article developed an analytical method for planning joint movements of two anthropomorphic manipulators. The performance of the proposed method has been tested using simulation. Further statistical studies are needed to verify the reliability of the method. The developed method is analytical, which makes it relatively low computational complexity about thousand of operations. Low computational complexity determines the prospects of using the method in control systems of robotic platforms with limited computing resources, for example, in mobile autonomous robots.

ACKNOWLEDGMENT

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REFERENCES


TABLE II. RESULTS OF MOTION PLANNING

<table>
<thead>
<tr>
<th>Variable</th>
<th>First manipulator</th>
<th>Second manipulator</th>
</tr>
</thead>
<tbody>
<tr>
<td>P_s;i,m</td>
<td>(−24.02, 75.76, 9.11)</td>
<td>(122.87, 76.27, −7.73)</td>
</tr>
<tr>
<td>P_e;i,e</td>
<td>(50.00, 86.46, 3.54)</td>
<td>(50.00, 93.54, −3.54)</td>
</tr>
<tr>
<td>P_w;i,w</td>
<td>(51.28, 87.99, 23.44)</td>
<td>(49.23, 95.13, −23.46)</td>
</tr>
<tr>
<td>P_h;i,h</td>
<td>(74.12, 29.96, 0.14)</td>
<td>(25.88, 29.96, 6.11)</td>
</tr>
</tbody>
</table>